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Influence of additive white noise forcing on solutions of mixed nls equations

Abstract. In this paper, the influence of an additive white noise forcing term on the numerical solution for a class of deterministic nonlinear one-dimensional Schrödinger equations with mixed concave convex was studied, sub-super nonlinearities, that is, the stationary states and the blowing-up solutions. Such a perturbation occurs when the size of the noise, described by the real-value parameter ε is positive. The size of the noise is controlled by the parameter $\varepsilon > 0$. We also proved that as ε approaches zero, the solution of the perturbed problem converges to the unique trajectory of the deterministic equation, which is the solitary wave. The stochastic model appears to be more realistic, and one can observe, for small values of ε, a similar evolution phenomena about the solution as that given by the deterministic case. However, an explosion of the solution and a blow-up phenomena can be noted as ε becomes bigger.
Key words: Nonlinear Schrödinger equation, Mixed nonlinearity, Blow-up phenomena, Finite difference scheme, White noise, Solvability.

1. Introduction

In this work, we are interested in the study of the one-dimensional stochastic nonlinear Schrödinger (NLS) equation with both subcritical and supercritical power nonlinearities in the presence of an additive noise. The resulting equation is a random perturbation of the dynamical system of the following form

$$\begin{cases} iu_t + u_{xx} + g_\alpha(u) + f_\varepsilon(u) = 0, t \geq 0, L_2 \geq x \geq L_1, \\ u(0,x) = u_0(x), L_1 \leq x \leq L_2, \\ u_x(t, L_1) = u_x(t, L_2) = 0, t \geq 0, \end{cases} \tag{1}$$

where $u = u(t,x)$ is a complex-valued function defined for $t \geq 0$ and $x \in \mathbb{R}$, α is a positive real parameter, L_1 and L_2 are reals such that $L_1 < L_2$, u_t is the derivative of u with respect to the time t and u_{xx} is its second derivative wiht respect to the position x. The function g_α is defined by

$$g_\alpha(u) = |u|^{p-1}u + \alpha|u|^{q-1}u.$$

The term $f_\varepsilon(u)$ includes the stochastic contribution. In our case, we are concerned with an additive noise. This means that it will be considered to be real-valued, Gaussian, white in time and either white or correlated in space. As an immediate consequence, the noise does not depend on the solution. The size of the noise is controlled by the parameter $\varepsilon > 0$.

The deterministic equation occurs as a basic model in many areas of physics, hydrodynamics, plasma physics, nonlinear optics, molecular biology. It describes the propagation of waves in media with both nonlinear and dispersive responses. It took the interest of some researchers like Ben Mabrouk et al. [3], Bratsos [4], Keraani [12] and Sulem and Sulem [15]. It is an idealized model and does not take into account many aspects such as in-homogeneities, high

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order terms, thermal fluctuations and external forces, which may be modeled as random excitations like it was the case in the works of Cheung and Mosincat [5], Falkovich et al. [9], Farlano et al. [10] and Oh et al. [13].

Here, we particularly treat the influence of a noise acting as a potential on the behavior of the deterministic solution. Such effects on solitary waves have already been studied for the NLS equation and for also for the Korteweg-de-Vries equation (see, for example, Printems [14]). This kind of noise has been considered by Garnier in [11], where the paths were smooth functions and the nonlinearity was subcritical. In the case of a white noise, this type of model has been introduced in the context of crystals by Bang et al. [1]. It is expected that such a noise has a strong influence on the solutions which blow-up. It may delay or even prevent the formation of a singularity. It has been shown in [6], by Debussche and Di Menza, via numerical simulations, that this is the case for a very irregular noise: for a space-time white noise. However, in the supercritical case and for a noise which is correlated in space but non degenerate, it has been observed, on the contrary, that any solution seems to blow-up in a finite time. We recall that in the deterministic case, only a restricted class of solutions blow-up.

The case of an additive noise has been considered in [7] by De Bouard and Debussche. It has been proved that for any initial data, blow-up occurs in the sense that, for arbitrary $t > 0$, the probability that the solution blows up before the time t is strictly positive. Thus, the noise strongly influences this blow-up phenomenon. In the present paper, the result is in perfect agreement with the numerical simulations. It represents, between others, a generalization of the results established by Ben Mabrouk et al. in [3] and by Debussche and Di Menza in [6].

The paper is organized as follows: In Section 2, we give a precise mathematical definition of the additive white noise and transform the continuous problem (1) to a discrete algebraic one. Then, we study the solvability of the difference scheme. The next section is devoted to the convergence of this scheme. Some numerical implementations are given in section 5 to validate the scheme. The paper is ended by a recapitulative conclusion.

2. Discretization of the stochastic Schrödinger equation

In this section, we start by giving a mathematical definition for an additive white noise. We follow the approach taken by Debussche and DiMenza in [6]. We consider a probability space $(\Omega, \mathcal{F}, \mathrm{P})$, endowed with a filtration $\big(\mathcal{F}(t)\big)_{t\geq 0}$. We also consider a cylindrical Wiener process $\big(W(t)\big)_{t\geq 0}$ on $L^2(\mathbb{R})$, which is adapted to this filtration. Then, we have

$$W(t,x,w) = \sum_{i=0}^{\infty} \beta_i(t,w) e_i(x) ,$$

where $(e_i)_{i\in\mathbb{N}}$ is an orthogonal basis of $L^2(\mathbb{R})$ and $(\beta_i)_{i\in\mathbb{N}}$ a sequence of independent real valued Brownian motions on $\mathbb{R}^+$, associated to $\big(\mathcal{F}(t)\big)_{t\geq 0}$. The white noise is the time derivative of W. This makes that the stochastic forcing term will be written in the following form

$$f_\varepsilon(u) = \varepsilon\dot{\chi} = \frac{\partial W}{\partial t} .$$

More details and generalizations of these notations can be found in [] and the references therein. Taking in account these notations, we then rewrite the first equation in (1) as follows

$$iu_t + u_{xx} + g_\alpha(u) = \varepsilon\dot{\chi}, t \geq 0, L_2 \geq x \geq L_1$$

Now, we are in position to establish the finite difference scheme corresponding to the problem (1). We consider a time step $l = \Delta t$ and denote the time discretization by

$$t^k = kl = \mathrm{k}\,(t^{k+1} - t^k).$$

We fix an integer M and consider, a space step

$$h = \Delta x = \frac{L_2 - L_1}{M+1} .$$

Then, we subdivide the interval $[L_1, L_2]$ into subintervals $[x_m, x_{m+1}]$, where

$$x_m = L_0 + mh, m \in \{0, \dots, M+1\}$$

This allows to consider the space grid

$$\Omega_h = \{x_m = L_0 + mh, m \in \{0, \dots, M+1\}\},$$

We, also, consider the space W_h of functions defined on Ω_h, and vanishing at zero. It is endowed with the inner product, defined for any given vectors

$$U = (U_0, U_1, \dots, U_{M+1})^t \text{ and } V = (V_0, V_1, \dots, V_{M+1})^t$$

of $\mathbb{R}^{M+2}$, by

$$< U, V >_h = h \sum_{m=0}^{M+1} U_m V_m ,$$

and the associated L^2-norm

$$||U||_{h,2} = (U, U)_h^{1/2} = \left(h \sum_{M=0}^{M+1} U_m \right)^{1/2}.$$

We denote by u_m^k the approximation of $u(t^k, x_m)$ and by U_m^k the numerical solution. We introduce the following notations

$$\delta_m^k U = \frac{U_m^{k+1} - U_m^{k-1}}{2l},$$

$$\Delta_m^k U = \frac{U_{m+1}^k - 2U_m^k + U_{m-1}^k}{h^2},$$

$$(U_t)_m^k = \lambda \delta_{m-1}^k U + (1 - 2\lambda)\delta_m^k U + \lambda \delta_{m+1}^k U,$$

$$(U_x)_m^k = \frac{U_{m+1}^k - U_{m-1}^k}{2h},$$

$$(U_{xx})_m^k = \mu \Delta_m^{k+1} U + (1 - 2\mu)\Delta_m^k U + \mu \Delta_m^{k-1} U.$$

We then discretize the problem (1) as follows

$$i(U_t)_m^k + (U_{xx})_m^k + g_\alpha\left(U_m^k\right) =$$

$$= \varepsilon f_m^{k+\frac{1}{2}}, \text{ m} = 0, \dots, \text{M+1}.$$

Since we treat an additive noise, we have

$$f_m^{k+\frac{1}{2}} = \frac{1}{l\sqrt{h}}\left(\beta_m(t_{k+1}) - \beta_m(t_k)\right),$$

$$\text{m} = 1, \dots, \text{M},$$

$$f_0^{k+\frac{1}{2}} = \frac{\sqrt{2}}{l\sqrt{h}}\left(\beta_0(t_{k+1}) - \beta_0(t_k)\right),$$

$$f_{M+1}^{k+\frac{1}{2}} = \frac{\sqrt{2}}{l\sqrt{h}}\left(\beta_M(t_{k+1}) - \beta_M(t_k)\right).$$

Moreover, as the random variables

$$\frac{1}{l}\left(\beta_m(t_{k+1}) - \beta_m(t_k)\right), \text{ k} \geq 0, \text{ m} = 0, \dots, \text{M+1}$$

are independent with normal law $\mathcal{N}$ (0,1), we can choose the

$$\left(\chi_m^{k+\frac{1}{2}}\right), \text{ k} \geq 0, \text{ m} = 0, \dots, \text{M+1}$$

to be a sequence of independent random variables with normal law $\mathcal{N}$ (0, 1). The numerical problem is considered under the initial data

$$\begin{cases} U_m^0 = u(0, x_m) = u_0(x_m), 0 \leq m \leq M+1, \\ U_m^1 = U_m^0 + il\left(u_0''(x_m) + g_\alpha\left(u_0(x_m)\right)\right), 0 \leq m \leq M+1, \\ U_1^k = U_0^k \text{ and } U_M^k = U_{M+1}^k, k \geq 0. \end{cases} \quad (2)$$

We consider the approximation

$$h_\alpha\left(U_m^k\right) = \left(v_1 U_m^k + (1 - v_1)U_m^{k-1}\right)\tilde{h}_\alpha, \quad v_1 \in [0,1],$$

where

$$\tilde{h}_\alpha = \max_{0 \leq m \leq M+1}\{|U_m^0|^{p-1} + \alpha\, |U_m^0|^{q-1}\}.$$

Next, we denote by σ the report

$$\sigma = \frac{l}{h^2}$$

and take the following notations,

$$a_1 = 2\mu\sigma + i\lambda,$$

$$a_2 = -4\mu\sigma + (1-2\lambda)i,$$

$$b_1 = -2\sigma(1-2\mu),$$

$$b_2 = 4\sigma(1-2\mu) - 2v_1 l\, \tilde{h}_\alpha\,,$$

$$c_1 = -2\mu\sigma + i\lambda,$$

$$c_2 = 4\mu\sigma + i(1-2\lambda) - 2(1-v_1)\, l\, \tilde{h}_\alpha\,,$$

This leads, for $1 \leq m \leq M$, to

$$a_1(U_{m-1}^{k+1} + U_{m+1}^{k+1}) + a_2 U_m^{k+1} =$$

$$= b_1\,(U_{m-1}^{k} + U_{m+1}^{k}) + b_2 U_m^k +$$

$$+c_1(U_{m-1}^{k-1} + U_{m+1}^{k-1}) + c_2 U_m^{k-1} + \varepsilon f_m^{k+\frac{1}{2}} \qquad (3)$$

The boundary conditions are expressed as follows

$$(a_1 + a_2)\, U_0^{k+1} + a_1 U_1^{k+1} =$$

$$= (b_1 + b_2)\, U_0^k + b_1 U_1^k +$$

$$+(c_1 + c_2)\, U_0^{k-1} + c_1 U_1^{k-1} + \varepsilon f_0^{k+\frac{1}{2}} \qquad (4)$$

and

$$a_1\, U_{M-1}^{k+1} + (a_1 + a_2)\, U_M^{k+1} =$$

$$= b_1\, U_{M-1}^{k+1} + (b_1 + b_2)\, U_M^{k+1} +$$

$$+c_1\, U_{M-1}^{k+1} + (c_1 + c_2)\, U_M^{k+1} + \varepsilon f_{M+1}^{k+\frac{1}{2}}. \qquad (5)$$

3. Solvability of the difference scheme

To prove the solvability of the dufference scheme, we need to write the problem (3)-(5) in its matrix form, i.e.,

$$AU^{k+1} = BU^k + CU^{k-1} + F, \qquad (6)$$

where A is the $(N+2)^2$-matrix defined as follows

$$A = \begin{pmatrix} a_1 + a_2 & a_1 & 0 & \cdots & \cdots & 0 \\ a_1 & a_2 & a_1 & & \ddots\ddots & \vdots \\ 0 & \ddots & \ddots & & \ddots\ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & a_1 & a_2 & a_1 \\ 0 & \cdots & \cdots & 0 & a_1 & a_1 + a_2 \end{pmatrix}.$$

B is also an $(M+2)^2$-matrix. It is obtained by replacing a_1 and a_2 respectively by b_1 and b_2 in the matrix A. Similarly, C is the $(M+2)^2$-matrix obtained by replacing respectively a_1 and a_2 by b_1 and b_2 in A. Finally, the matrix F represents the white noise vector. It is expressed as follows

$$F = \begin{pmatrix} f_0^{k+\frac{1}{2}} \\ \vdots \\ f_m^{k+\frac{1}{2}} \\ \vdots \\ f_{N+1}^{k+\frac{1}{2}} \end{pmatrix}.$$

The solvability of the difference scheme (3)-(5) is related to the determinant of the matrix A. This is based on a result developed by El-Mikkawy and Karawia in [8] and treating the invertibility of a general tri-diagonal matrix. We recall the basic result in what follows,

Lemma 1 *[8] Consider the following real matrix A,*

$$E = \begin{pmatrix} d_1 & y_1 & 0 & \cdots & \cdots & 0 \\ z_2 & d_2 & y_2 & & \ddots\ddots & \vdots \\ 0 & \ddots & \ddots & & \ddots\ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & z_{M-1} & d_{M-1} & y_{M-1} \\ 0 & \cdots & \cdots & 0 & z_M & d_M \end{pmatrix}$$

and define the real vector

$$\tau = (\tau_0, \tau_1, \ldots, \tau_n)^t$$

as follows

$$\tau_j = \begin{cases} 1 \; if\; j = 0, \\ d_1 \; if\; j = 1, \\ d_j \tau_{j-1} - z_j y_{j-1} \tau_{j-2} \; if\; j = 2, 3, \ldots, n. \end{cases} \qquad (7)$$

Then, there holds

$$Det(E) = \tau_n.$$

Now, we are in position to state the main result of this section.

Theorem 1 *The difference scheme (3)-(5) is uniquely solvable.*

Proof Denote by $Det_{M+2}(A)$ the determinant of the matrix A. Then, we have the following recursive equation

$$Det_{M+2}(A) - \\ -(a_1 + a_2)\, Det_{M+1}(A) + a_1^2\, Det_N(A) = 0.$$

Thanks to Lemma 1, we deduce $Det_{M+2}(A)$ in three cases.

First case: $\mu = 0$ and $\lambda = \frac{1}{3}$: Standard computations yield,

$$Det_{M+2}(A) = (M+3)\left(\frac{i}{3}\right)^{M+2} \neq 0.$$

Second case: $\mu = 0$ and $\lambda \neq \frac{1}{3}$. We denote by

$$\delta = \sqrt{1 - 2\lambda - 3\lambda^2},$$

$$X_1 = \frac{i}{2}(1 - \lambda + \delta),$$

$$X_2 = \frac{i}{2}(1 - \lambda - \delta),$$

$$C_1 = -\frac{1-\lambda}{\delta} + \frac{iX_2(-2 + 5\lambda - 2\lambda^2)}{\delta\lambda^2},$$

$$C_2 = -\frac{1-\lambda}{\delta} + \frac{iX_1(-2 + 5\lambda - 2\lambda^2)}{\delta\lambda^2}.$$

Then, we have

$$Det_{M+2}(A) = C_1 {X_1}^{M+2} + C_2 {X_2}^{M+2} \neq 0 \quad (8)$$

Third case: $\mu \neq 0$. We consider the following complex values,

$$\delta = \sqrt{(a_1 + a_2)^2 - 4{a_1}^2},$$

$$X_1 = \frac{1}{2}(a_1 + a_2 + \delta),$$

$$X_2 = \frac{1}{2}(a_1 + a_2 - \delta).$$

Computations similar to the second case lead also to the equation (8).

It follows that the system (3)-(5) is uniquely solvable.

4. Convergence of the difference scheme

The main result of this section can be stated as follows,

Theorem 2 *Suppose that $l = o(h^2)$, is small enough. Then, the difference scheme (3)-(5) is convergent.*

Proof We set,

$$X = e^{i\psi} \text{ and } Z = e^{i\theta},$$

$$\theta \in \mathbb{R}, \psi \in \mathbb{C},$$

and write

$$U_m^k = e^{ik\psi} e^{im\theta} = X^k Z^m,$$

$$k \geq 0,\ m = 0, \dots, M+1.$$

By replacing, first in (3), we obtain, for k ≥ 1 and $1 \leq m \leq M$,

$$A_m(Z) X^{k+1} + B_m(Z) X^k + \\ + C_m(Z) X^{k-1} - \varepsilon f_m^{k+\frac{1}{2}} = 0, \quad (8)$$

where

$$\begin{cases} A_m(Z) = \left[(2\mu\sigma + i\lambda)(1 + Z^2) + \left(-4\mu\sigma + i(1 - 2\lambda)\right) Z\right] Z^{m-1}, \\ B_m(Z) = \left[-2\sigma(1 - 2\mu)(1 + Z^2) + \left(4\sigma(1 - 2\mu) - 2\nu l \tilde{h}_\alpha\right) Z\right] Z^{m-1}, \\ C_m(Z) = \left[(-2\mu\sigma + i\lambda)(1 + Z^2) + \left(4\mu\sigma + i(1 - 2\lambda) - 2(1 - \nu_1) l \tilde{h}_\alpha\right) Z\right] Z^{m-1}. \end{cases}$$

Then, replacing in (4), we obtain for k ≥ 1 and $m = 0$,

$$A_0(Z)X^{k+1} + B_0(Z)X^k + C_0(Z)X^{k-1} - \varepsilon f_0^{k+\frac{1}{2}} = 0,$$

with

$$\begin{cases} A_0(Z) = -2\mu\sigma + i(1-\lambda) + (2\mu\sigma + i\lambda)\, Z, \\ B_0(Z) = 2\sigma(1-2\mu) - 2v_1 l\tilde{h}_\alpha - 2\sigma(1-2\mu)\, Z, \\ C_0(Z) = 2\mu\sigma + i(1-\lambda) - 2(1-v_1)l\tilde{h}_\alpha + (-2\mu\sigma + i\lambda)\, Z. \end{cases}$$

Finally, replacing in (5), we obtain for k ≥ 1 and $m = M + 1$,

$$A_{M+1}(Z)X^{k+1} + B_{M+1}(Z)X^k + C_{M+1}(Z)X^{k-1} - \varepsilon f_{M+1}^{k+\frac{1}{2}} = 0,$$

where

$$\begin{cases} A_{M+1}(Z) = \left[2\mu\sigma + i\lambda + \left(-2\mu\sigma + i(1-\lambda)\right) Z\right] Z^M, \\ B_{M+1}(Z) = \left[-2\sigma(1-2\mu) + \left(2\sigma(1-2\mu) - 2v_1 l\tilde{h}_\alpha\right) Z\right] Z^M, \\ C_{M+1}(Z) = \left[-2\mu\sigma + i\lambda + \left(2\mu\sigma + i(1-\lambda) - 2(1-v_1)l\tilde{h}_\alpha\right) Z\right] Z^M. \end{cases}$$

When ψ is real, it is obvious from the equation (9) that U_m^k remains bounded. Otherwise, a sufficient condition for the convergence of the scheme is that

$$\left|e^{i\psi}\right| \leq 1. \quad (9)$$

In that case, one has

$$\left|e^{i\psi}\right| = |X| \leq \min\left\{\frac{|B_m|}{|A_m|}, 0 \leq m \leq M+1\right\}. \quad (10)$$

Taking $m = 0$, supposing that $l = o(h^2)$ and following the calculations given by Ben Mabrouk et al. in [2], the equations (9) and (10) lead to,

$$|B_0|^2 - |A_0|^2 \leq 0,$$

and the result follows.

5. Numerical implementations

We want to investigate the noise effects on stationary solutions in a concrete situation. We recall that the deterministic solutions take the following form

$$u(x,t) = \sqrt{\frac{2a}{q_s}} \exp\left(i\left(\frac{1}{2}cx - \theta t + \varphi\right) + sech\left(\sqrt{a}(x - ct) + \phi\right)\right).$$

where a, q_s, $\theta = \frac{c^2}{4} - a$, φ and ϕ are appropriate constants. It is a soliton-type disturbance which travals with speed c and with a governed amplitude.

In the treated example, the time and space partial derivative parameters are fixed to the particular case where

$$\lambda = \frac{1}{5} \; and \; \mu = \frac{1}{3}.$$

For the numerical scheme (3)-(5), the computations are done in the space domain $[L_1, L_2]$, with $L_1 = -80$ and $L_2 = 100$. The space step was $h = 1$. The considered time interval was $[0, 10]$, with a time step $l = 0.01$. The soliton parameters were fixed as follows,

$$a = 0.01,\ q_s = 1 \text{ and } c = 0.1$$

and the phase parameters

$$\phi = \varphi = 0.$$

For the nonlinearity, we took the values

$$q = 0.73, p = 1.5 \text{ and } v_1 = 0.5.$$

It was numerically proved that the asymptotic limit of the solution $u(\varepsilon)$ of the problem (1), as ε goes to 0, is in fact, the stationary wave $u(0)$, which corresponds to the the deterministic case (see Figure 1), and physically interpreted by the absence of noise.

For small amplitudes of the noise, corresponding to small values of the parameter ε, we can see that the solitary wave is not strongly perturbed and that the noise does not prevent its propagation. This is clearly expressed in Figure 2, where the values $\varepsilon = 0.1$, $\varepsilon = 0.05$ *and* $\varepsilon = 0.04$ were respectively drawn in the parts (a), (b) and (c) of this figure.

However, as the noise level becomes higher, the wave is progressively destroyed. This is the subject of Figure 3, in which the values $\varepsilon = 0.1$, $\varepsilon = 0.15$ *and* $\varepsilon = 0.25$ correspond respectively to the parts (a), (b) and (c).

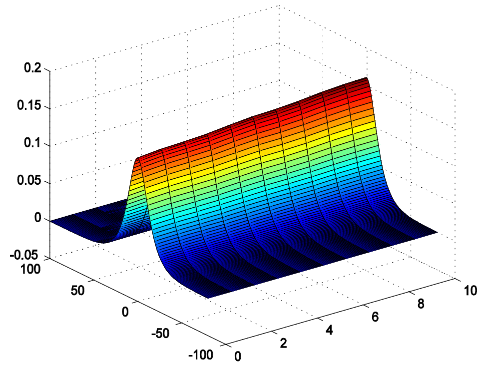
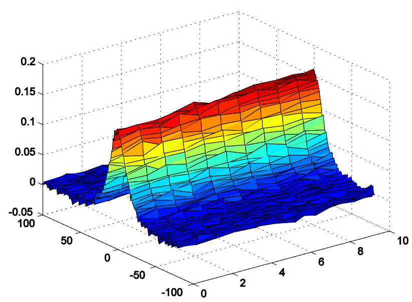
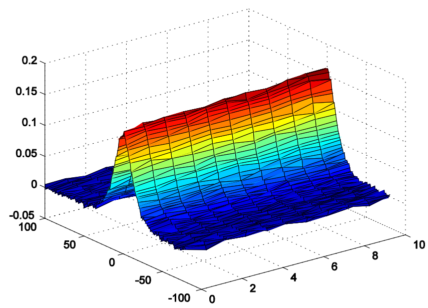


Figure 1 – Plots in the (t,x)-plane of the stationary wave corresponding to the deterministic case: $\varepsilon = 0$

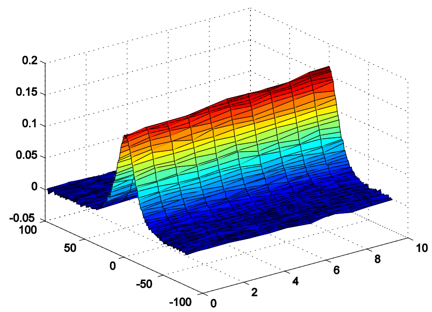
Now, taking the amplitude ε of the noise greater than 0.3, it is clearly seen that the wave explodes under the influence of the additive noise. This blow-up phenomenon appears in Figure 4, (a) and (b), respectively for $\varepsilon = 0.45$ and $\varepsilon = 0.35$.



(a)

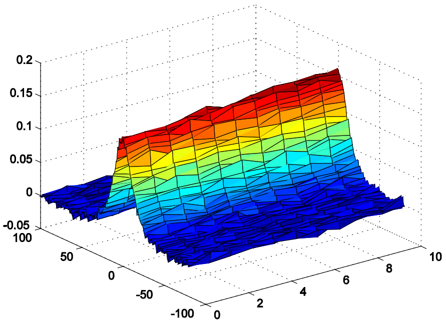


(b)

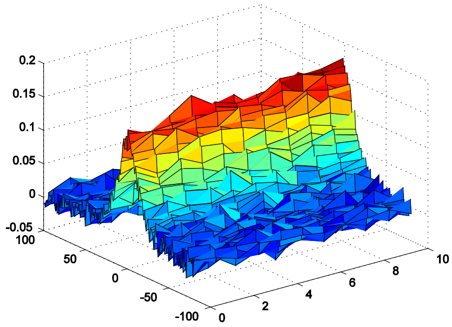


(c)

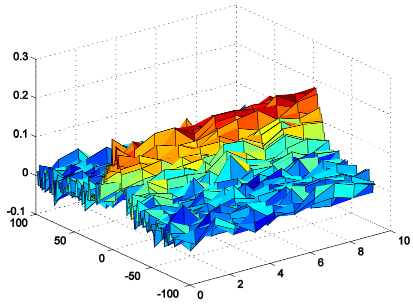
Figure 2 – Plots in the (t,x)-plane of |u| for one trajectory for small values of the amlitude of the noise.
(a) $\varepsilon = 0.1$, (b) $\varepsilon = 0.05$, (c) $\varepsilon = 0.04$



(c)

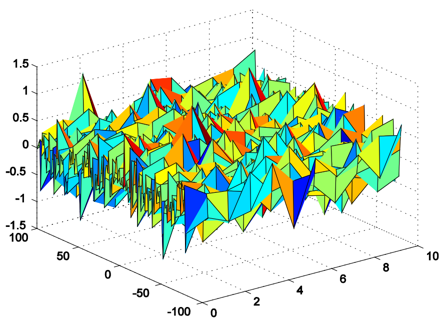


(d)

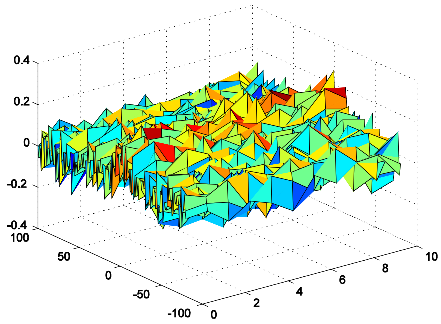


(c)

Figure 3 – Progressive destruction of the stationary wave,
plotted in the (t,x)-plane, as the amplitude of the noise becomes bigger.
(a) $\varepsilon = 0.1$, (b) $\varepsilon = 0.15$, (c) $\varepsilon = 0.25$



(a)



(b)

Figure 4 – Explosion of the wave, plotted in the (t,x)-plane,
under the effect of big values of the noise.
(a) $\varepsilon = 0.45$, (b) $\varepsilon = 0.35$.

6. Conclusion

It is noted that the stochastic nonlinear equation (1) can be considered as an additive white noise random perturbation of the deterministic equation, defined for $\varepsilon = 0$. Such a perturbation occurs when the size of the noise, described by the real-value parameter ε, is positive. We proved that as ε approaches zero, the solution of the perturbed problem converges to the unique trajectory of the deterministic equation, which is the solitary wave. The stochastic model appears to be more realistic, and one can observe, for small values of ε, a similar evolution phenomena about the solution as that given by the deterministic case. However, an explosion of the solution and a blow-up phenomena can be noted as ε becomes bigger.

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