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Soft intersection almost subsemigroups of semigroups

Abstract. Semigroups are the building blocks of algebra as they have application in automata, coding theory, formal languages, and theoretical computer science. They are also used in the solutions of graph theory and optimization theory. For the advanced study of algebraic structures and their applications, ideals are essential. The generalization of ideals in algebraic structures is necessary for further research on algebraic structures. The main purpose of this paper is to present the notion of soft intersection almost subsemigroup of a semigroup, which is a generalization of soft intersection subsemigroup and investigate its basic properties in detail. In this context, we also obtain many striking relationships between almost subsemigroups and soft intersection almost subsemigroups concerning minimality, primeness, semiprimeness and strongly primeness.
Key words: semigroups, soft intersection, algebraic structures, almost subsemigroup of a semigroup, graph theory, optimization theory, minimality, simplicity, semi-simplicity and strong simplicity.

I. Introductıon

Semigroups are the building blocks of algebra as they have applications in automata, coding theory, formal languages, and theoretical computer science. They are also used in the solutions of graph theory and optimization theory. For the advanced study of algebraic structures and their applications, ideals are essential. The generalization of ideals in algebraic structures is necessary for further research on algebraic structures. Many mathematicians introduced different expansions of the notion of ideals in algebraic structures, demonstrating important results and characterizing algebraic structures. Grosek and Satko [1] introduced the concept of almost left, right, and two-sided ideals of semigroups for the first time in 1980. Later in 1981, Bogdanovic [2] proposed the notion of almost bi-ideals in semigroups as an extension of bi-ideals. Using the concepts of semigroup quasi-ideals and almost ideals, Wattanatripop et al. (2018) introduced the idea of almost quasi-ideals. Kaopusek et al. [4], using the concepts of almost ideals and interior ideals of semigroups, proposed the concepts of almost interior ideals and weakly almost interior ideals of semigroups and investigated their characteristics in 2020. The concepts of almost bi-interior ideals of semigroups, almost subsemigroups of semigroups, and almost bi-quasi-interior ideals of semigroups were first presented by Iampan et al. [5] in 2021, Chinram and Nakkhasen [6] in 2022, and Gaketem [7] in 2022, respectively. In addition, various forms of fuzzification of almost ideals were studied in [3, 5-9].

To model uncertainty, Molodtsov [10] introduced the concept of soft set in 1999. Soft set is defined as a function from the parameter set E to the power set of U. Since then, scholars from a wide range of domains have become interested in soft sets. Soft set operations, which form the foundation of the theory, are examined in [11–26]. The definition of soft set and soft set operations were modified by Çağman and Enginoğlu [27]. Furthermore, Çağman et al. [28] developed the idea of soft intersection groups, which served as inspiration for a variety of soft algebraic structures. The idea of soft intersection substructures of semigroups originated with the use of soft sets in semigroups. Soft intersection subsemigroups, left (right/two-sided ideals), (generalized) bi-ideals, interior ideals, and quasi-ideals of semigroups were developed and explored by Sezer et al. [29, 30]. Sezgin and Orbay [31] characterized semisimple semigroups, duo semigroups, right(left) zero semigroups, right(left) simple semigroups, semilattice of left(right) simple semigroups, semilattice of left(right) groups, and semilattice of groups. Lately, Rao [32–35] presented a variety of new semigroup ideals, including weak-interior ideals, bi-quasi-interior ideals, bi-interior ideals, and quasi-interior ideals. Baupradist [36] established the essential semigroup ideals. A variety of algebraic structures, including soft sets, were examined in [37–46] which can be handled as regards graph applications together with network analysis with the inspiration of divisibility of determinants as in [47].

In this study, we introduced the notion of soft intersection almost subsemigroup of a semigroup, which is a generalization of the nonnull soft intersection subsemigroup of a semigroup. We obtain that the union of soft intersection almost subsemigroups is again soft intersection almost subsemigroup; but their intersection is not. Moreover, we revealed the relation between almost subsemigrops and soft intersection almost subsemigroups of a semigroup with respect to minimality, primeness, semiprimeness, and strongly primeness.

II. PRELIMINARIES

In this paper, we give some fundamental notions related to semigroups and soft sets.

Definition 2.1. Let U be the universal set, E be the parameter set, $P(U)$ be the power set of U, and $K \subseteq E$. A soft set f_K over U is a set-valued function such that $f_K: E \to P(U)$ such that for all $x \notin K$, $f_K(x) = \emptyset$. A soft set over U can be represented by the set of ordered pairs

$$f_K = \{(x, f_K(x)) : x \in E, f_K(x) \in P(U)\}$$

[10,27]. Throughout this paper, the set of all the soft sets over U is designated by $S_E(U)$.

Definition 2.2. Let $f_A \in S_E(U)$. If $f_A(x) = \emptyset$ for all $x \in E$, then f_A is called a null soft set and denoted by $\emptyset_E$. If $f_A(x) = U$ for all $x \in E$, then f_A is called an absolute soft set and denoted by U_E [27].

Definition 2.3. Let $f_A, f_B \in S_E(U)$. If for all $x \in E$, $f_A(x) \subseteq f_B(x)$, then f_A is a soft subset of f_B and denoted by $f_A \tilde{\subseteq} f_B$. If $f_A(x) = f_B(x)$ for all $x \in E$, then f_A is called soft equal to f_B and denoted by $f_A = f_B$ [27].

Definition 2.4. Let $f_A, f_B \in S_E(U)$. The union of f_A and f_B is the soft set $f_A \tilde{\cup} f_B$, where $(f_A \tilde{\cup} f_B)(x) = f_A(x) \cup f_B(x)$ for all $x \in E$. The intersection of f_A and f_B is the soft set $f_A \tilde{\cap} f_B$, where $(f_A \tilde{\cap} f_B)(x) = f_A(x) \cap f_B(x)$ for all $x \in E$ [27].

Definition 2.5. For a soft set f_A, the support of f_A is defined by

$$supp(f_A) = \{x \in A: f_A(x) \neq \emptyset\} \text{ [15].}$$

It is obvious that a soft set with an empty support is a null soft set, otherwise the soft set is nonnull.

Note 2.6. If $f_A \tilde{\subseteq} f_B$, then $supp(f_A) \subseteq supp(f_B)$.

A semigroup S is a nonempty set with an associative binary operation. Throughout this paper, S denotes a semigroup and all the soft sets are the elements of $S_S(U)$. A nonempty subset K of S is called a subsemigroup of S if $KK \subseteq K$; and is called an almost subsemigroup (briefly almost SS) if $K^2 \cap K \neq \emptyset$ $(KK \cap K \neq \emptyset)$. Let A and B be any almost subsemigoups of S such that $B \subseteq A$. If $A = B$, then A is a minimal almost subsemigroup (briefly minimal almost SS) of S. An almost subsemigroup P of S is called a prime almost subsemigroup (briefly prime almost SS) if for any almost subsemigroup A and B of S such that $AB \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$. An almost subsemigroup $P$of S is called a semiprime almost subsemigroup (briefly semiprime almost SS) if for any almost subsemigroup A of S such that $AA \subseteq P$ implies that $A \subseteq P$. An almost subsemigroup P of S is called a strongly prime almost subsemigroup (briefly strongly prime almost SS) if for any almost subsemigroups A and B of S such that $AB \cap BA \subseteq P$ implies that $A \subseteq P$ or $B \subseteq P$.

Definition 2.7. Let f_S and g_S be soft sets over the common universe U. Then, soft intersection product $f_S \circ g_S$ is defined by [29]

$$(f_S \circ g_S)(m) = \begin{cases} \bigcup_{m=nr} \{f_S(n) \cap g_S(r)\}, if\ \exists n, r \in S\ such\ that\ m = nr \\ \emptyset, \quad otherwise \end{cases}$$

Theorem 2.8. Let $f_S, g_S, h_S \in S_S(U)$. Then,

i) $(f_S \circ g_S) \circ h_S = f_S \circ (g_S \circ h_S)$.

ii) $f_S \circ g_S \neq g_S \circ f_S, generally.$

iii) $f_S \circ (g_S \tilde{\cup} h_S) = (f_S \circ g_S) \tilde{\cup} (f_S \circ h_S)$ and $(f_S \tilde{\cup} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cup} (g_S \circ h_S)$.

iv) $f_S \circ (g_S \tilde{\cap} h_S) = (f_S \circ g_S) \tilde{\cap} (f_S \circ h_S)$ and $(f_S \tilde{\cap} g_S) \circ h_S = (f_S \circ h_S) \tilde{\cap} (g_S \circ h_S)$.

v) If $f_S \tilde{\subseteq} g_S$, then $f_S \circ h_S \tilde{\subseteq} g_S \circ h_S$ and $h_S \circ f_S \tilde{\subseteq} h_S \circ g_S$.

vi) If t_S, $k_S \in S_S(U)$ such that $t_S \tilde{\subseteq} f_S$ and $k_S \tilde{\subseteq} g_S$, then $t_S \circ k_S \tilde{\subseteq} f_S \circ g_S$ [29].

Lemma 2.9. Let f_S and g_S be soft sets over U. Then, $f_S \circ f_S = \emptyset_S \Leftrightarrow f_S = \emptyset_S$.

Definition 2.10. Let A be a subset of S. We denote by S_A the soft characteristic function of A and define as

$$S_A(x) = \begin{cases} U, if\ x \in A \\ \emptyset, if\ x \in S\backslash A \end{cases}$$

The soft characteristic function of A is a soft set over U, that is, $S_A: S \longrightarrow P(U)$ [29].

Corollary 2.11. $supp(S_A) = A$.

Theorem 2.12. Let X and Y be nonempty subsets of S. Then, the following properties hold [29]:

i) $X \subseteq Y$ if and only if $S_X \tilde{\subseteq} S_Y$

ii) $S_X \tilde{\cap} S_Y = S_{X\cap Y}$ and $S_X \tilde{\cup} S_Y = S_{X\cup Y}$

iii) $S_X \circ S_Y = S_{XY}$

Proof: In [29], (i) is given as if $X \subseteq Y$, then if $S_X \tilde{\subseteq} S_Y$. Now, we also show that if $S_X \tilde{\subseteq} S_Y$, then $X \subseteq Y$. Let $S_X \tilde{\subseteq} S_Y$ and $x \in X$. Then, $S_X(x) = U$ and this implies that $S_Y(x) = U$ since $S_X \tilde{\subseteq} S_Y$. Hence, $x \in Y$ and so $X \subseteq Y$. Now let $x \notin Y$. Then, $S_Y(x) = \emptyset$, and this implies that $S_X(x) = \emptyset$ since $S_X \tilde{\subseteq} S_Y$. Hence, $x \notin X$ and so $Y' \subseteq X'$, implying that $X \subseteq Y$.

Definition 2.13. A soft set over U is called a soft intersection subsemigroup of S over U if $f_S(xy) \supseteq f_S(x) \cap f_S(y)$ for all $x, y \in S$ [29].

Here note that in [29], the definition of "soft intersection subsemigroup of S" is given as "soft intersection semigroup of S"; however in this paper, without loss of generality, we prefer to use "soft intersection subsemigroup" by the abbreviation of SI-SS [29].

Theorem 2.14. Let f_S be a soft set over U. Then, f_S is an SI-SS if and only if $f_S \circ f_S \tilde{\subseteq} f_S$ [29].

III. Soft Intersection Almost Subsemigroups of Semigroups

Definition 3.1. Let f_S be a soft set over U. f_S is called soft intersection almost subsemigroup of S if

$$(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S.$$

Hereafter, soft intersection almost subsemigroup is denoted by SI-almost SS for brevity.

Example 3.2. Let $S = \{\mathscr{s}, \mathscr{n}, \mathscr{v}, \mathscr{j}\}$ be the semigroup with the following Cayley Table.

	$\mathscr{s}$	$\mathscr{n}$	$\mathscr{v}$	$\mathscr{j}$
$\mathscr{s}$	$\mathscr{s}$	$\mathscr{n}$	$\mathscr{v}$	$\mathscr{j}$
$\mathscr{n}$	$\mathscr{n}$	$\mathscr{s}$	$\mathscr{v}$	$\mathscr{j}$
$\mathscr{v}$	$\mathscr{j}$	$\mathscr{v}$	$\mathscr{j}$	$\mathscr{v}$
$\mathscr{j}$	$\mathscr{v}$	$\mathscr{j}$	$\mathscr{v}$	$\mathscr{j}$

Let f_S, g_S, and h_S be soft sets over $U = \mathbb{N}$ as follows:

$f_S = \{(\mathscr{s}, \{0,1,2\}), (\mathscr{n}, \{1,5\}), (\mathscr{v}, \{2,4,6,8\}), (\mathscr{j}, \{7,9\})\}$,

$g_S = \{(\mathscr{s}, \{3,5,7\}), (\mathscr{n}, \{3,6,9\}), (\mathscr{v}, \{12,13\}), (\mathscr{j}, \{2,8\})\}$,

$h_S = \{(\mathscr{s}, \emptyset), (\mathscr{n}, \{0\}), (\mathscr{v}, \{1\}), (\mathscr{j}, \emptyset)\}$.

Here, f_S and g_S are SI-almost SSs. Let's show that $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$.

$[(f_S \circ f_S) \tilde{\cap} f_S](\mathscr{s}) = (f_S \circ f_S)(\mathscr{s}) \cap f_S(\mathscr{s}) = \left[\left(f_S(\mathscr{s}) \cap f_S(\mathscr{s})\right) \cup \left(f_S(\mathscr{n}) \cap f_S(\mathscr{n})\right)\right] \cap f_S(\mathscr{s}) = \{0,1,2\}$

$[(f_S \circ f_S) \tilde{\cap} f_S](\mathscr{n}) = (f_S \circ f_S)(\mathscr{n}) \cap f_S(\mathscr{n}) = \left[\left(f_S(\mathscr{s}) \cap f_S(\mathscr{n})\right) \cup \left(f_S(\mathscr{n}) \cap f_S(\mathscr{s})\right)\right] \cap f_S(\mathscr{n}) = \{1\}$

$[(f_S \circ f_S) \tilde{\cap} f_S](\mathscr{v}) = (f_S \circ f_S)(\mathscr{v}) \cap f_S(\mathscr{v}) = \left[\left(f_S(\mathscr{s}) \cap f_S(\mathscr{v})\right) \cup \left(f_S(\mathscr{n}) \cap f_S(\mathscr{v})\right) \cup \left(f_S(\mathscr{v}) \cap f_S(\mathscr{n})\right) \cup \left(f_S(\mathscr{v}) \cap f_S(\mathscr{j})\right) \cup \left(f_S(\mathscr{j}) \cap f_S(\mathscr{s})\right) \cup \left(f_S(\mathscr{j}) \cap f_S(\mathscr{v})\right)\right] \cap f_S(\mathscr{v}) = \{2\}$

$[(f_S \circ f_S) \tilde{\cap} f_S](\mathscr{j}) = (f_S \circ f_S)(\mathscr{j}) \cap f_S(\mathscr{j}) = \left[\left(f_S(\mathscr{s}) \cap f_S(\mathscr{j})\right) \cup \left(f_S(\mathscr{n}) \cap f_S(\mathscr{j})\right) \cup \left(f_S(\mathscr{v}) \cap f_S(\mathscr{s})\right) \cup \left(f_S(\mathscr{v}) \cap f_S(\mathscr{v})\right) \cup \left(f_S(\mathscr{j}) \cap f_S(\mathscr{n})\right) \cup \left(f_S(\mathscr{j}) \cap f_S(\mathscr{j})\right)\right] \cap f_S(\mathscr{j}) = \{7,9\}$

Therefore,

$$(f_S \circ f_S) \tilde{\cap} f_S = \{(\mathscr{s}, \{0,1,2\}), (\mathscr{n}, \{1\}), (\mathscr{v}, \{2\}), (\mathscr{j}, \{7,9\})\} \neq \emptyset_S.$$

It is seen that f_S is an SI-almost SS. Similarly,

$$(g_S \circ g_S) \tilde{\cap} g_S = \{(\mathscr{s}, \{3,5,7\}), (\mathscr{n}, \{3\}), (\mathscr{v}, \emptyset), (\mathscr{j}, \{2,8\})\} \neq \emptyset_S.$$

That is to say, g_S is an SI-almost SS. However, h_S is not an SI-almost SS. In fact;

$$[(h_S \circ h_S) \tilde{\cap} h_S](\mathscr{s}) = (h_S \circ h_S)(\mathscr{s}) \cap h_S(\mathscr{s}) = \left[\left(h_S(\mathscr{s}) \cap h_S(\mathscr{s})\right) \cup \left(h_S(\mathscr{n}) \cap h_S(\mathscr{n})\right)\right] \cap h_S(\mathscr{s}) = \emptyset$$

$$[(h_S \circ h_S) \tilde{\cap} h_S](\mathscr{n}) = (h_S \circ h_S)(\mathscr{n}) \cap h_S(\mathscr{n}) = \left[\left(h_S(\mathscr{s}) \cap h_S(\mathscr{n})\right) \cup \left(h_S(\mathscr{n}) \cap h_S(\mathscr{s})\right)\right] \cap h_S(\mathscr{n}) = \emptyset$$

$$\begin{aligned}[(h_S \circ h_S) \tilde{\cap} h_S](\mathscr{v}) &= (h_S \circ h_S)(\mathscr{v}) \cap h_S(\mathscr{v}) \\ &= \left(h_S(\mathscr{s}) \cap h_S(\mathscr{v})\right) \cup \left(h_S(\mathscr{n}) \cap h_S(\mathscr{v})\right) \cup \left(h_S(\mathscr{v}) \cap h_S(\mathscr{n})\right) \cup \left(h_S(\mathscr{v}) \cap h_S(\mathscr{j})\right) \\ &\cup \left(h_S(\mathscr{j}) \cap h_S(\mathscr{s})\right) \cup \left(h_S(\mathscr{j}) \cap h_S(\mathscr{v})\right)] \cap h_S(\mathscr{v}) = \emptyset\end{aligned}$$

$$\begin{aligned}[(h_S \circ h_S) \tilde{\cap} h_S](\mathscr{j}) &= (h_S \circ h_S)(\mathscr{j}) \cap h_S(\mathscr{j}) \\ &= \left[\left(h_S(\mathscr{s}) \cap h_S(\mathscr{j})\right) \cup \left(h_S(\mathscr{n}) \cap h_S(\mathscr{j})\right) \cup \left(h_S(\mathscr{v}) \cap h_S(\mathscr{s})\right) \cup \left(h_S(\mathscr{v}) \cap h_S(\mathscr{v})\right) \right. \\ &\left. \cup \left(h_S(\mathscr{j}) \cap h_S(\mathscr{n})\right) \cup \left(h_S(\mathscr{j}) \cap h_S(\mathscr{j})\right)\right] \cap h_S(\mathscr{j}) = \emptyset\end{aligned}$$

Therefore,

$$(h_S \circ h_S) \tilde{\cap} h_S = \{(\mathscr{s}, \emptyset), (\mathscr{n}, \emptyset), (\mathscr{v}, \emptyset), (\mathscr{j}, \emptyset)\} = \emptyset_S.$$

It is seen that h_S is not an SI-almost SS.

Proposition 3.3. If f_S is an SI-SS such that $f_S \neq \emptyset_S$, then f_S is an SI-almost SS.

Proof: Let $f_S \neq \emptyset_S$ be an SI-SS, then, $f_S \circ f_S \tilde{\subseteq} f_S$. By Lemma 2.9, since $f_S \neq \emptyset_S$ it follows that $f_S \circ f_S \neq \emptyset_S$. We need to show that

$$(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S.$$

Since $f_S \circ f_S \tilde{\subseteq} f_S$, then $(f_S \circ f_S) \tilde{\cap} f_S = (f_S \circ f_S) \neq \emptyset_S$. Thus, f_S is an SI-almost SS.

Here it is obvious that $\emptyset_S$ is an SI-SS as $\emptyset_S \circ \emptyset_S \tilde{\subseteq} \emptyset_S$; but it is not an SI-almost SS since $(\emptyset_S \circ \emptyset_S) \tilde{\cap} \emptyset_S = \emptyset_S \tilde{\cap} \emptyset_S = \emptyset_S$.

Here note that if f_S is an SI-almost SS, then f_S needs not to be an SI-SS as shown in the following example:

Example 3.4. In Example 3.2, it is shown that f_S and g_S are SI-almost SSs; however f_S and g_S are not SI-SSs. In fact,

$$(f_S \circ f_S)(\mathscr{s}) = \left(f_S(\mathscr{s}) \cap f_S(\mathscr{s})\right) \cup \left(f_S(\mathscr{n}) \cap f_S(\mathscr{n})\right) = \\ = \{0,1,2,5\} \nsubseteq f_S(\mathscr{s})$$

and so f_S is not an SI-SS. Similarly, g_S is not an SI-SS. In fact;

$$\begin{aligned}(g_S \circ g_S)(\mathscr{j}) = &\left[\left(g_S(\mathscr{s}) \cap g_S(\mathscr{j})\right)\right. \\ &\cup \left(g_S(\mathscr{n}) \cap g_S(\mathscr{j})\right) \\ &\cup \left(g_S(\mathscr{v}) \cap g_S(\mathscr{s})\right) \\ &\cup \left(g_S(\mathscr{v}) \cap g_S(\mathscr{v})\right) \\ &\cup \left(g_S(\mathscr{j}) \cap g_S(\mathscr{n})\right) \\ &\left.\cup \left(g_S(\mathscr{j}) \cap g_S(\mathscr{j})\right)\right] \\ &= \{2,8,12,13\} \nsubseteq g_S(\mathscr{j})\end{aligned}$$

and so g_S is not an SI-SS.

Theorem 3.5. Let $f_S \tilde{\subseteq} h_S$. If f_S is an SI-almost SS, then h_S is an SI-almost SS.

Proof: Assume that f_S is an SI-almost SS. Hence, $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$. We need to show that $(h_S \circ h_S) \tilde{\cap} h_S \neq \emptyset_S$. In fact,

$$(f_S \circ f_S) \tilde{\cap} f_S \tilde{\subseteq} (h_S \circ h_S) \tilde{\cap} h_S.$$

Since $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, it is obvious that $(h_S \circ h_S) \tilde{\cap} h_S \neq \emptyset_S$. This completes the proof.

Theorem 3.6. Let f_S and h_S be SI-almost SSs. Then, $f_S \tilde{\cup} h_S$ is an SI-almost SS.

Proof: Since f_S is an SI-almost SS and $f_S \tilde{\subseteq} f_S \tilde{\cup} h_S$, $f_S \tilde{\cup} h_S$ is an SI-almost SS by Theorem 3.5.

Corollary 3.7. The finite union of SI-almost SSs is an SI-almost SS.

Corollary 3.8. Let f_S or h_S be SI-almost SS. Then, $f_S \tilde{\cup} h_S$ is an SI-almost SS.

Here note that if f_S and h_S are SI-almost SSs, then $f_S \tilde{\cap} h_S$ needs not to be an SI-almost SS.

Example 3.9. Consider the SI-almost SSs f_S and g_S in Example 3.2. Since,

$f_S \tilde{\cap} g_S = \{(s,\emptyset),(n,\emptyset),(v,\emptyset),(j,\emptyset)\} = \emptyset_S$

$f_S \tilde{\cap} g_S$ is not an SI-almost SS.

Proposition 3.10. Let f_S be an idempotent soft set such that $f_S \neq \emptyset_S$. Thus, f_S is an SI-almost SS.

Proof: Assume that $f_S \neq \emptyset_S$ and f_S is an idempotent soft set. Then, $f_S \circ f_S = f_S$. We need to show that

$$(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S.$$

Since $(f_S \circ f_S) \tilde{\cap} f_S = f_S \tilde{\cap} f_S = f_S \neq \emptyset_S$, f_S is an SI-almost SS.

Now, we give the relationship between almost SS and SI-almost SS of S.

Theorem 3.11. Let A be a subset of S. A is an almost SS if and only if S_A, the soft characteristic function of A, is an SI-almost SS, where $\emptyset \neq A \subseteq S$.

Proof: Assume that $\emptyset \neq A$ is an almost SS. Then, $AA \cap A \neq \emptyset$, so there exist $k \in S$ such that $k \in AA \cap A$. Since,

$$\big((S_A \circ S_A) \tilde{\cap} S_A\big)(k) = (S_{AA} \tilde{\cap} S_A)(k) = \\ = (S_{AA\cap A})(k) = U \neq \emptyset,$$

it follows that $(S_A \circ S_A) \tilde{\cap} S_A \neq \emptyset_S$. Thus, S_A is an SI-almost SS.

Conversely assume that S_A is an SI-almost SS. Hence, we have $(S_A \circ S_A) \tilde{\cap} S_A \neq \emptyset_S$. In order to show that A is an almost SS, we should prove that $A \neq \emptyset$ and $AA \cap A \neq \emptyset$. $A \neq \emptyset$ is obvious from assumption. Now,

$$\emptyset_S \neq (S_A \circ S_A) \tilde{\cap} S_A \Rightarrow \exists k \in S;\ \big((S_A \circ S_A) \tilde{\cap} S_A\big)(k) \neq \emptyset$$
$$\Rightarrow \exists k \in S;\ (S_{AA} \tilde{\cap} S_A)(k) \neq \emptyset$$
$$\Rightarrow \exists k \in S;\ (S_{AA\cap A})(k) \neq \emptyset$$
$$\Rightarrow \exists k \in S;\ (S_{AA\cap A})(k) = U$$
$$\Rightarrow k \in AA \cap A$$

Hence, $AA \cap A \neq \emptyset$. Consequently, A is an almost SS.

Lemma 3.12. Let f_S be a soft set over U. Then, $f_S \tilde{\subseteq} S_{supp(f_S)}$.

Proof: We need to show that for all $x \in S$, $f_S(x) \tilde{\subseteq} S_{supp(f_S)}(x)$. By definition of soft characteristic function of $supp(f_S)$;

$$S_{supp(f_S)}(x) = \begin{cases} \emptyset, x \notin supp(f_S) \\ U, x \in supp(f_S) \end{cases}$$

Let, $x \in supp(f_S)$. Then, $f_S(x) \neq \emptyset$ and so $S_{supp(f_S)}(x) = U$. Thus,

$$\emptyset \neq f_S(x) \subseteq S_{supp(f_S)}(x) = U$$

Hence, $f_S \tilde{\subseteq} S_{supp(f_S)}$. Now assume that $x \notin supp(f_S)$. Then, $f_S(x) = \emptyset$ and so,

$$\emptyset = f_S(x) \subseteq S_{supp(f_S)}(x)$$

So, $f_S \tilde{\subseteq} S_{supp(f_S)}$. Therefore, in all circumstances, $f_S \tilde{\subseteq} S_{supp(f_S)}$.

Theorem 3.13. If f_S is an SI-almost SS, then $supp(f_S)$ is an almost SS.

Proof: Assume that f_S is an SI-almost SS. Thus, $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$. In order to show that $supp(f_S)$ is an almost SS, by Theorem 3.11, it is enough to show that $S_{supp(f_S)}$ is an SI-almost SS. By Lemma 3.12,

$$(f_S \circ f_S) \tilde{\cap} f_S \tilde{\subseteq} \big(S_{supp(f_S)} \circ S_{supp(f_S)}\big) \tilde{\cap} S_{supp(f_S)}$$

and $(f_S \circ f_S) \tilde{\cap} f_S \neq \emptyset_S$, it implies that $\big(S_{supp(f_S)} \circ S_{supp(f_S)}\big) \tilde{\cap} S_{supp(f_S)} \neq \emptyset_S$. Consequently, $S_{supp(f_S)}$ is an SI-almost SS and by Theorem 3.11, $supp(f_S)$ is an almost SS.

Here note that the converse of Theorem 3.13 is not true in general as shown in the following example.

Example 3.14. We know that h_S is not an SI-almost SS in Example 3.2 and it is obvious that $supp(h_S) = \{n, v\}$. Since,

$$\big[\big(supp(h_S)\big)\big(supp(h_S)\big)\big] \cap supp(h_S) = [\{n,v\}\{n,v\}] \cap \{n,v\} \\ = \{s,v,j\} \cap \{n,v\} \\ = \{v\} \neq \emptyset$$

$supp(h_S)$ is an almost SS; although h_S is not an SI-almost SS.

Definition 3.15. Let f_S and h_S be SI-almost SSs such that $h_S \tilde{\subseteq} f_S$. If $supp(h_S) = supp(f_S)$, then f_S is called a minimal SI-almost SS.

Theorem 3.16. A is a minimal almost SS if and only if S_A, the soft characteristic function of A, is a minimal SI-almost SS, where $\emptyset \neq A \subseteq S$.

Proof: Assume that A is a minimal almost SS. Thus, A is an almost SS, and so S_A is an SI-almost SS by Theorem 3.11. Let f_S be an SI-almost SS such that $f_S \tilde{\subseteq} S_A$. By Theorem 3.13, $supp(f_S)$ is an almost SS and by Note 2.6 and Corollary 2.11,

$$supp(f_S) \subseteq supp(S_A) = A.$$

Since A is a minimal almost SS, $supp(f_S) = supp(S_A) = A$. Thus, S_A is a minimal SI-almost SS by Definition 3.15.

Conversely, let S_A be a minimal SI-almost SS. Thus, S_A is an SI-almost SS and A is an almost SS by Theorem 3.11. Let B be an almost SS such that $B \subseteq A$. By Theorem 3.11, S_B is an SI-almost SS, and by Theorem 2.12 (i), $S_B \tilde{\subseteq} S_A$. Since S_A is a minimal SI-almost SS,

$$B = supp(S_B) = supp(S_A) = A$$

by Corollary 2.11. Thus, A is a minimal almost SS.

Definition 3.17. Let f_S, g_S, and h_S be any SI-almost SSs. If $h_S \circ g_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-prime almost SS.

Definition 3.18. Let f_S and h_S be any SI-almost SSs. If $h_S \circ h_S \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$, then f_S is called an SI-semiprime almost SS.

Definition 3.19. Let f_S, g_S, and h_S be any SI-almost SSs. If $(h_S \circ g_S) \tilde{\cap} (g_S \circ h_S) \tilde{\subseteq} f_S$ implies that $h_S \tilde{\subseteq} f_S$ or $g_S \tilde{\subseteq} f_S$, then f_S is called an SI-strongly prime almost SS.

It is obvious that every SI-strongly prime almost SS is an SI-prime almost SS and every SI-prime almost SS is an SI-semiprime almost SS.

Theorem 3.20. If S_P, the soft characteristic function of P, is an SI-prime almost SS, then P is a prime almost SS, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-prime almost SS. Thus, S_P is an SI-almost SS and thus, P is an almost SS by Theorem 3.11. Let A and B be almost SSs such that $AB \subseteq P$. Thus, by Theorem 3.11, S_A and S_B are SI-almost SSs, and by Theorem 2.12 (i) and (iii), $S_A \circ S_B = S_{AB} \tilde{\subseteq} S_P$. Since S_P is an SI-prime almost SS and $S_A \circ S_B \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$ or $S_B \tilde{\subseteq} S_P$. Therefore, by Theorem 2.12 (i), $A \subseteq P$ or $B \subseteq P$. Consequently, P is a prime almost SS.

Theorem 3.21. If S_P, the soft characteristic function of P, is an SI-semiprime almost SS, then P is a semiprime almost SS, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-semiprime almost SS. Thus, S_P is an SI-almost SS and thus, P is an almost SS by Theorem 3.11. Let A be an almost SS such that $AA \subseteq P$. Thus, by Theorem 3.11, S_A is an SI-almost SS, and by Theorem 2.12 (i) and (iii), $S_A \circ S_A = S_{AA} \tilde{\subseteq} S_P$. Since S_P is an SI-semiprime almost SS, and $S_A \circ S_A \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$. Therefore, by Theorem 2.12 (i), $A \subseteq P$. Consequently, P is a semiprime almost SS.

Theorem 3.22. If S_P, the soft characteristic function of P, is an SI-strongly prime almost SS, then P is a strongly prime almost SS, where $\emptyset \neq P \subseteq S$.

Proof: Assume that S_P is an SI-strongly prime almost SS. Thus, S_P is an SI-almost SS and thus, P is an almost SS by Theorem 3.11. Let A and B be almost SSs such that $AB \cap BA \subseteq P$. Thus, by Theorem 3.11, S_A and S_B are SI-almost SSs and by Theorem 2.12,

$$(S_A \circ S_B) \tilde{\cap} (S_B \circ S_A) = S_{AB} \tilde{\cap} S_{BA} = S_{AB \cap BA} \tilde{\subseteq} S_P$$

Since S_P is an SI-strongly prime almost SS and $(S_A \circ S_B) \tilde{\cap} (S_B \circ S_A) \tilde{\subseteq} S_P$, it follows that $S_A \tilde{\subseteq} S_P$ or $S_B \tilde{\subseteq} S_P$. Thus, by Theorem 2.12 (i), $A \subseteq P$ or $B \subseteq P$. Therefore, P is a strongly prime almost SS.

Conclusıon

In this paper, we established the notion of soft intersection almost subsemigroup, which is an extension of the nonnull soft intersection subsemigroup of semigroups. Given the collection of almost subsemigroups of a semigroup, we observe that a semigroup may be constructed under the binary operation of union for soft sets, but not under the binary of operation of intersection for soft sets. Additionally, we demonstrated how minimality, primeness, semiprimeness, and strongly primeness relate to the soft intersection almost subsemigroup of a semigroup and the almost subsemigroup of a semigroup. Future research can be conducted on a variety of soft intersection almost ideals, such as the left (right/two-sided) ideal, quasi-ideal, interior ideal, bi-ideal, bi-interior ideal, bi-quasi ideal, quasi-interior ideal, weak-interior ideal, bi-quasi-interior ideal of semigroups.

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