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On the development of a numerical method of calculation speed wind turbine darrieus

Abstract. In this paper the mathematical model of vertical-axis Darrieus wind turbine (carousel type etc.). The paper presents a mathematical formulation of the problem, method of calculating the angular velocity of the Darrieus wind power generating apparatus when exposed to the incoming flow and the results obtained from the calculation. According to the calculations, below the established methodology, the reliable results that well represents the physics of the phenomenon. The mathematical model and its numerical implementation and the results may be useful to further improve the mathematical description of the problem in the design and construction of vertical-axis wind turbines.
Keywords: wind turbine, angular velocity, calculation, iteration, mathematical models, moment of inertia, the lift, drag force analysis.

Introduction

The most known and widely used propeller wind turbines with a horizontal axis of rotation. Along with them have been proposed various types of devices with a vertical axis of rotation.

The main advantages of the two vertical-axis wind turbines (wind turbine called " Darrieus ", invented by French engineer Darya in 1925.) Compared to horizontal-axis:

a) the wind direction does not matter (for which there is no need for a mechanism for the orientation of the wind rose is facilitated construction and reduced gyroscopic loads simplified transmission system for electricity, which can be located at the ground level);

b) power generator, and other major components of the unit are located on the ground level, which reduces the requirements for the tower, and easy maintenance and repair. Thus, there is free access to the electrical parts for maintenance and repair works [1-3].

Method of research

The rotor has a relatively small initially, but most of rapidity, because of this - a relatively high power density, referred to its weight or value. Darrieus wind turbine is operated by the lifting force of the working vane as aircraft wings symmetrical with respect to the type of the chord profiles. The blades are at a distance r0 from the rotation shaft and connected thereto in one of two ways: with or swings "troposkino". Furs are called flat wings span, the ends of which are attached workers wings (turbine blades) with the letter "T" or "Γ" so that the chord of the blades was tangential to a circle of radius r0 (Figure 1a). Method of attachment "troposkino" is that the elastic blade is folded flat to form a bow and both ends attached to the top and bottom of the rotation shaft. When the turbine blades are forced to take the form of a rotatable slack rope - troposkino (Figure 1b).

Vertical-axis wind generators can be classified by their aerodynamic and mechanical characteristics. By definition, all vertical axis machines have a common characteristic, which consists in that the aerodynamic bearing elements forming rotors move around a vertical axis, and their points describe trajectories lying generally horizontal planes [4-5].

Consider the interaction of wind turbines with fixed air flow [4-7]. Figure 2 schematically shows the four most important positions of the working blades rotating at a constant angular velocity ω.

The angle $\theta \in [0,2\pi]$ is measured from the x coordinate as the zero position of swing. Thus, the angle θ determines the position of the working of the blade and the forces acting on it along the circle described by the workers during the rotation of turbine blades.

As seen in Figure 2 at points A and C, the angle of attack (α) is zero because of the parallelism of the wind velocity vector $\vec{V}$ and the vector of the linear velocity $\vec{U}$ of rotation of the turbine.

The velocity vector of attack can be written as the sum of the vector wind speed $\vec{V}$ and inductive linear velocity vector $\vec{U}$ of the working of the blade with the opposite sign

$$\vec{W} = V\sin\theta\vec{e}_1 + (\omega r_o + V\cos\theta)\vec{e}_e \quad (1)$$

where, ω - angular speed of rotation of the turbine, r - the length of the swing H - rotor, V - inductive wind speed. Below it will be shown how to determine its value.

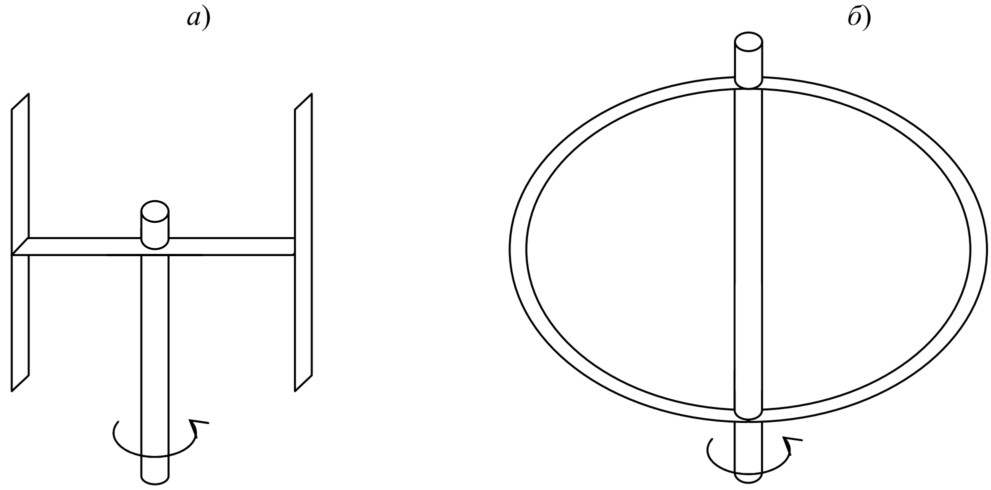

Figure 1 – Schematic of the wind turbine carousel

The angle of attack is expressed by the following formula

$$tg\alpha = \frac{(\vec{W},\vec{e}_1)}{(\vec{W},\vec{e}_e)} = \frac{V\sin\theta}{V\cos\theta + r_o\omega}$$

or

$$\alpha = arctg\left(\frac{V\sin\theta}{V\cos\theta + r_o\omega}\right) \quad (2)$$

Introducing the parameter rapidity $Z = \frac{r_o\omega}{V}$, we can get

$$\alpha = arctg\left(\frac{\sin\theta}{\cos\theta + Z}\right) \quad (3)$$

a) lifr fors of the working blade profil

$$\vec{R}_L = C_L(\alpha)\rho\frac{W^2}{2}hdz\vec{e}_L, \quad (4)$$

where in $C_L(\alpha)$ - lift coefficient, h - height of the section chord, dz - blade adjustment element, $\vec{e}_L -$ the unit vector in the direction of lift of the wing,

b) the resistance force

c) $$\vec{R}_D = C_D(\alpha)\rho\frac{W^2}{2}hdz\vec{e}_w, \quad (5)$$

Coefficients $C_D(\alpha)$, $C_L(\alpha)$ can be life represented by the formulas related to the angle of attack and are determined experimentally.

Let's write down the elementary moment of aerodynamic forces operating on an element of the dz blade at change $\theta \in [0, 2\pi]$ it is equal

$$dM = r_o\left[\left(\vec{R}_D,\vec{e}_\theta\right)+\left(\vec{R}_L,\vec{e}_\theta\right)\right] = \rho\frac{W^2}{2}hdzr_o\left[\square C_D\cos(\alpha)+C_L\sin(\alpha)\right] \quad (6)$$

where z – the third axis of orthogonal Cartesian system of coordinates. Working blades of the N-rotor are located parallel to the third axis z.

$$W^2 = \left(\vec{W}\right)^2 = V^2\sin^2\theta + \left(\omega r_o + V\cos\theta\right)^2 \quad (7)$$

Average value of force $\overline{R}_x(\theta)$ operating on an element, it is equal

$\overline{R}_x(\theta) = R'(\theta)dx$ here

Value W^2 we will receive, using a formula (2)

$$dx = r_o \sin\theta d\theta; \quad (8)$$

where is determined by a formula (9'), or in more detail

$$R_x = \frac{1}{2\pi}\int_0^{2\pi}\overline{R}_x d\theta \quad (9)$$

$$R_x = \frac{1}{2\pi}\int_0^{2\pi}\overline{R}_x d\theta = \frac{1}{2\pi}\int_0^{2\pi}\rho\frac{W^2}{2}\frac{hB(\theta)}{r_o\sin(\theta)}d\theta = \frac{\rho h}{4\pi r_o}\int_0^{2\pi}\frac{W^2B(\theta)}{\sin(\theta)}d\theta \quad (10)$$

here expression (8) it is used for R_x and $B(\theta) = C_L\sin(\theta-\alpha) - C_D\cos(\theta-\alpha)$

Having used determination of coefficient of C_p, are fair for $0<\theta<\pi$, it is possible to write down

$$C_P = \frac{R_x}{\frac{1}{2}\rho V^2 S} = \frac{h}{2\pi r_o}\int_0^{2\pi}\left(\frac{W}{V}\right)^2\frac{B(\theta)}{\sin(\theta)}d\theta \quad (11)$$

Let's choose in a wind stream a tube of the current interacting from AVSD (figure 2) moving on a circle of the working blade. Formula for inductive speed through aerodynamic characteristics wind turbine

$$\frac{V}{V_\infty} = \frac{1}{1+\frac{h}{8\pi r_o}\int_0^{2\pi}\left(\frac{W}{V}\right)^2\frac{B(\theta)}{\sin(\theta)}d\theta} \quad (12)$$

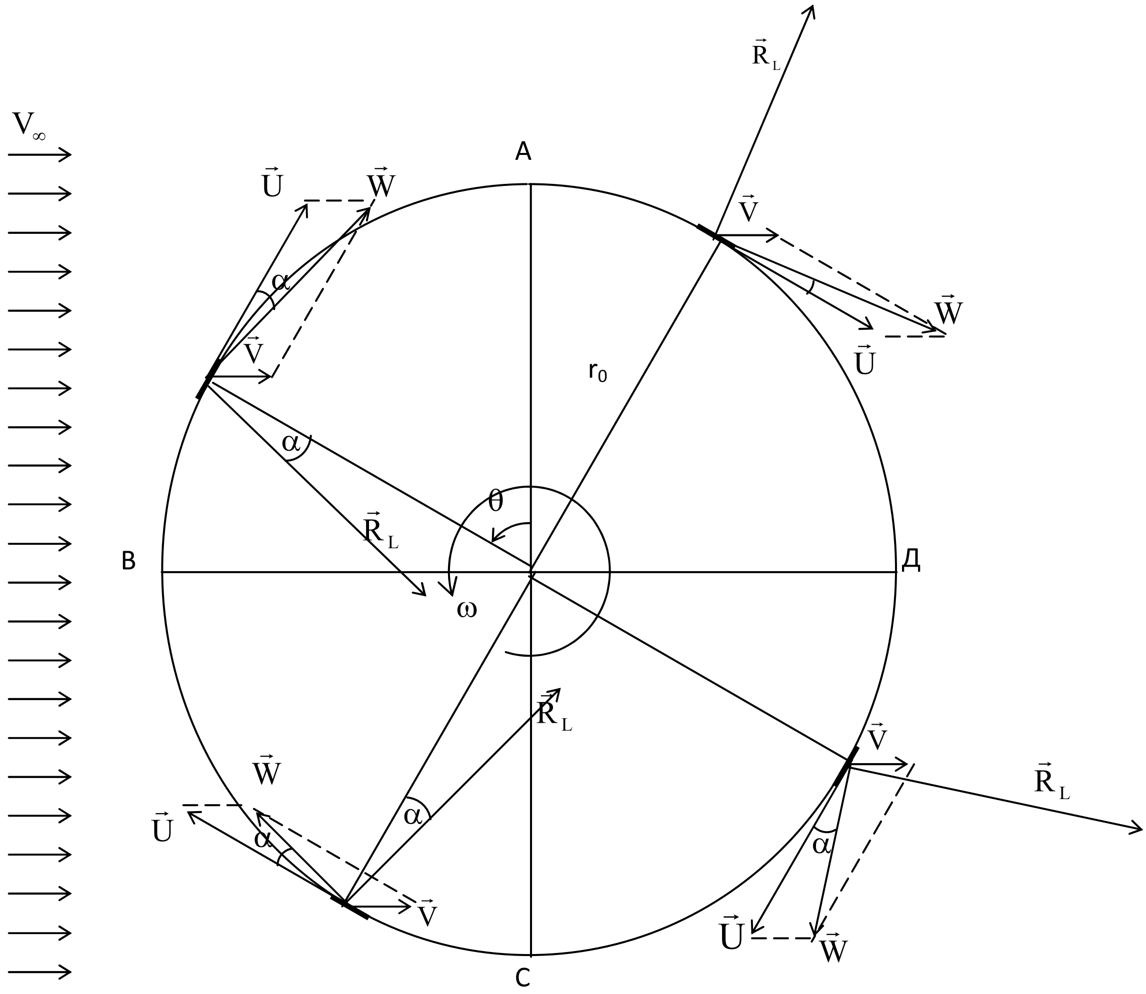

Figure 2 – rotating counterclockwise by a working wind turbine blades

The formula (12) in combination with (1) and (2) gives iterative algorithm for determination of inductive speed by V at windward part of a rotor to Darya, in the case under consideration, when the turbine as a whole is clasped by one tube. When carrying out calculations by the method considered above the iterative program in the Fortran language was used and realized on the computer. For carrying out calculations by us on the basis of aerodynamic characteristics of the alary NASA-0021 processing was carried out and empirical formulas of dependence of coefficients of carrying power (C_L) and force of resistance (C_D) from an angle of attack are received,

$$C_L = -0{,}00011\cdot\alpha^3 + 0{,}0023\cdot\alpha^2 + 0{,}0633\cdot\alpha \quad (13)$$

$$C_D = 0{,}0005\cdot\alpha^2 - 0{,}002\cdot\alpha + 0{,}0129 \quad (14)$$

Let's write down expressions for the rotating moment of the turbine for windward and lee side propeller at $\theta\in[0,2\pi]$

$$M_1 = \frac{nhr_oH}{\pi}\times \int_0^{\pi}\rho\frac{W^2}{2}\left(C_L(\alpha)\sin\alpha - C_D(\alpha)\cos\alpha\right)d\theta \quad (15)$$

For determination of angular speed of rotation of a rotor to Darya, at influence of a wind stream we apply the theorem of change of the kinetic moment of mechanical system [4,7] and looks like

$$\frac{dL_z}{dt} = M_{\text{турб}} + \sum M_i \tag{16}$$

where L_z – the kinetic moment of the wind turbine concerning an axis z. M_{turb} – the rotary moment created by working blades of the turbine, M_i – the moment of various forces of resistance.

For the turbine to Darya with two direct blades it is had

$$\mathrm{I} = \frac{2}{3} r_0^2 m_{\text{м}} + r_0^2 m_{\text{л}} + r_{\text{в}}^2 m_{\text{в}} \tag{17}$$

where r_o – the distance from the rotational axis to the blades, r_B – radius of the shaft, $m_М$, $m_л$, m_B – the masses of strides, blades, shaft rotation.

The time difference will express

$$\omega = \frac{2\pi}{T} = \frac{d\theta}{dt} \tag{18}$$

where dθ - corresponds to the angle of rotation of the working of the blade to the z axis in the time interval dt;

In (16), substituting (18) and writing in difference form, we get

$$\omega^{n+1} = \omega^n + \frac{(R_L \sin\alpha - R_D \cos\alpha) r_o + \sum M_i}{I\omega^n} \times (\theta^{n+1} - \theta^n) \tag{19}$$

and where ω^{n+1} and ω^n - respectively, the angular velocity of the turbine at the time t^{n+1} and t^n. Thus, the determination of the angular velocity ω will continue until it converges to its single value (see Fig. 3.4). As seen in Figure 3, without resistance, based can not be ignored, the turbine makes enough vibrational motion, and given the resistance and bring it to its limit, the oscillation almost vanishes (see Fig. 4). The results of this work and the development of his technique will be useful for the design work for the establishment of industrial designs windmill carousel.

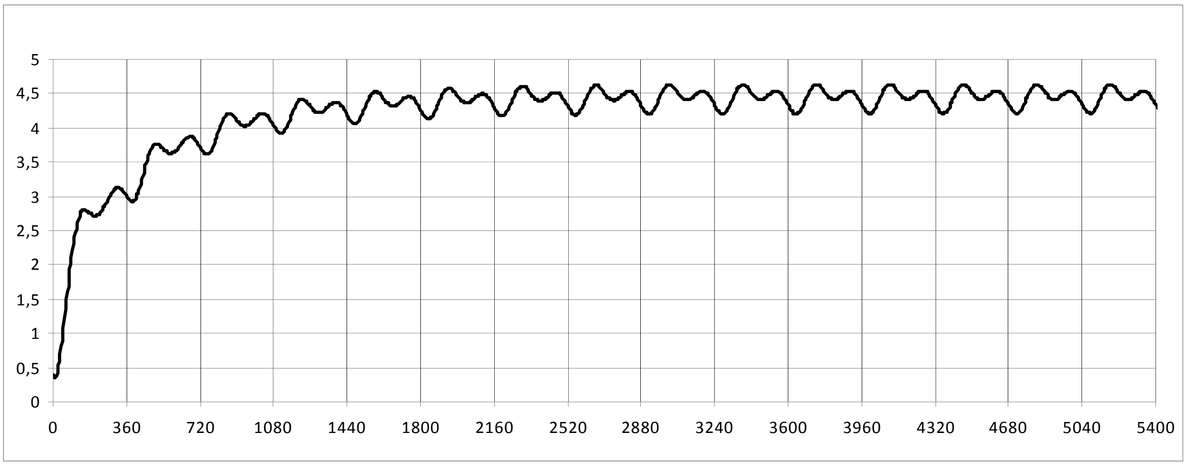

Figure 3 - Graph of the angular velocity ω on the position θ of the moving blades working at a relatively low value of I = 0,5 and without resistance to the turbine

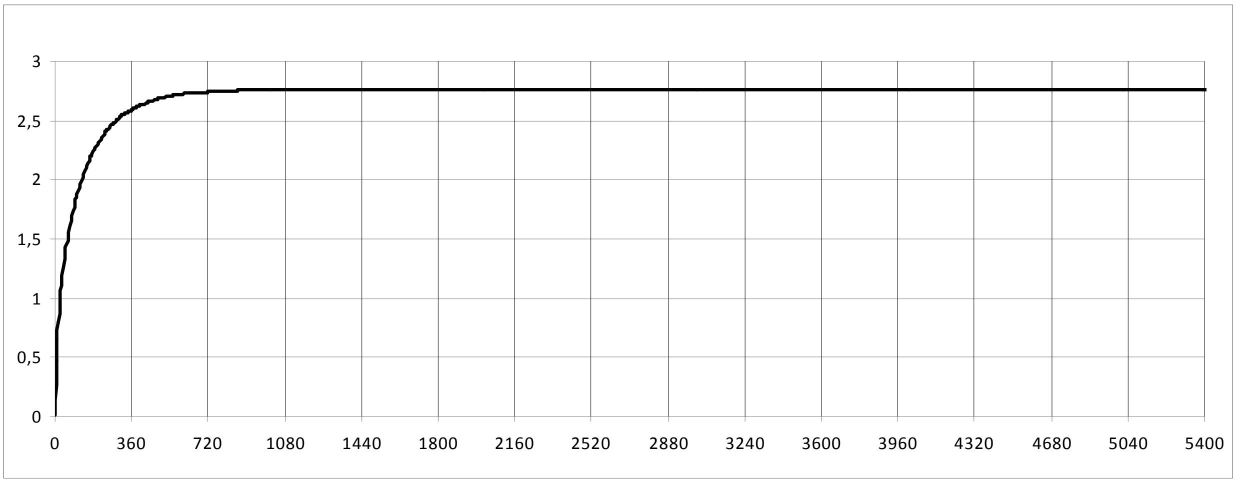

Figure 4 – Graph of the angular velocity ω on the position θ of the moving blade at work I = 0,5, and taking into account the resistance of the turbine 10%

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