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Interaction of dust grains in a plasma under quasineutrality conditions

Abstract. Pseudo potential model of dust particles interaction in the plasma is proposedto take into account the finite-size, the polarization and the screening effects under total quasineutrality conditions. The interaction micro potentials of plasma constituents include the polarization phenomena within the charge image method under the assumption that the grains are made of a conductive material. The original insight of accounting for the finite size effect is to start counting distances from the grains' surfaces rather than from their centers. The derivation procedurestems entirely fromthe renormalization theory of plasma particles interaction which results in the so-called generalized Poisson-Boltzmann equation. The main idea is to apply the renormalization theory in order to treat the dust grains as one component plasma with the specific interaction potentialderived in such a way so as to naturallyincorporate the number densities of electrons and ions of the buffer plasma andto exclude the grain number density. The pseudo potential model developed can further be utilized in theoretical considerations and computer simulations.
Key words: Dusty plasma, pseudo potential model, generalized Poisson-Boltzmann equation, finite size effects, polarization phenomena.

Introduction

These days strongly coupled Coulomb systems are still attracting much interest of the plasma physics community since they frequently appear in contemporary context ranging from nanotechnology [1-3] and Penning traps [4] to astrophysics [5-7]. One common feature that unifies such intrinsically various objects is strong antiparticle interactions caused by the long-range electrostatic forces with the latter being responsible for notorious difficulties in theoretical description. It is, thus, rather obvious that the realm of strongly coupled plasmas cannot be theoretically covered by one single approach. Moreover, the entire apparatus of theoretical physics [8,9] and the simulation techniques [10,11] are engaged to correctly describe the whole range of phenomena occurring at such different scales.

A dusty plasma takes a very special place among the strongly coupled Coulomb systems since it is a fundamentally classical system whose behavior is readily visualized in rather sophisticated experimental researches. This provides a good opportunity to test theoretical approaches worked out for the past decades and to shed some light on what effect the strong interactions have on thermodynamic [12,13] and transport [14,15] properties of plasmas. Until very recently almost all investigations have been adapting the Yukawa potential to describe the interaction between the dust particles [16,17]. As it will be shown below such an assumption implicitly implies that the dust particles are point-like charges which cannot be always true especially if the grain number density grows. This work solely focuses on the influence of finite dimensions of dust particles on such measurable macroscopic characteristics as the radial distribution function and the static structure factor.

Dimensionless plasma parameters

Let the dust particles, called grains, be merged into a two-component hydrogen plasma consisting of free electrons with the electric charge e and the number density n_e and free protons with the electric

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charge e and the number density n_p. It is assumed hereinafter that the dust particles are metallic hard balls of radius R and possess the negative electric charge $-Z_d e$ with Z_d being the grain charge number.

Since an ordinary plasma medium must remain quasineutral, the equality

$$n_p = n_e + Z_d n_d \tag{1}$$

should essentially be held with n_d being the dust particle number density.

The state of the buffer hydrogen plasma is described by the density parameter

$$r_s = \frac{a_e}{a_B}, \tag{2}$$

where $a_e = (3/4\pi n_e)^{1/3}$ stands for the average distance between free electrons in the buffer plasma, $a_B = \hbar^2 / m_e e^2$ is the very well known first Bohr radius, $\hbar$ designates the reduced Planck constant and m_e denotes the electron mass.

The coupling parameter of the buffer plasma

$$\Gamma = \frac{e^2}{a_e k_B T} \tag{3}$$

represents the ratio of the average Coulomb interaction and thermal kinetic energies of free electrons. Here k_B denotes the Boltzmann constant and T stands for the plasma temperature.

The interrelation between the dust component and the buffer plasma is introduced by the Havnes parameter

$$P = \frac{Z_d n_d}{n_e} \tag{4}$$

that determines the ratio of the charge densities of the dust and electron components [18], as well as by the screening parameter

$$\kappa = \frac{a_d}{\lambda_D} \tag{5}$$

with $\lambda_D = (k_B T / 4\pi(n_e + n_p)e^2)^{1/2}$ being the Debye screening length and $a_d = (3/4\pi n_d)^{1/3}$ being the mean intergrain spacing.

The main objective in the sequel is to consistently treat the finite dimensions of dust particles that are to be characterized by the grain size parameter

$$D = \frac{a_d}{R}. \tag{6}$$

It has to be strictly emphasized that numerical values of plasma parameters, i.e. all the number densities and the plasma temperature, are simply retrieved if all magnitudes of the dimensionless parameters (2)-(6) mentioned above are thoroughly specified.

Intergrain interaction potential

As it has already been mentioned above the grains are assumed to be metallic charged balls such that the interaction micro potentials between the dusty plasma constituents are found in the framework of the image charge method as follows [19]:

$$\varphi_{ee}(r) = \varphi_{pp}(r) = -\varphi_{ep}(r) = \frac{e^2}{r}, \tag{7}$$

$$\varphi_{de}(r) = \frac{Z_d e^2}{r} - \frac{e^2 R^3}{2r^2(r^2 - R^2)},$$

$$\varphi_{id}(r) = -\frac{Z_d e^2}{r} - \frac{e^2 R^3}{2r^2(r^2 - R^2)}, \tag{8}$$

$$\varphi_{dd}(r) = \frac{Z_d^2 e^2}{R}\left[\frac{1}{\operatorname{sh}\beta \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\operatorname{sh} n\beta}} - 1\right], \tag{9}$$

where $\operatorname{ch}\beta = r/2R$.

It is seen from (8) that the electrostatic induction results in that the interaction of a charged particle with a metallic charged ball comprises, together with the pure Coulomb interaction, an additional term corresponding to the attraction of the charged particle with an induced image charge of opposite sign. It can be learnt from formula (9) an infinite number of image charges must be taken into account in describing the interaction between two charged metallic balls.

It makes no sense to consider the penetration of the electrons and the protons into the dust particle since it only alters the grain charge already accounted for by the introduction of the grain charge number Z_d. The same can be said about the mutual penetration of grains. Thus, to treat the finite size effects in interaction potentials (8) and (9) the substitutions $\varphi_{d(i,e)}(r) \to \varphi_{d(i,e)}(r+R)$ and $\gamma_{dd}(r) \to \varphi_{dd}(r+2R)$ are made to imply that the distances between the particles involved are now counted from the grain surface. Such a shift in distance counting is introduced in order to correctly derive the Fourier transforms of interaction potential (8) and (9) needed for all further consideration. At the same time this means that the system of hard balls is virtually substituted by the system of point-like charges with the appropriately modified interaction potential.

When the finitesizeof the dust particles is taken into account, the interaction between the electron and the dust particle remains pure repulsive in nature, but it turns out finite at the origin. Polarization effects due to the advent of induced charges lead to a change in the character of the interaction at small distances, i.e.at some separation close to the dust particle surface therepulsion gives place to attraction and the micro potential turns out infinite. Since the proton is positively charged, it is attracted by a dust particle and the polarization phenomena only increase this effect. Again if onlyfinite sizeof the dust particlesis singly taken into account the micropotential remains finite at the origin. As for the dust particles they are negatively charged and therefore repel each other. The finite dimension of grains leads to finiteness of the corresponding micropotential at some very close distances between the dust particles whereas the polarization effects only weaken their mutual repulsion.

The Fourier transforms of micro potentials (7)-(9) take the form:

$$\tilde{\varphi}_{ee}(k) = \tilde{\varphi}_{pp}(k) = -\tilde{\varphi}_{ep}(k) = \frac{4\pi e^2}{k^2}, \quad (10)$$

$$\tilde{\varphi}_{pd}(k) = -\frac{4\pi z_d \Gamma_R}{k^2} + \frac{4\pi z_d \Gamma_R}{k}[(\mathrm{Ci}(k)\sin(k) + \frac{1}{2}\cos(k)(\pi - 2\,\mathrm{Si}(k)))] -$$

$$-\frac{\pi \Gamma_R}{k}[2\,\mathrm{Ci}(k)\sin(k) - 2\,\mathrm{Ci}(2k)\sin(2k) + \cos(k)(\pi - 2\,\mathrm{Si}(k)) - \cos(2k)(\pi - 2\,\mathrm{Si}(2k))], \quad (11)$$

$$\tilde{\varphi}_{ed}(k) = \frac{4\pi z_d \Gamma_R}{k^2} - \frac{4\pi z_d \Gamma_R}{k}[Ci(k)\sin(k) + \frac{1}{2}\cos(k)(\pi - 2\,\mathrm{Si}(k))] -$$

$$-\frac{\pi \Gamma_R}{k}[2\,\mathrm{Ci}(k)\sin(k) - 2\,\mathrm{Ci}(2k)\sin(2k) + \cos(k)(\pi - 2\,\mathrm{Si}(k)) - \cos(2k)(\pi - 2\,\mathrm{Si}(2k))], \quad (12)$$

$$\tilde{\varphi}_{dd}(k) = \frac{4\pi z_d^2 \Gamma_R}{k^2} + \frac{4\pi z_d^2 \Gamma_R}{k^2}\mathrm{f}(k) - \frac{8\pi z_d^2 \Gamma_R}{k}[\mathrm{Ci}(2k)\sin(2k) + \frac{1}{2}\cos(2k)(\pi - 2\,\mathrm{Si}(2k)))], \quad (13)$$

where $\mathrm{f}(k)$ is a known interpolating function.

To account for the screening effects in the interaction of two isolated dust particles the following generalized Poisson-Boltzmann equation is applied [20]:

$$\Delta_i \Phi_{ij}(\mathbf{r}_i^a, \mathbf{r}_j^b) = \Delta_i \varphi_{ij}(\mathbf{r}_i^a, \mathbf{r}_j^b) - \sum_{c=e,p} \frac{n_c}{k_B T} \int \Delta_i \varphi_{ik}(\mathbf{r}_i^a, \mathbf{r}_k^c) \Phi_{jk}(\mathbf{r}_j^b, \mathbf{r}_k^c) d\mathbf{r}_k^c \quad (14)$$

with n_C being the number density of particle species c. Note that in equation (14) the summation is only taken over the free electrons and protons of the buffer plasma, $c = e, p$, whereas the number density of dust particles is kept to be zero. It is deliberately done since of interest is the interaction of two isolated grains whose screening is realized by the electrons and protons of the buffer plasma.

In virtue of (14), the microscopic potentials φ_{ab} determine the intergrain potential Φ_{dd} which takes into account the screening phenomena due to the electrons and protons of the buffer plasma by incorporating the corresponding number densities. In the Fourier space, the set of equations (14) turns into a set of linear algebraic equations whose solution for the intergrain interaction potential is found as:

$$\tilde{\Phi}_{dd}(k) = \tilde{\varphi}_{dd}(k) - \frac{A_p\tilde{\varphi}_{pd}^2(k) + A_e\tilde{\varphi}_{ed}^2(k) - A_eA_p\tilde{\varphi}_{ee}(k)[\tilde{\varphi}_{ed}^2(k) + \tilde{\varphi}_{pd}^2(k) + 2\tilde{\varphi}_{ed}(k)\tilde{\varphi}_{pd}(k)]}{1 + (A_e + A_p)\tilde{\varphi}_{ee}(k)}, \quad (15)$$

where $A_{(e,p)} = n_{(e,p)}/k_BT$.

The expression for the intergrain potential in the configuration space is utterly obtained from (15) by the inverse Fourier transform

$$\Phi_{dd}(\mathbf{r}) = \int \tilde{\Phi}_{dd}(\mathbf{k}) \exp(-i\mathbf{kr}) d\mathbf{k}, \quad (16)$$

It is worth noting that under the present consideration the dust particles have absolutely no effect on the interaction of the buffer plasma particles, i.e. the free electrons and protons. At the same time the well used Yukawa potential is a limiting case of the above expressions when $R \to 0$, i.e. when the dust particles are assumed to be point-like charges to entirely ignore their finite dimensions.

As it has been mentioned above, expressions (15) and (16) simultaneously take into account the finite size of grains, the shielding of their electric field due to the buffer plasma and the polarization phenomena. A theoretical analog of the present approach is the dielectric medium approximation [21] in which the screening in the intergrain interaction is accounted for by the dielectric function of the buffer plasma. The only drawback of both methods is that the dusty plasma is meant to be in equilibrium state which somehow restricts the applicability of the results to real dusty plasma experiments. Nevertheless, deviations of the particle distribution functions from the Maxwellian, caused by the so-called absorption and shadowing effects. can easily be handled to show that the Yukawa potential turns invalid at rather large distances between grains [22-24]. Moreover, under certain external conditions the free flight paths of the buffer plasma particles may turn less in magnitude than the Debye shielding length and, thus, an increasingly important role is played by interparticle collisions that result in ion trapping in the vicinity of dust particles [25] or even requires an application of hydrodynamics to correctly treat the plasma shielding effect [26]. It has to be admitted that real dust grains are hardly spherical in shape which leads to non-zero dipole moments of dust particles and, as a consequence, to anisotropic interactions between them [27]. The intergrain interaction potential can yet be determined experimentally with the aid of some theoretical arguments which, for example, was done in [28] for the rf discharge dusty plasma in the framework of the Langevin dynamics.

Figures 1 and 2 shows the interaction potentials between two isolated dust grains immersed in the hydrogen plasmas. Two different cases are considered. The first one corresponds to properly taken into account polarization phenomena whereas the second drops them out. As it is expected the electrostatic induction results in the weakening of intergrain interaction since they cause a new mechanism of attraction. Another fact that is obvious from the analysis of both figures is that the finite size effects are responsible for substantial decrease in the interaction potential.

It should be mentioned that the screening phenomena is introduced via the renormalization procedure of plasma particle interactions. Indeed, the dust particles are negatively charged, which means that they repelelectronsand attract protons, thereby forming positivelycharged clouds around them. Such cloudsappear becausethe probability density of finding a proton at some distance from the dust particle is higher than the corresponding probability density for an electron. Maximum of the probability density lies just in between the dust particles and the positively charged cloud formed may play the same role as the paired electrons in the formation of covalent bonds in molecules.

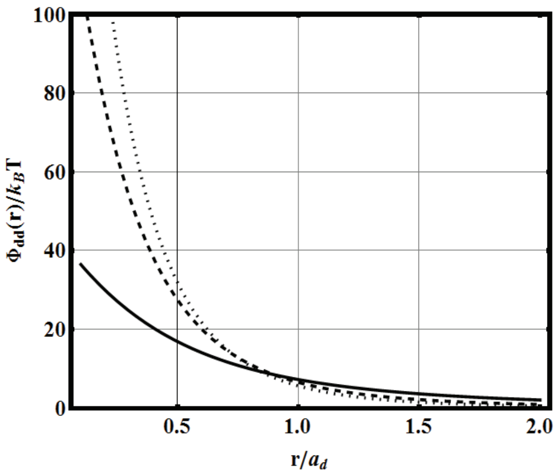

Figure 1 – The interaction potential between two isolated dust grains in plasmas with parameters Γ=0,2, P=5 and κ=4. Solid line: D=2; dashed line: D=5; dotted line: D=8. Polarization effects are properly taken into account.

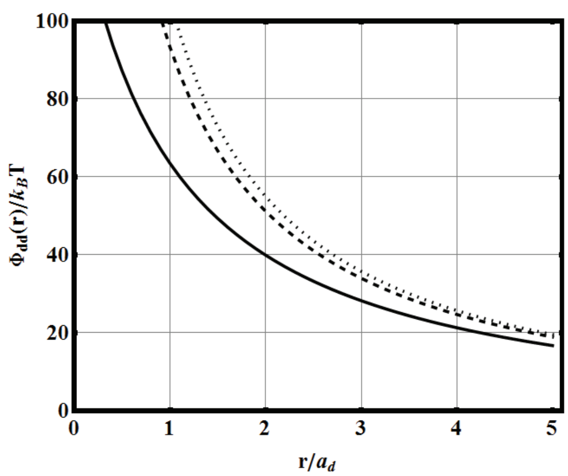

Figure 2 – The interaction potential between two isolated dust grains in plasmas with parameters Γ=0,2, P=5 and κ=4. Solid line: D=2; dashed line: D=5; dotted line: D=8. Polarization effects are not taken into account.

Conclusions

In this work micro potentials of structural elements of dusty plasmas have been chosen to construct the pseudopotential interaction model of two isolated dust particles. Two fundamentally new effects have been taken into account's) the finite size of the dust particles; ii)the polarization effects in the interaction of dust particles due to the phenomenon of electrostatic induction. The latter has been accounted for by the electrostatic charge image method with a subsequent conversion of distance to avoid the penetration of micro particles into each other. Comparison with the pure Coulomb potential demonstrates that both effects actually lead to some effective attraction mechanism which had previously not been studied in the literature.

The analytical expressions for the Fourier transforms of the micro potentials have been obtained and their dependence on the wave number has been thoroughly analyzed. After that the Fourier transform of the interaction potential of two isolated dust grains has been determined on the basis of the renormalization theory and the corresponding pseudo potential has been restored in configuration space. The increase in the coupling parameter leads to an increase in the corresponding values of the interaction pseudopotential of dust particles. In complete accord with its name, the increase in the screening parameter yields awakening of the interaction between dust particles due to the shielding of the electric field.

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