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Analysis of the linearized quantum hydrodynamic model

Abstract. In this paperthe analytical analysis of the linearized quantum hydrodynamics modelby matching with random phase approximation for degenerate electrons is done.The equivalence of the twoapproacheson the treatmentof spatialcorrelationbetween electronsdemonstrated by dividing all effects on quantum fluctuations on short distance and long range collective effects. It is shown that linearized quantum hydrodynamic model takes into account electron exchange and correlation effects only by shifting Fermi level. Also, the advantages of the linearized quantum hydrodynamics model in comparison with random phase approximation areclearly illustrated.

Keywords: quantum plasma, pseudopotential, quantum hydrodynamics model, degenerate electrons, electron exchange and correlation effects, random phase approximation.

Introduction

At present, there are numerous methods and models to study quantum and classical many particle systems. These approaches are basis of the condensed mater physics and plasma physics. For instance, the method of the Green function, density functional theory, Hartree-Fok method, dielectric response function method [1,2] and much more approximations within. Thus,it is very important the clear physical understanding of the advantages and disadvantages of them in comparison with each other.

Currently,quantum hydrodynamics [3] isattracting attentionas a convenient methodfor thesemianalytical study ofthe manyparticle systems. The strengths and weaknesses of the linearized quantum hydrodynamic model (LQHD) in comparison with other approaches, recently, actively discussed [3-6]. In thesediscussions a numerous quantum effects such us diffraction effects, spin effects (Pauli principle), exchange and correlation effects considered. Obviously, that all of them taken into account in some approximation and some of them more important and others have character of the second order corrections. Also, it is crucial to understand the characteristic length scale of the given approximation. Bellow static dielectric function (SDF) obtained using LQHD [4] is analytically analyzed and equivalence,on the treatmentof spatialcorrelationbetween electrons, to random phase approximation (RPA) in long wave length limitis shown.

Analysis of the static dielectric function

To begin with, the SDF obtained within LQHD model has the following form [4] (D, in their notation):

$$\varepsilon(\mathrm{k}) = \frac{1 + k^2 / k_{TF}^2 + \alpha k^4 / k_{TF}^4}{k^2 / k_{TF}^2 + \alpha k^4 / k_{TF}^4}, \qquad (1)$$

where $k_{TF} = \omega_{pe}\left(\frac{\upsilon_F^2}{3} + \upsilon_{ex}^2\right)^{-1/2}$ is the inverse Thomas-Fermi screening length,$\alpha = h^2\omega_{pe}^2/(4m^2((\upsilon_{F_0}^2)/3 + \upsilon_{ex}^2)^2), \upsilon_{F_0} = h(3\pi^2)^{1/3}/mr_0$ is the electron Fermi speed, r_0 represents the Wigner-Seitz radius,ω_{pe} is the electron plasma frequency, $\upsilon_{ex} = (0.328e^2/m\epsilon r_0)^{1/2}[1 + \frac{0.62}{}(1 + 18.36a_B n^{1/3})]^{1/2}$, ϵ the relative dielectric permeability of the material, a_B is the first Bohr radius of the electron, and n is the electron number density.

The formula (1) was obtained by linearization the system of the equations which consists quantum hydrodynamics [4]. There is classical continuity equation, Poisson equation and modified momentum equation. The modified momentum equation which takes into account quantum effects is as follows[3]:

$$m\left(\frac{\partial\vec{u}}{\partial t}+\vec{u}\,\mathrm{div}\vec{u}\right)=e\,\mathrm{grad}\,\phi-n^{-1}\mathrm{grad}\,P+\mathrm{grad}\,V_{xc}+\mathrm{grad}\,V_B, \quad (2)$$

here$P$ is a quantum statistical pressure, $V_{xc}(n(r))$is an exchange and correlation potential, and $V_B(n(r))$ is the quantum Bohm potential [3,4].

In order to show that an exactly same formula for the SDF as (1) can be obtained using RPA in long wave limit ($k\to 0$) we divide all effects on short range and long range. It is well known that due to quantum fluctuations the singularity of the Coulomb potentialat short distance smoothes. As the consequence, interaction potential has limit value at zero point. In our analysis, all quantumfluctuations which important at short distance have been taken into account by introducing parameter b which limits pair interaction potential at zero and the following pseudopotentialhas been chosen:

$$V(r)=\frac{e^2}{4\pi\epsilon r}(1-\exp[-r/b]). \quad (3)$$

When $r\to 0$ the interaction potential (3) has the limit value equal to e^2/b.Usually, the potential (3) uses at high temperature to realize semiclassical approximation [7]. There are exists other pseudopotentialswhich treat quantum effects more accurately [8]. For our qualitativeanalysis we choose pseudopotential (3) because pseudopotential (3) have the simple formand useable for analytical manipulations. Moreover, it will be seen that pseudopotential (3) turns out the most optimal one for our purpose.

The SDF in the case of none interacting particles equal to one and due to interaction between particles provided to be different from unity. All collective effects due to the long range Coulomb interaction can be correctly taken into account by the accurate calculation of the dielectric function. In general there is no analytical solution of this problem. We consider collective effects in the system of the degenerate electrons in the simplest so called random phase approximation which valid only when $r_S\ll 1$ and does not take into account correlation between electrons.

The formula for dielectric function inRPA:

$$\varepsilon(k,\omega)=1+\tilde{V}(k)\Pi(k,\omega), \quad (4)$$

where$\tilde{V}(k)$ is the Fourier transform of the interaction potential and for static case when $k\to 0$:

$$Re\Pi(0,0)=\frac{mp_F}{\pi\hbar^2}, \quad (5)$$

wherep_F is the momentum of the electron on the Fermi surface.

The Fourier transformation of the pseudopotential (3):

$$\tilde{V}(k)=\frac{e^2}{\epsilon(b^2k^2+1)k^2}. \quad (6)$$

The real part of the SDF by substituting Eq.(6) and Eq.(5) in the formula (4) has been obtained for the small values of k:

$$\varepsilon(k,0)=1+\frac{k_{TF}^2}{(b^2k^2+1)k^2}, \quad (7)$$

where $k_{TF}=\frac{e^2mp_F}{\epsilon\pi\hbar^2}=3\omega_{pe}^2/\vartheta_F^2$.

Finally, if we formally take $\vartheta_F^2=(\upsilon_{F_0}^2)+3\upsilon_{ex}^2$ and put b equal to the de Broglie wavelength $\lambda=h/p_F$ then $b^2/(k_{TF}^2)\equiv\alpha/k_{TF}^4$ and the formula (7) exactly coincides with the formula (1):

$$\varepsilon(k,0)=\frac{(b^2k^2+1)k^2+k_{TF}^2}{(b^2k^2+1)k^2}=\frac{1+k^2/k_{TF}^2+\alpha k^4/k_{TF}^4}{k^2/k_{TF}^2+\alpha k^4/k_{TF}^4}.$$

According to the pseudopotential (3) the equality of the parameter$b$ to the de Broglie wavelength means that the interaction potential smoothes due to the diffraction effect. Remembering that RPA valid when $r_S\ll 1$ and does not take into account local field correction following summaries about LQHD can be done:

- LQHD valid just in long wavelength limit $k\to 0$;
- does not take into considerationspatial-correlationbetween electrons;
- reproduce quantum diffraction effect (without using any pseudopotential);

- takes into account electrons exchange and correlation effects only by shifting Fermi momentum $p_F \rightarrow p_F + \delta p$;

where according to the performed analysis $\delta p = \sqrt{3} m \vartheta_{ex}$.

Obviously, SDF obtained on the basis of LQHD inherits main features from the equations of the quantum hydrodynamics.Where, the quantum corrections included solely in the momentum equation, while continuity equation and Poisson equation formally remains classical. Therefore, the quantum diffraction effect at small distance reproduces by LQHD as the result of using the smooth function of particle distribution in Poisson equation. Shifting of the Fermi momentum is due to adding exchange and correlation terms in the momentum equation. Neglecting of the spatial correlations between particles is the feature of the any hydrodynamic model. Thus, main futureswhich have been pinpointed can be generalized entirely to the quantum hydrodynamics.

Conclusions

The application domain of the LQHD and quantum hydrodynamics in general has been discussed. It is shown that quantum hydrodynamics model does not consider correlations at small distance and in this regard equivalent to the RPAin long wave length limit. By dividing all effects to quantum fluctuations and long range collective behavior it shown that quantum hydrodynamics reproduce quantum diffraction with modified Fermi surface.

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