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Form factors for $B\to K\, l^+l^-$ decay

Abstract. The exclusive $B\to V(P)l^+l^-$ and $B\to V\gamma$ decays, with a vector $(V=K^*$, ρ, ...) or pseudo scalar $(P=K,\ \pi,\ ...)$ meson in the final state are important for the search for flavour-changing new physics.
In this paper we study one of the exclusive rare decays of pseudo scalar B mesons: $B\to Kl^+l^-$ by using a relativistic constituent quark model, including infrared confinement. We calculate the relevant form factors of the $(B\to P)$ transitions in the full kinematical range of momentum transfer. The calculated form factors are used to evaluate differential decay rates and polarization observables.
We present numerical results on these observables using the covariant quark model. This model can be viewed as an effective quantum field theory approach based on an interaction Lagrangian of hadrons interacting with their constituent quarks. Universal and reliable predictions for exclusive processes involving both mesons composed from a quark and antiquark and baryons composed from three quarks result from this approach.
Key words: exclusive rare decays, bottom and charm mesons, kaons, leptons.

Introduction

In the last years the flavor-changing neutral current transition $B\to K+X$ with $X=l^+,\ l^-,\ \bar{\nu}\nu$ are of special interest because they proceed at the loop level in the Standard Model (SM) involving also the top quark. The study of heavy-flavor physics may therefore be used for a determination of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements V_{tg} $(q = d, s, b)$. One of the main purposes of heavy-flavor experiments is to look for new physics beyond the standard model. The subject of this paper is to study heavy hadrons consisting of b and c quarks and their weak decays.

The rare semileptonic decay $B\to Kl^+l^-$ $(l = e,\ \mu)$ has been observed by the BELLE Collaboration [1] with an available experimental measurement of the branching ratio of

$$Br\ (B\to K+l^+l^-) = (0.75^{+0.25}_{-0.21}\pm 0.09)\times 10^{-6}\,. \tag{1}$$

In this paper we study the exclusive rare decay $B\to Kl^+l^-$. We employ a relativistic quark model [2, 3] to calculate the decay form factors. This model is based on an effective Lagrangian which describes the coupling of a meson $H(q_1,\ q_2)$ to its constituent quarks. The coupling strength is determined by the compositeness condition $Z_H = 0$ [4, 5] where Z_H is the wave function renormalization constant of the hadron field.

The calculation of the hadronic matrix elements of local operators between initial and final states require nonperturbative methods. One needs to know how hadrons are constructed from quarks. Technically, any matrix element of a local operator may be expressed in terms of a set of scalar functions which are referred to as form factors.

A variety of theoretical approaches have been used to evaluate the hadronic form factors. The least model dependent among these is the light-cone sum rule (LCSR) approach (see Refs. [6, 7]). In the LCSR approach, one can access the form factors in the large recoil (small momentum transfer) region which are then extrapolated to the near-zero recoil region using some model-dependent pole-type parameterizations. Grinstein and Pirjol have developed a systematic approach to the rare decay $B \to K l^+ l^-$ in the low-recoil region using the heavy-quark effective theory framework [7]. One of a few model approaches for the calculation of the form factors is the covariant constituent quark model. In the covariant quark model, meson transitions are described by covariant Feynman diagrams.

In section 2 we briefly discuss the theoretical framework underlying the covariant quark model and in section 3 we calculate the transition form factors of the B meson to pseudoscalar meson.

Constituent quark model

A starting point of this model is an effective Lagrangian written down in terms of quark and hadron fields. Then, by using Feynman rules, the S-matrix elements describing the hadronic interactions are given in terms of a set of quark diagrams. In particular, the compositeness condition enables one to avoid a double counting of hadronic degrees of freedom. The approach is self-consistent and universally applicable. All calculations of physical observables are straight-forward. The model has only a small set of adjustable parameters given by the values of the constituent quark masses and the scale parameters that define the size of the distribution of the constituent quarks inside a given hadron. The shape of the vertex functions and the quark propagators can in principle be found from an analysis of the Bethe- Salpeter and Dyson-Schwinger equations as was done e.g. in [8]. In this paper, however, we choose a phenomenological approach where the vertex functions are modelled by a Gaussian form, the size parameter of which is determined by a fit to the leptonic and radiative decays of the lowest lying charm and bottom mesons. For the quark propagators we use the local representation. In the present calculations we do not employ the so-called impulse approximation used previously [3].

We will employ the relativistic constituent quark model [2,3] to calculate the form factors relevant to the decay $B \to K l^+ l^-$. This model is based on an effective interaction Lagrangian which describes the coupling between hadrons and their constituent quarks.

The coupling of the meson H to its constituent quarks q_1 and q_2 is given by the effective Lagrangian

$$L_{\mathrm{int}}(x) = g_H H(x) \cdot \int dx_1 \int dx_2 F_M(x; x_1, x_2) \bar{q}_2(x_2) \Gamma_M q_1(x_1). \tag{2}$$

Here, Γ_M are Dirac matrices which entail the spin quantum numbers of the meson field $H(x)$. In the present case the Dirac structures involved are γ_5 for the pseudo scalar mesons. The function F_M is related to the scalar part of the Bethe-Salpeter amplitude and characterizes the finite size of the meson. This function must be invariant under the translation

$$F_M(x + a,\ x_1 + a,\ x_2 + a) = F_M(x, x_1, x_2).$$

In previous papers [2, 3] it was used the so-called impulse approximation for the evaluation of the Feynman diagrams.

In the impulse approximation one omits a possible dependence of the vertex functions on external momenta. The impulse approximation therefore entails a certain dependence on how loop momenta are routed through the diagram at hand. This problem no longer exists in the present full treatment where the impulse approximation is no longer used. In the present calculation we will use a specific form of the vertex function given by

$$F_M(x; x_1, x_2) = \delta(x - x_1\omega_1 - x_2\omega_2)\Phi_M\left((x_1 - x_2)^2\right), \tag{3}$$

where $\omega = m_i / (m_1 + m_2)$ $(i=1,2)$ and m_1, m_2 are the constituent quark masses. The vertex function F_M evidently satisfies the above translational invariant-ce condition. As mentioned before we no longer use the impulse approximation in the present calculation.

The coupling constants g_H in *Eq*. (2) are determi-ned by the so called compositeness condition proposed in [2] and extensively used in [3]. The compositeness condition means that the renormalization constant of the elementary $H(x)$ meson field is set equal to zero.

$$Z_H = 1 - \frac{3g_H^2}{4\pi^2} \tilde{\Pi}'_H(m_H^2) = 0 \ , \tag{4}$$

where $\tilde{\Pi}'_H(m_H^2)$ is the derivative of the meson mass operator. The physical meaning of the compositeness condition in Eq. (2), we first remind the reader that the renormalization constant $Z_H^{1/2}$ is also interpreted as the matrix element between the physical and the corresponding bare state. The condition $Z_H = 0$ implies that the physical state does not contain the bare state and is appropriately described as a bound state. The interaction Lagrangian of Eq. (2) and the corresponding free parts of the Lagrangian describe both the constituents (quarks) and the physical particles (hadrons) which are viewed as the bound states of the quarks. As a result of the interaction, the physical particle is dressed, i.e. its mass and wave function have to be renormalized.

For the pseudoscalar and vector mesons the derivative of the meson mass operator appearing in Eq. (4) can be calculated from the one-loop two-point function given by

$$\begin{aligned} \tilde{\Pi}'_P(p^2) &= \frac{1}{2p^2} p^\alpha \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_P^2(-k^2)\ tr[\gamma^5 S_1(k+w_1p)\gamma^5 S_2(k-w_1p)] \\ &= \frac{1}{2p^2} \int \frac{d^4k}{4\pi^2 i} \tilde{\Phi}_P^2(-k^2)\{w_1\, tr[\gamma^5 S_1(k+w_1p)\, \not{p}\, S_1(k+w_1p)\gamma^5 S_2(k-w_1p)] \\ &\quad - w_2\, tr[\gamma^5 S_1(k+w_1p)\gamma^5 S_2(k-w_1p)\, \not{p}\, S_2(k-w_1p)]. \end{aligned} \tag{5}$$

$\tilde{\Phi}_M(-k^2)$ is the Fourier-transform of the correlation function $\Phi_M\left((x_1-x_2)^2\right)$ and $S_i(\not{k})$ is the quark propagator. It is given by

$$S_i(\not{k}) = \frac{1}{m_i - \not{k}} \cdot \tag{6}$$

We will choose a simple Gaussian form of the vertex function

$$\overline{\Phi}_M(-k^2) = \exp(k^2/\Lambda_M^2)\,, \tag{7}$$

where the parameter Λ_H characterizes the size of the respective bound state meson H. Since k^2 turns into $-k^2$ in Euclidean space the function (7) has the appropriate fall-off behavior in the Euclidean region.

We write the local quark propagators in the following way according to [9]

$$S(k) = (m+\not{k})\int_0^\infty d\beta\ e^{-\beta(m^2-k^2)}\ . \tag{8}$$

The loop momentum now appears in the exponent which allows one to deal very efficiently with tensor loop integrals by converting loop momenta into derivatives via the identity

$$k_i^\mu\ e^{2kr} = \frac{1}{2}\frac{\partial}{\partial r_{i\mu}} e^{2kr}. \tag{9}$$

After doing the loop integration and turning parameters β_i into a simplex by an additional t integration via the identity

$$q1 = \int_0^\infty dt\, \delta(t - \sum_{i=1}^n \beta_i)\,, \tag{10}$$

we obtain

$$\Pi = \int_0^\infty dt\, t^{n-1} \int_0^1 d^n\alpha\, \delta(t - \sum_{i=1}^n \alpha_i) F(t\alpha_1,...,t\alpha_n)\,. \tag{11}$$

Here we introduce an infrared cut off on the upper limit of the t integration to remove all singularities. Finally

$$\Pi^c = \int_0^{1/\lambda^2} dt\, t^{n-1} \int_0^1 d^n\alpha\, \delta(t - \sum_{i=1}^n \alpha_i) F(t\alpha_1,...,t\alpha_n)\,. \tag{12}$$

The numerical evaluation of the integrals have been done by a numerical program written in FORTRAN code.

Model form factors

Let us first define the transition form factors of B meson to pseudoscalar meson in the framework of covariant quark model.

The leptonic decay constant f_p is calculated from

$$\frac{3g_P}{4\pi^2}\int\frac{d^4k}{4\pi^2 i}\tilde{\Phi}_P(-k^2)tr[O^\mu S_1(\not{k}+w_{21}\not{p})\gamma^5 S_2(\not{k}-w_{12}\not{p})]=f_P p^\mu. \tag{13}$$

The transition form factors ($P(p_1)\to P(p_2)$ can be calculated from the Feynman integral corresponding to the diagram of figure 1:

$$\Lambda^{\Gamma^\mu}(p_1,p_2)=\frac{3g_P g_{P'}}{4\pi^2}\int\frac{d^4k}{4\pi^2 i}\tilde{\Phi}_P(-(k+w_{13}p_1)^2)\tilde{\Phi}_{P'}(-(k+w_{23}p_2)^2)$$
$$\times tr[S_2(\not{k}+p_2)\Gamma^\mu S_1(\not{k}+p_1)\gamma^5 S_3(\not{k})\Gamma_{out}], \tag{14}$$

where $\Gamma^\mu=\gamma^\mu,\ \gamma^\mu\gamma^5,\ i\sigma^{\mu\nu}q_\nu,$ or $i\sigma^{\mu\nu}q_\nu\gamma^5$ and $\Gamma_{P,V}=\gamma^5,\ \gamma_\nu\in_2^\nu$.

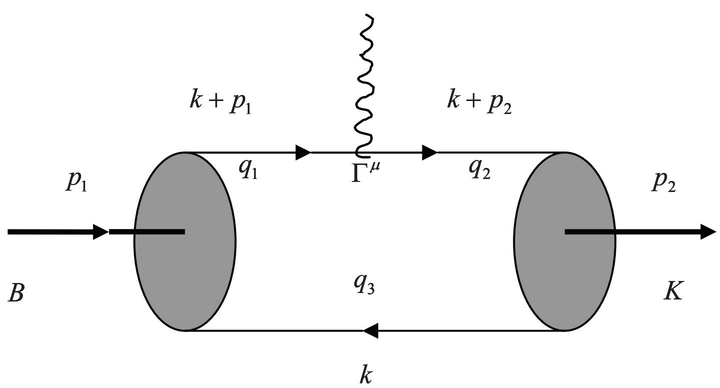

Figure 1 – Diagram describing the form factors of the decay $B\to Kl^+l^-$.

The fit values for the constituent quark masses are taken [2, 3]

m_u	m_s	m_c	m_b	
0.235	0.333	1.67	5.06	GeV

The numerical values for Λ_H are $\Lambda_B=2.25\ GeV$ and $\Lambda_K=1.6\ GeV$ for all K and B partners, respectively.

The calculated values of the form factors are given in figure 2.

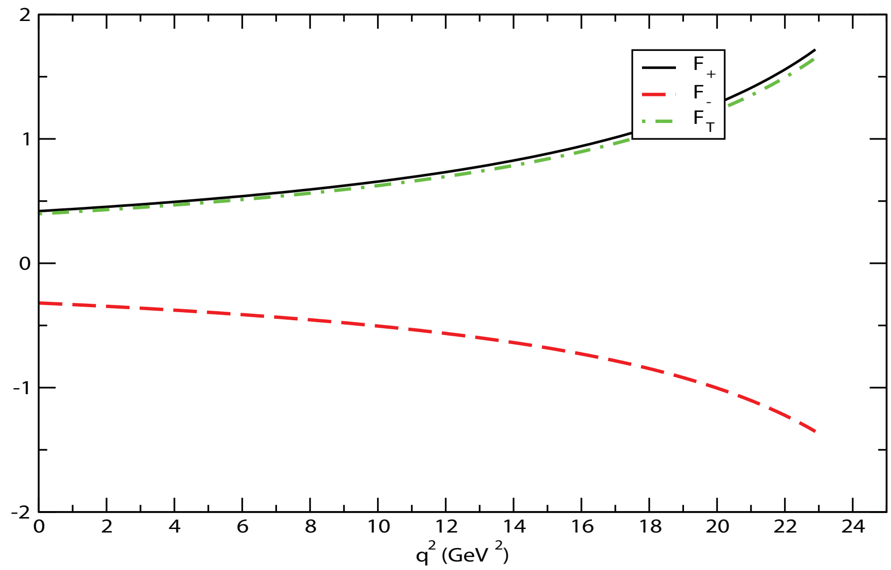

Figure 2 – Results for the form factors

Conclusion

We have given a brief discussion of the theoretical framework underlying the covariant quark model with infrared confinement is incorporated in the model. The model parameters of the covariant quark model are fixed. We have calculated the transition form factors of the heavy-B meson to pseudoscalar meson K, which are needed as ingredients for the calculation of the semileptonic rare decays of the B meson. Our form factor results hold in the full kinematical range of momentum transfer.

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