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Evolution of isolated white dwarfs

Abstact. We consider evolution of isolated general relativistic uniformly-rotating white dwarfs which lose angular momentum via magnetic dipole braking. We show for selected constant mass sequences how the main parameters of white dwarfs such as the central density, mean radius and magneticfield change with time by fulfilling all stability criteria of the general relativistic uniformly-rotating configurations. Namely, we explicitly demonstrate that all isolated white dwarfs by angular momentum loss will be shrinking in order to reach stable equilibrium states.
Key words: rotating white dwarfs, equilibrium configurations, angular momentum loss, magnetic dipole braking, constant mass sequence.

Introduction

Rotating white dwarfs (RWDs), depending on their mass, i.e., whether they are sub-Chandrasekhar white dwarfs (WDs) or super-Chandrasekhar WDs, display different behavior. Namely, both uniformly and differentially rotating super-Chandrasekhar white dwarfs (SCWDs) spin-up by angular momentum loss whereas sub-Chandrasekhar WDs only spin-down by angular momentum loss. We should mention that the spin-up of rapidly-rotating stars was first described by Shapiro et al. [1] and later by Geroyannis and Papasotiriou [2]. In both references [1] and [2] the authors performed computations in classical physics without taking into account theeffects of general relativity (GR) although GR is verycrucial in investigating the stability of RWDs [3].

In our recent work [4], we computed general relativistic configurations of uniformly RWDs within Hartle's formalism [5]. We used the relativistic Feynman-Metropolis-Teller equation of state [6, 7] for WD matter, which generalizes the traditionally-used equations of state of Chandrasekhar [8] and Salpeter [9] as follows: (1) in order to guarantee self-consistency with a relativistic treatment of the electrons, the point like assumption of the nucleus is abandoned, introducing a finite-sized nucleus; (2) the Coulomb interaction energy is fully calculated without any approximation by solving numerically the relativistic Thomas-Fermi equation for each given nuclear composition; (3) the inhomogeneity of the electrondistribution inside each Wigner-Seitz cell is accountedfor; (4) the energy density of the system is calculatedtaking into account the contributions of the nuclei, ofthe Coulomb interactions, as well as of the relativistic electrons to the energy of the Wigner-Seitz cells; and (5) the β equilibrium among neutrons, protons, and electrons is also taken into account, leading to a self-consistent calculation of the threshold density for triggeringthe inverse β decay of a given nucleus. The stabilityof rotating WDs [4] was analyzed taking into account themass-shedding limit, inverse β-decay instability, and secular axisymmetric instability, with the last being determined by using the turning point method of Friedman et. al. [10].

In this work, we consider compression of isolated rotating WDs by angular momentum loss based on the results of Boshkayev et al. [4, 11-17]. Particularly, by fulfilling all the stability criteria for RWDs in GR we estimate the change intime of the basic parameters of rotating WDs. We consider two cases: (1) we assume that dipolar magnetic isconstant throughout the entire evolution of WDs, and (2) for the sake of comparison, we adopt magnetic fluxconservation relaxing the constancy of the magnetic field.

Our paper is organized as follows: In Section II, we calculate the main physical parameters of isolated rotating WDs by angular momentum loss. In Section III, we summarize ourmain results, discuss their significance, and draw our conclusions.

Evolution of isolated rotatingWDs

To investigate the evolution of isolated white dwarfswith time we made use of the equation for the rotational energy loss of pulsars via magnetic dipole brakingand depending on whatparameter we are interested in, this equation was slightly-modified. For example, if we want to see how the centraldensity of the WD evolves with time we need to rewritethe equation for the rotational energy lossin the following form

$$dt = -\frac{3}{2}\frac{c^3}{B^2}\frac{1}{\langle R\rangle^2}\frac{1}{\Omega^3}dJ = -\frac{3}{2}\frac{c^3}{B^2}\frac{1}{\langle R\rangle^2}\frac{1}{\Omega^3}\frac{\partial J}{\partial \rho}d\rho \quad (1)$$

where

$$\langle R\rangle = \langle R\rangle(\rho), \;\; \Omega = \Omega(\rho), \;\; J = J(\rho), \quad (2)$$

and c is the speed of light in vacuum, B is the dipole magnetic field, $\langle R\rangle = (1/3)(R_{polar} + 2R_{equatorial})$ is the mean radius, Ω is angular velocity, J is the angular momentum, ρ is the central density of a white dwarf .The values of $\langle R\rangle = \langle R\rangle(\rho)$,$\Omega = \Omega(\rho)$ and $J = J(\rho)$are calculated along a given constant baryon (rest) masssequence. Here we adopt carbon white dwarfs using the Feynman-Metropolis-Teller equation of state like in reference [17]. According to [11-17] there are limiting values of all the parameter of rotatingWDs on the borderof the stability region. These limiting values determinethe range of integration of the central density ρ. Performing numerical integration of Eq. 1for each momentof time we obtain the evolution of the central densitywith time. To account for the magnetic flux conservation relaxing the constancy of the magnetic field we use following expression

$$B = B_0\frac{\langle R_0\rangle^2}{\langle R\rangle^2}, \quad (3)$$

where B_0 is the surface dipole magnetic field corresponding to the initial value of B at $t = 0$, $\langle R_0\rangle$ is the mean radius corresponding to the initial value of $\langle R\rangle$ at $t = 0$. Substituting B in Eq. 1we obtain an equation that describes the evolution of ρ with time taking into account the magnetic flux conservation. Clearly, in figure 1 we see the main difference between two cases. In both cases WDs will increase their central density and will be compressed by losing angular momentum. However in the case with the magnetic flux conservation the value of B will be increasing due to the compression thus causing more torque and evolving faster with respect to the B=constant case.

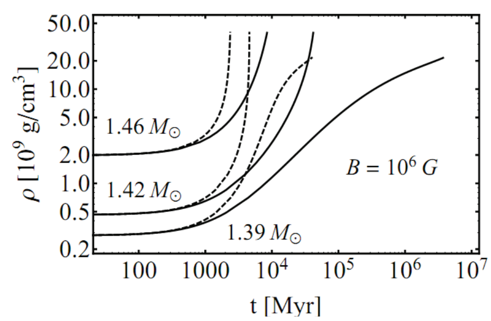

Figure 1 – Central density versus time. Solid curves are the evolution path for selected constant mass sequences when B is constant. Dashed curves are the evolution path for constant mass sequences when the magnetic flux is constant with $B_0 = 10^6$G.

In order to estimate how the mean radius of the WD evolves with time we need to rewrite Eq. 1as follows

$$dt = -\frac{3}{2}\frac{c^3}{B^2}\frac{1}{\langle R\rangle^2}\frac{1}{\Omega^3}\frac{\partial J}{\partial\langle R\rangle}d\langle R\rangle \quad (4)$$

where now

$$\Omega = \Omega(\langle R\rangle), \;\; J = J(\langle R\rangle). \quad (5)$$

The values of $\Omega = \Omega(\langle R\rangle)$, and $J = J(\langle R\rangle)$ are calculated along a given constant rest mass sequence like in the previous case. Here the range of integration of $\langle R\rangle$ is also defined along the constant rest mass sequence on the border of the stability region.

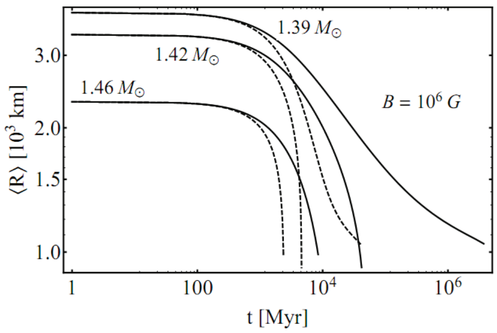

Figure 2 –Mean radius versus time. Solid curves are the evolution path for selected constant mass sequences when B is constant. Dashed curves are the evolution path for constant mass sequences when the magnetic flux is constant with $B_0 = 10^6$G.

The procedure to estimate the evolution of other parameters of rotating WDs is analogous to the central density with Eq. 1and to the mean radius with Eq. 4. To take into consideration the magnetic flux conservation we need to use Eq. 3. In figure 2 the mean radius is plotted as a function of evolution time. Over the course of time the mean radius decreases, hence WDs shrink with time. For the case of the conserved magnetic flux it decreases faster than for the case with constant surface magnetic field. Note, that for isolated rotating white dwarfs the rest mass remains unchanged over the course of time.

Isolated WDs regardless of their masses will always lose their angular momentum via magnetic dipole braking. By losing angular momentum WDs tend to reach more stable configurations by increasing their central density and by decreasing their mean radius. Eventually super-Chandrasekhar WDs will spin-up and sub-Chandrasekhar WDs spin-down [12].

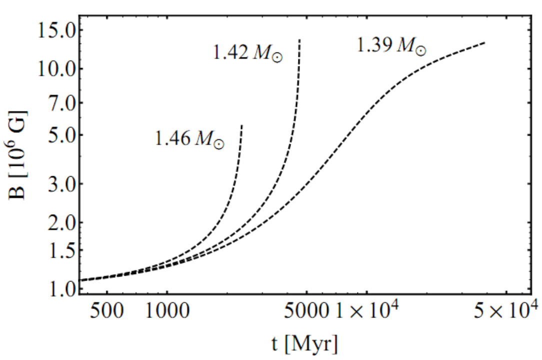

Figure 3 – Magnetic field versus time. The field is obtained through the magnetic flux conservation law

Figure 3 shows how the surface dipolar magnetic field changes with time for the conserved magnetic flux. Here we selected three different constant rest masses. B increases as a result of the WD compression. This means that at certain point of their evolution WDs with “smaller” masses, but still super-Chandrasekhar WDs can possess high magnetic fields.

Conclusions

In this work, we showed how the basic parameters of isolated (solitary) rotating WDs evolve with time by angular momentum loss. For the sake of comparison we considered two cases with constant magnetic field and constant magnetic flux. For the magnetic flux conservation the evolution time (life time) turned out to be shorter than for the constant magnetic field.

We showed that WDs regardless of masses by angular momentum loss via electro-magnetic radiation will tend to get more stable configurations. Thus their central densities increase, and mean radii decrease. Hence all solitary WDs will shrink over their entire evolution. Super-Chandrasekhar WDs will end up exploding like supernovae type Ia or collapse into neutron stars, and sub-Chandrasekhar WDs will be slowing down until they completely stop rotating.

Throughout the paper, we performed computations in GR by using the Hartle formalism for uniformly-rotating configurations. We considered mainly WDs consisting of carbon, although the typical white dwarfs are known not to consist of a pure chemical element, but a mixture of elements such as carbon, oxygen, neon, magnesium, etc. It would be interesting to consider the chemical profiles of Renedo et al. [18] in our future investigations.

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