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Radial distribution functions and thermodynamics of macroparticles in dusty plasmas

Abstract. Starting from the previously developed pseudopotential model of dust particles interaction in the plasma, which takes into account both the finite-size and the screening effects, equilibrium distribution functionsarewell studied. Consideration is entirely based on the renormalization theory of plasma particles interaction which results in the so-called generalized Poisson-Boltzmann equation. The main idea is to apply the renormalization theory in order to treat the dust grains as a one component plasma with a specific interaction potential. The interaction potential derived in such a way does naturallyincorporate the number density of dust particles and can be utilized for further theoretical considerations. Such an approach provides quite effectivecalculation scheme for the radial distribution function of the dust particles whosenon-monotonic behavior is observed at different values of plasma parameters to prove short-range or even long-range order formation in the system. The correlation energy is investigated in a wide range of plasma parameters and the results are examined from the viewpoint of the thermodynamic stability.

***Keywords*:** Dusty plasma, pseudopotential model,generalized Poisson-Boltzmann equation,radial distribution function, correlation energy.

Introduction

Study of systems with strong interparticle interaction has always been of great interest from the point of view of fundamental physics. For over the past two centuries, researches from all over the world have been looking for answers to the question about how the interaction between the structural elements determines the physical properties of bodies being in different states of matter. The plasma physics is no exception and in recent decades a great boost in studying strongly coupled Coulomb systems has been witnessed. Many approachesto the experimental investigations of various properties of dense plasmas have been developed and significant progress has been made in the theoretical understanding of the elementary processes caused by the strong interaction between the particles.

It has to be noted that in recent years a whole series of studies has been attempted on the static and dynamic properties of the so-called dusty plasmas appearing when micro-sized particles are injected into an ordinary (buffer) plasma. Such systems are not only artificially created in the laboratory, but also occur in astrophysical objects, appear in tokamak wall regions due to contact of the plasma with the first wall of the reactor, is generatedin etching of circuits in electronics.

Interest in the study of dusty plasmas is prescribed to an extremely simple observation technique of microscopic sized particle behavior which is visualized by standard optical methods employing a camcorder. This provided an important practical tool for testing the existing theoretical approaches and findings, put a lot of important new issues and stimulated further development of experimental techniques.

It was almost immediately found that under certain external conditions, dust particles in the plasma arrange themselves in a kind of ordered structure [1,2], which is called a plasma dust crystal. A long-range order in the location of dust particles was actually discovered, which means that most of the observation time grains are located at the sites of a quasi-lattice, occasionally breaking away from their equilibrium positions [3-5].

Being immersed into the buffer plasma dust grains begin to absorb electrons and ions, however, the flow of electrons is much greater than the flow of ions becausethe former have greater mobility and speed due to the difference in the masses. Thus, microscopic particles usually acquire quite a high negative charge [6-8] which can reach hundreds or even thousands of the elementary charge.This inevitably results in strong coupling effects due to the interparticle interactions. For a correct theoretical description of the microscopic and macroscopic properties of the dust component of the plasma it I therefores very important to establish the exact form of the interaction potential between dust particles and ions with the buffer gas [9-11]. It should be noted that theoretical analysis often impliesthatthe interaction potential is chosen in the form of the screened Yukawa potential [12-14], whichis called the Debye- Huckel potential in the physics of strongly coupled plasmas. The use of such a potential automatically suggests that the microscopic particles are considered to be point-like [15-17], i.e. the mean distance between them is much higher than their average size which is not always true in practice.

*Dimensionless plasma parameters.*In the following the buffer plasma is assumed to be afully ionized hydrogen consisting of free electrons with the electric charge $-e$ and the number density n_e and of free protons with the electric charge e and the number density $n_p = n_e = n$. The microparticlesare consideredto be solid spheres with the radius R and the electric charge $-Z_d e$, where Z_d stands for the charge number of dust particles. It has to be stressed that the number density n_d of dust grains has to be so small to satisfy the inequality $n_d Z_d << n$ which guarantees the total plasma neutrality/

To describe the state of the buffer plasma it is of convenienceto introduce the effective coupling parameter as

$$\Gamma_R = \frac{e^2}{R k_B T}, \tag{1}$$

where k_B is theBoltzmann constant, T denotes the plasmatemperature.

Coupling parameter (1) is not conventional since it representsthe ratio of the Coulombic interaction energy of two electrons located at a distance R from each otherto their average kinetic energy of chaotic thermal motion.

Another dimensionless parameter involved is the screening parameter defined as

$$\kappa = \frac{R}{\lambda_D}, \tag{2}$$

where $\lambda_D = (k_B T / 8\pi n e^2)^{1/2}$ designates theDebye screening radius.

Knowledge of the charge number of dust particles Z_d and the dimensionless parameters (1) and (2) is perfectly enough to describe the interaction of two isolated hard spheres in a buffer plasma. However, the coupling parameter for the dust particles Γ_D is related to the above defined effective coupling parameter Γ_R as follows

$$\Gamma_D = \frac{Z_d^2 e^2}{a_d k_B T} = \frac{Z_d^2 \Gamma_R}{D}, \tag{3}$$

wherethe new dimensionless parameter $D = a_d / R$ represents the ratio of the average distance between the dust particles $a_d = (3 / 4\pi n_d)^{1/3}$ to their radius.

Radial distribution functions. In the previous serious of papers [18,19] a new approach to taking into account two important effects in the interaction of dust particles was proposed. The first effect mentioned is the finite size of the dust particles, and the second isthe polarization phenomenon which is most simply incorporated in the method of electrostatic images under the assumption that the dust particles are made of a conductive material and can acquire a dipole moment due to electrostatic induction. Such a pseudopotential model Φ_{dd} does not include the number density of dust particles, it actually represents the interaction energy of two isolated dust grains and the screening is made by the electrons and ions of the buffer plasma. This allows one to use that pseudopotential model in a well-tested theoretical approaches and computer simulation methods. In particular, it is reasonable to reapply the renormalization theory [20], which in this case leads to the following generalized Poisson-Boltzmann equation:

$$\Delta_i \Psi_{dd}(\mathbf{r}_i,\mathbf{r}_j) = \Delta_i \Phi_{dd}(\mathbf{r}_i,\mathbf{r}_j) - \frac{n_d}{k_B T}\int \Delta_i \Phi_{dd}(\mathbf{r}_i,\mathbf{r}_k)\Psi_{dd}(\mathbf{r}_j,\mathbf{r}_k)\,d\mathbf{r}_k\,, \tag{4}$$

where Ψ_{dd} stands for the pseudopotential that already takes into account the collective events in the interaction of dust particles by incorporating the dust particle number density n_d.

Solution to (4) is found as the following expression for the Fourier transform

$$\tilde{\Psi}_{dd}(\mathbf{k}) = \frac{\tilde{\Phi}_{dd}(\mathbf{k})}{1+\frac{n_d}{k_B T}\tilde{\Phi}_{dd}(\mathbf{k})}. \tag{5}$$

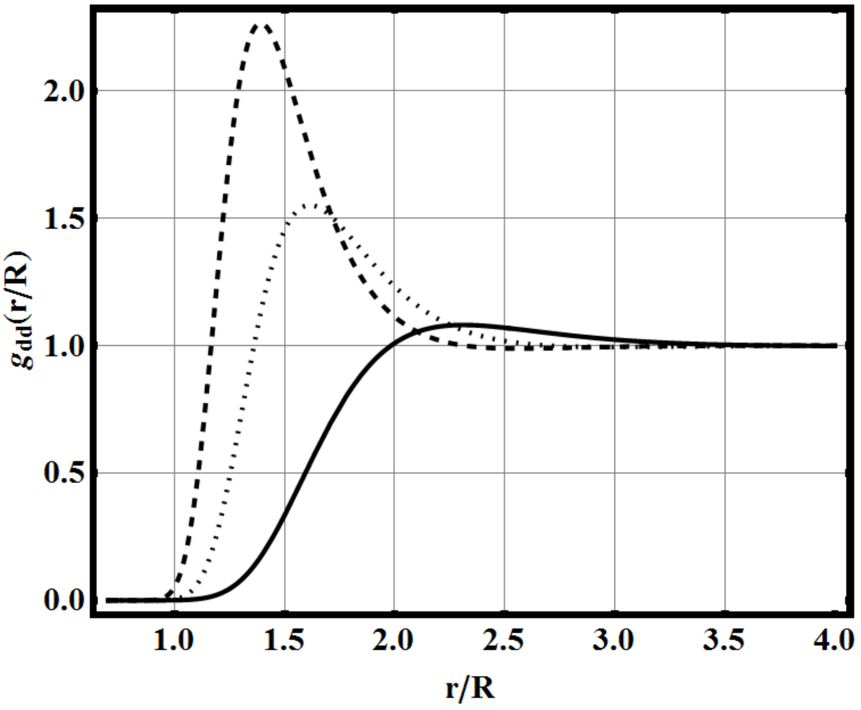
The interaction pseudopotential in the configurational space is calculated from equation (5) by using the backward Fourier transform

$$\Psi_{dd}(\mathbf{r}) = \frac{1}{(2\pi)^3}\int \tilde{\Psi}_{dd}(\mathbf{k})\exp(-i\mathbf{k}\mathbf{r})\,d\mathbf{k}\,. \tag{6}$$

It is well known that the expression for the radial distribution function is then simply found in the following form

$$g_{dd}(r) = \exp\left(-\frac{\Psi_{dd}(r)}{k_B T}\right). \tag{7}$$

Figures 1-3 show the corresponding radial distribution functions calculated at different values of the dimensionless plasma parameters.



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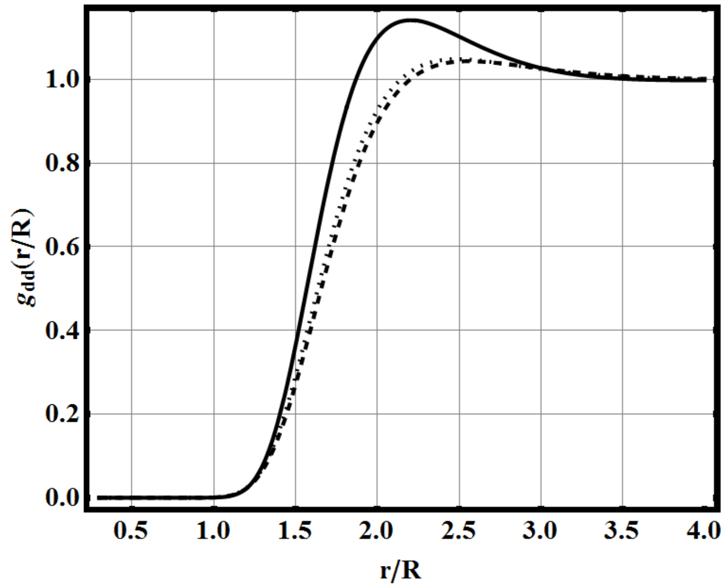
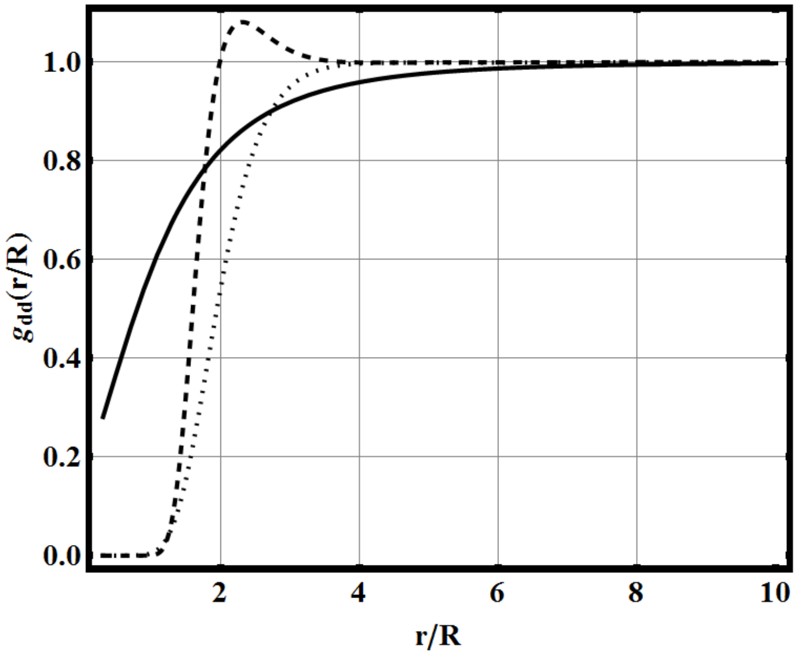
Solid line: $\Gamma_R = 0.1$; dotted line: $\Gamma_R = 0.3$; dashed line: $\Gamma_R = 0.5$, the dimensionless plasma parameters are $\kappa = 3$, $d = 5$ and $Z_d = 100$.

Figure 1 – Radial distribution function of dust particles



Solid line: $\kappa = 2$; dotted line: $\kappa = 5$; dashed line: $\kappa = 7$, the dimensionless plasma parameters are $\Gamma_R = 0.1$, $d = 5$ and $Z_d = 100$.

Figure2 – Radial distribution function of dust particles



Solid line: $Z_d = 10$; dotted line: $Z_d = 50$; dashed line: $Z_d = 100$, the dimensionless plasma parameters are $\Gamma_R = 0.1$, $\kappa = 3$ and $d = 5$.

Figure3 – Radial distribution function of dust particles

Analysis of figures 1-3drives us to the following conclusions:

- an increase in the coupling parameter leads to a sharp growth in the value of the first peak on the radial distribution function, which can be clearly associated with the strengtheningof dust particle interaction and the formation of ordered structures;

- a similar pattern is observed with a decrease in the screening parameter, however, the maximum on the curve of the radial distribution function turns less pronounced the interaction potential of dust particles depends hardly on the screening parameter at short distances;
- the most strong dependence of the radial distribution function is observed when the charge of the dust particles varies, such that the behavior of the curve fundamentally changes from monotonicto non-monotonic which corresponds to the short-range order formation in the liquid-like phase.

It should be noted that quite a similar behavior of the radial distribution functionwas previously observed in other types of plasma [21-25], such as the semiclassical and partially ionized plasma, which, however, are inherently multicomponent systems .

Correlation energy

It is well known that among all of the thermodynamic characteristics, which determine the plasma properties, a special role is played by the internal energy that allows one to calculate some other thermodynamic functions such as the heat capacity, the free energy etc.

In the statistical theory of equilibrium states it has been rather well established that in systems, consisting of a large number of particles, the internal energy can be calculated using the following relation

$$E = \frac{3}{2} N_d k_B T + U_N , \quad (8)$$

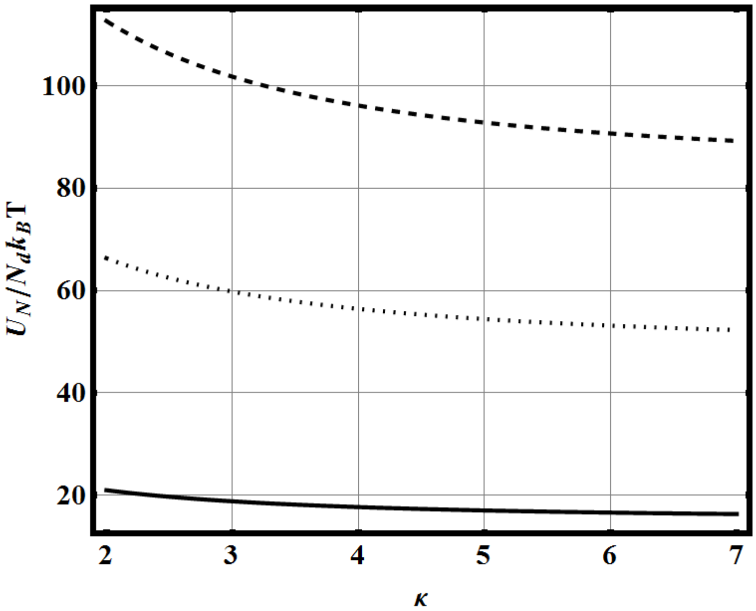
where N_d is the total number of particles in the system, and U_N denoted the so-called correlation energy, which is evaluated with the aid of the radial distribution function of dust particles as:

$$U_N = 2\pi n_d^2 V \int_0^\infty \left(\Phi_{dd}(r) - T \frac{\partial \Phi_{dd}(r)}{\partial T} \right) g(r) r^2 dr , \quad (9)$$

with V being the system volume.

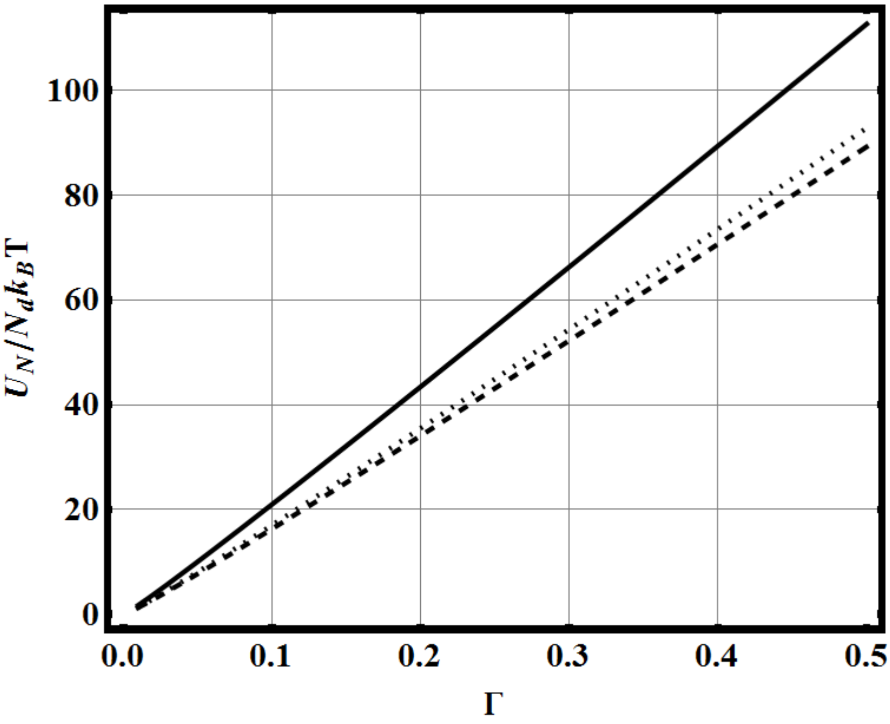
It should be noted that expression (9) represents only the contribution to the internal energy of the plasma due to interaction of dust grains. This fact will be used below to explain the results obtained.

Figures 4-6 demonstrates the dependence of the dimensionless correlation energy $U_N / N_d k_B T$ on the plasma parameters introduced above.



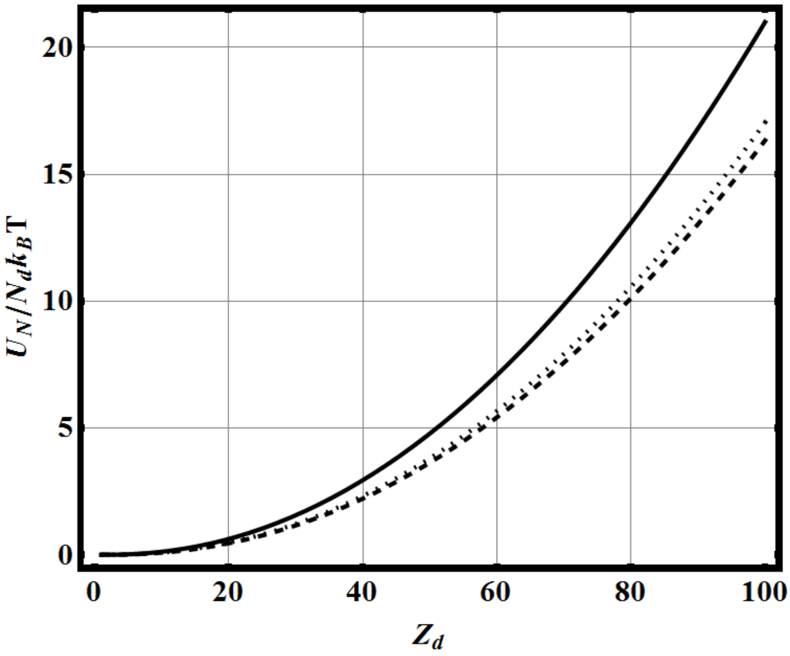
Solid line: $\Gamma_R = 0.1$; dotted line: $\Gamma_R = 0.3$; dashed line: $\Gamma_R = 0.5$,
the dimensionless plasma parameters are $d = 5$ and $Z_d = 100$.

Figure 4 – Dimensionless correlation energy of the dust component of the plasma



Solid line: $\kappa = 2$; dotted line: $\kappa = 5$; dashed line: $\kappa = 7$, the dimensionless plasma parameters are $d = 5$ and $Z_d = 100$.

Figure 5 – Dimensionless correlation energy of the dust component of the plasma



Solid line: $\kappa = 2$; dotted line: $\kappa = 5$; dashed line: $\kappa = 7$, the dimensionless plasma parameters are $\Gamma_R = 0.1$ and $d = 5$.

Figure 6 – Dimensionless correlation energy of the dust component of the plasma

Analysis of figures 4-6 drives us to the following conclusions:

- the correlation energy hardly depends on the screening parameter, while the dependence on the coupling parameter is well pronounced. This is explained by the behavior of the interaction potential, which is a linear function of the coupling parameter and only asymptotically changes when the screening parameter varies;

- the dependence of the correlation energy on the particle charge is almost quadratic which is also prescribed to specific features of the proposed interaction potential of dust particles.

Let us consider the results obtained from the viewpoint of the thermodynamic stability of the system. It is clearly seen that the calculated values of the correlation energy are positive and increase while the coupling parameter grows. At first glance, this contradicts the theory of thermodynamic stability of the equilibrium state of matter. Indeed , the increase of the coupling parameter corresponds to the decrease in temperature. Thus, it turns out that a decrease in temperature results in a growth of the internal energy which should not happen in a thermodynamically stabilized system. The answer to this controversy lies in the fact that the studied correlation energy is just a contribution to thetotal internal energy, there areother contributions arising from the interaction of the he buffer plasma particles with the dust grainsand with each other. With those contributions properly taken into accountthe total correlation energy should stay negative and, thus, the question on the thermodynamic stability of the entire system drops out.

Conclusions

Using the interaction potential of dust particles proposed in earlier papers [18,19] the behavior of the radial distribution functions in the pair correlation approximation has been studied at different value of the plasma parameters. An increase in the coupling parameter results in a sharp growth in the magnitude of the first peak on the radial distribution function, which is clearly explained bystrengthening of the interaction betweenthe dust particles and can be interpreted as the ordered structure formation. A similar pattern is seen with a decrease in the screening parameter, however, the maximum on the curve of the radial distribution function turns less pronouncedsince the interaction potential of the dust grains depends hardly on the screening parameter at short distances. The strongest dependence of the radial distribution function is witnessed when the charge of the dust particles vary such that the behavior of the curve of the radial distribution function fundamentally changes from monotonictonon-monotonic which is treated asthe short-range order formation in the liquid-like phase.

The correlation energy of dust particles has been studied and it has been shown that it only slightly depends on the screening parameter, while the dependence on the coupling parameter remains rather essential. On the other hand, the dependence of the correlation energy on the particle charge is almost parabolic. All these fact are easily explained by the characteristic behavior of the interaction potential, which depends linearly on the coupling parameter, quadratic relative to the particle charge and only asymptotically changed by varying the screening parameter.

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