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Effective potential and pressure of two-component plasma at high temperature

Abstract. An effective pair interaction potential of particles for ideal and weakly non-ideal plasma were obtained. It takes into account not only screening and dynamic quantum effects, but also statistical quantum effects. Comparison between other kinds of potentials was conducted. The results for this potential show that our potential tends to a finite value at small distances and lies below Deutsch and Kelbg potentials. At large distances it tends to zero as Debye-Huckel potential. We used this potential for the calculation of plasma pressure and compared our results with simulation data. The calculated pressure values are in a good agreement with simulation one at high temperatures.
Key words: pseudo potential, dielectric response formalism, ideal plasma, shielding.

Introduction

It is known, that the interaction of charged particles in plasmas is different from the Coulomb interaction. If we use Coulomb potential, we will have problems with the divergence of collision integrals at scattering angles, because this potential does not take into account the collective effects (screening and quantum effects, for example). In order to overcome these problems, some kinds of potentials (effective potentials), which take into account the collective effects, were introduced to the plasma physics. Let us consider the main ones. Potential of the self-consistent field (Debye - Huckel potential corresponds to the approximation of pair correlations and it is applicable for small plasma densities. Deutsch potential takes into account quantum mechanical effects of diffraction and symmetry. Special effective potential, based on the theory of perturbations, was proposed in the paper [1], it is so called Kelbg potential. Deutsch and Kelbg potentials take into account only quantum effects. Authors of [1] proposed a potential which takes into account both screening and quantum mechanical effects.

Effective potential

The purpose of this article is to show how an effective potential of charged particles interaction can be used for calculating of plasma pressure. This effective potential can be obtained in the framework of the linear dielectric response theory. The main idea is as follows. We chose some microscopic potential, in which we want to add a new property and determine its Fourier transform. Next, the Fourier-image of new potential is determined via the ratio of Fourier-images of microscopic potential and model of dielectric function. Finally, new effective potential with necessary properties is obtained by reverse Fourier transformation. Let's see this scheme in practice:

The equation of this potential in Fourier space then:

$$\varphi_{ab}\left(q\right) = \phi_{ac}\left(q\right)\varepsilon_{cb}^{-1}\left(q,0\right), \qquad (1)$$

where $\varepsilon_{cb}^{-1}(q,0)$ is the static dielectric function (SDF) of the plasma, $\phi_{ac}(q)$ is the Fourier transform of a micropotential and $\varphi_{ab}(q)$ - Fourier transform of the desired potential looked for.

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Further, it is necessary to select a micropotential which possesses a part of the required properties, as well as a model for SDF, which has other required properties.

As one of this micro potentials, Deutsch potential were chosen, which takes into account dynamic quantum-mechanical effects (without symmetric part). Its Fourier transform:

$$\phi\left(q\right)=\left(\frac{4\pi\alpha}{q^2}-\frac{4\pi\alpha}{k_{ab}^2+q^2}\right). \quad (2)$$

A model for the SDF was taken from [2]:

$$\varepsilon\left(q,0\right)=1+\frac{1}{\left(q/k_D\right)^2+\left(q/k_q\right)^4}. \quad (3)$$

Here $k_D^2=4e^2\beta n_{tot,}n_{tot}=n_e+\sum_i n_i Z_i^2$, $k_q^4=16\,\pi e^2/h^2(n_e m_e+\sum_i n_i m_i Z_i^2)$.

Now, using these formulas, it is necessary to obtain the equation for the potential in ordinary space by inverse transform of equation (2) using the formula:

$$\varphi_{ab}\left(r\right)=\frac{1}{\left(2\pi\right)^3}\int \boldsymbol{dk}\varphi_{ab}\left(q\right)\exp(i\boldsymbol{qr}). \quad (4)$$

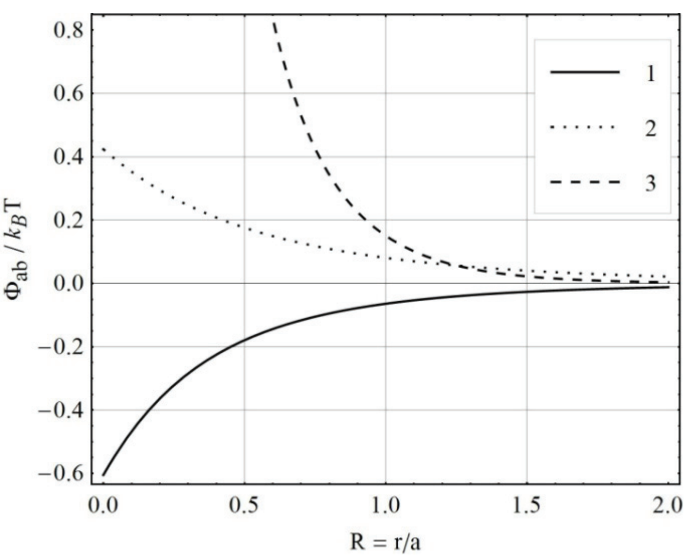
We have obtained the following expression for the potential:

$$\varphi_{ab}\left(r\right)=\frac{e_a e_b}{r}\left(A\ exp\left(-r\ k_{ab}\right)+B_1\ exp\left(-r\ K_1\right)-B_2\ exp\left(-r\ K_2\right)\right), \quad (5)$$

$$A=\frac{1-\beta}{\Delta}\ ,\ B_{1,2}=\frac{C_{1,2}(C_{1,2}-\beta)}{(2C_2-1)\Delta}\ ,\ K_{1,2}=k_{ab}\sqrt{\frac{C_{2,1}}{\beta}},$$

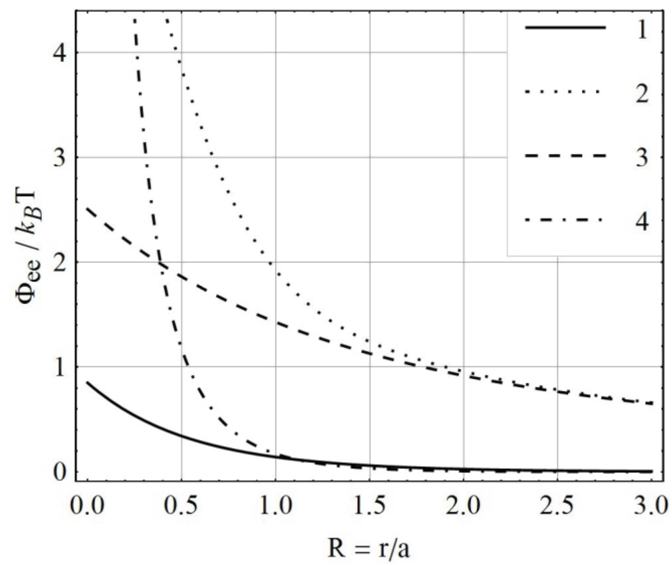
$$\Delta=\gamma+\beta-1,\ C_{1,2}=\frac{1\mp Rad}{2},\ Rad=\sqrt{1-4\,\alpha}$$

$$\alpha=\frac{k_D^4}{k_q^4},\quad \beta=\frac{k_{ab}^2 k_D^2}{k_q^4},\ \gamma=\frac{k_D^2}{k_{ab}^2}.$$



1 – electron-electron interaction; 2 – electron-proton interaction;
3 – proton-proton interaction at parameters

Figure 1 – The hydrogen plasma potentials (5) at $\Gamma = 3$, $r_s = 0.5$.



1 - potential (5); 2 – Kelbg potential; 3 – Deutsch potential; 4– Debye – Hueckel potential

Figure 2 – Potentials comparison.

PlasmaPressure

A system in the thermodynamic equilibrium can be described by using the measured macroscopic parameters, such as pressure and internal energy. The pressure of the plasma or equation of state (EOS) can be written in next way [1]:

$$P = P_{id} - \frac{2\pi}{3}\int_0^\infty \sum_{a,b} n_a n_b \frac{d\varphi_{ab}(r)}{dr} g_{ab}(r) r^3 dr, \tag{6}$$

where $P_{id} = \sum_a n_a k_B T$ is the pressure of the ideal gas.

We implemented this formula in order to calculate pressure of hot hydrogen plasma and compare with results of [7].

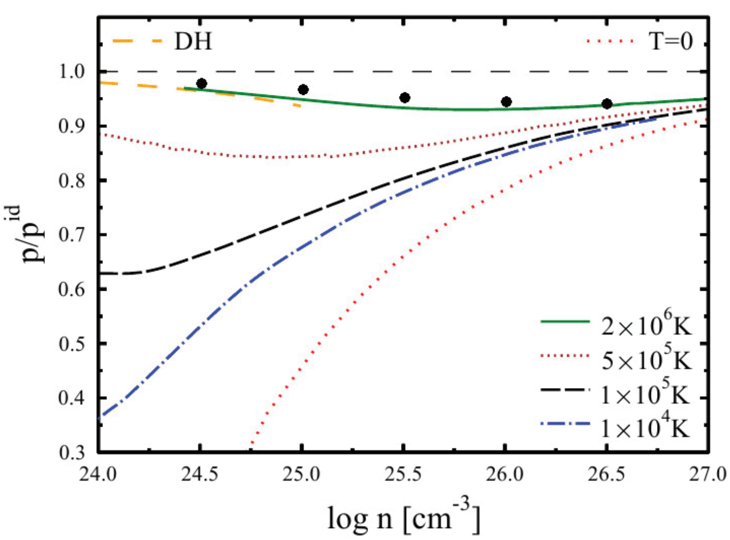


Figure3 –Pressure of hydrogenas predicted by the two-fluid model normalized by the ideal contribution [7], where black circles – formula (6) with potential (5) at T=2*10^6K.

Conclusions

An effective pair interaction potential of particles for ideal and weakly non-ideal plasma were obtained. It takes into account not only screening and dynamic quantum effects, but also statistical quantum effects. Comparison between other kinds of potentials was conducted. We used this potential for the calculation of plasma pressure and compared our result with [7]. It is in a good agreement at high temperatures.

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