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Lifetime of Uniformly Rotating Super-Chandrasekhar Mass White Dwarfs

Abstract. We present recent results on general relativistic uniformly rotating white dwarfs. Namely, on the basis of the general relativistic Feynman-Metropolis-Teller theory for white dwarfs we focus on delayed type Ia supernova explosions and gravitational collapses of super-Chandrasekhar mass white dwarfs, where we estimate so called «spinning down» time scale (lifetime) due to the electromagnetic radiation.

Keywords: relativistic white dwarfs, delayed supernova explosion, delayed collapse.

Introduction

We have recently computed in Ref. [1] general relativistic configurations of uniformly rotating white dwarfs (RWDs) within Hartle's formalism [2, 3]. We have used the relativistic Feynman-Metropolis-Teller equation of state [4] for cold WD matter, which we have shown generalizes the traditionally used equation of state of Salpeter [5-7]. It has been there shown that rotating WDs can be stable up to rotation periods of ~ 0.28 s (see [1] for details). This range of stable rotation periods for WDs amply covers the observed rotation rates of Soft Gamma Repeaters (SGRs) and Anomalous X-Ray Pulsars (AXPs) $P \sim$ 2-12 s [8].

The stability of rotating WDs was analyzed taking into account the mass-shedding limit, inverse β-decay instability, and secular axisymmetric instability, with the latter determined by the turning point method of Friedman et. al [9].

In this work we consider the astrophysical implycation of rotating WDs based on the results of Boshkayev et. al. [1]. We namely focus on the lifespan of super-Chandrasekhar mass white dwarf before type Ia supernova explosion and gravitational collapse into a neutron star as a consequence of the secular and inverse β-decay instabilities, correspondingly.

Delayed Type Ia Supernova Explosions and Gravitational Collapse of Super Chandrasekhar Mass White Dwarfs

Most of the observed magnetic WDs are massive; for instance REJ 0317-853 with $M \sim 1.35\ M_{\odot}$ and $B \sim (1.7\text{-}6.6)*10^8$ G (see e.g. [10, 11]); PG 1658+441 with $M \sim 1.31\ M_{\odot}$ and $B \sim 2.3*10^6$ G (see e.g. [12, 13]) and PG 1031+234 with the highest magnetic field ~ 10^9 G (see e.g. [14, 15]). However, they are generally found to be slow rotators [16]. It is worth mentioning that it has been recently shown by Garcia-Berro et. al [17] that such a magnetic WDs can be indeed the result of the merger of double degenerate binaries; the misalignment of the final magnetic dipole moment of the newly born RWD with the rotation axis of the star depends on the difference of the masses of the WD components of the binary.

Magnetic braking radiation of super Chandrasekhar white dwarfs (SCWDs) has been recently involved as a possible mechanism to explain the delayed time distribution of type Ia supernovae (SNe) [18], where a type Ia SN explosion is delayed for a time typical of the spin-down time scale t due to magnetic braking, providing the result of the merging process of a WD binary system is a magnetic SCWD rather than a sub-Chandrasekhar one. The characteristic timescale t of SCWD has been estimated to be $10^7 < t < 10^{10}$ yr for magnetic fields comprised in the range $10^6 < B < 10^8$ G. A constant moment of inertia ~ 10^{49} g cm^2 and a fixed critical (maximum) rotation angular velocity

$$\Omega_{crit} \sim 0.7\Omega_K^{J=0} = 0.7\sqrt{\frac{GM^{J=0}}{R^3_{M^{J=0}}}}. \qquad (1)$$

have been adopted in [18], where G is the gravitational constant, $M^{J=0}$ is the static mass, $R_{M^{J=0}}$ is the static radius corresponding to the static mass.

It should be mentioned here that type Ia SNe are widely believed to be the thermonuclear detonations of carbon-oxygen WDs, whose masses are near the Chandrasekhar limit. The route to achieve this mass and the condition for explosion are in dispute. Three main routes are discussed in the literature: single degenerate, double degenerate and core degenerate scenarios (see for instance [18] and references therein). Observations and theoretical studies cannot tell us yet whether these models for SNe Ia are viable or not. Anyway all three models allow us to obtain super-Chandrasekhar mass WDs what we exactly need for our theoretical studies.

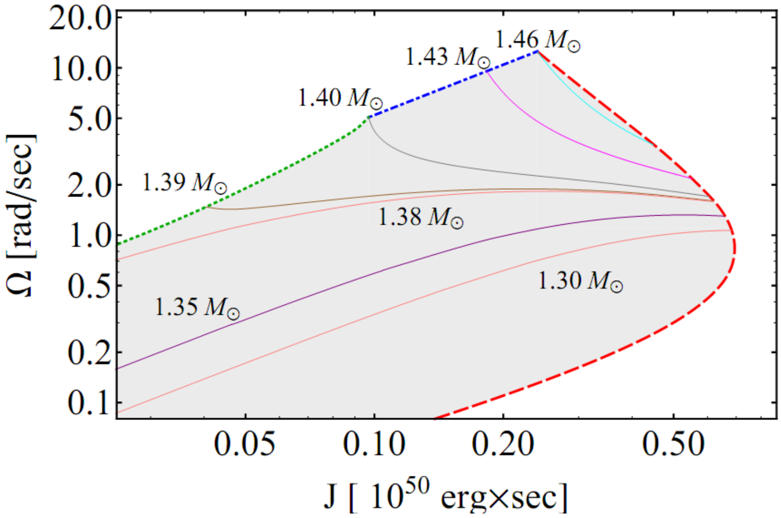

Figure 1 – Contours of constant rest mass in the $J-\Omega$ plane: rotating white dwarfs that evolve along a track with $\partial\Omega/\partial J > 0$ spin-down by angular momentum loss while the ones with $\partial\Omega/\partial J < 0$ spin-up. The Keplerian sequence is the red thick dashed curve, the blue thick dotted-dashed curve is the inverse β-decay instability curve, and the green thick dotted curve is the axisymmetric secular instability boundary.

It is important to recall here that, SCWDs spin-up by angular momentum loss see Fig 1., and therefore the reference to a «spin-down» time scale for them is a just historical term [19]. SCWDs then evolve toward the mass-shedding limit, which determines in this case the critical angular velocity Ω_{crit} for rotational instability.

Note, according to Fig.1 angular momentum loss corresponds to the direction from right to left. One can easily see that super-Chandrasekhar mass WDs of masses (1.40-1.46)$M_\odot$ by angular momentum loss will certainly reach the inverse β-decay instability density. Thus they will spin up before collapsing into a neutron star. The WDs of masses (1.38-1.40) $M_\odot$ by angular momentum loss will spin down and explode as type SNe Ia [19], where 1.38$M_\odot$ is the maximum static mass of carbon WDs. Sub-Chandrasekhar WDs will live forever if they are not in a binary system.

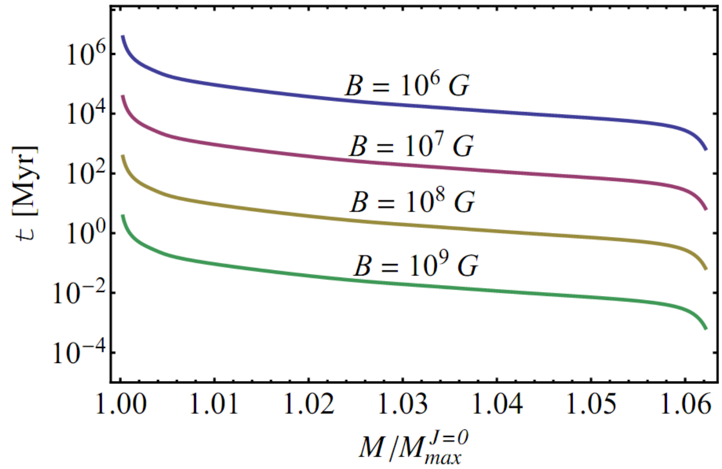

Figure 2 - The characteristic life time t in mega years (Myr) versus WD mass in units of $M_{\max}^{J=0}$ for carbon ^{12}C rotating WDs. The magnetic field B is in Gauss

If we express $\Omega_K^{J\neq 0}$ in terms of $\Omega_K^{J=0}$, taking into account the values of the dimensionless angular momentum j and dimensionless quadrupole moment q from the numerical integration, we find for RWDs $\Omega_K^{J\neq 0} = \sigma\, \Omega_K^{J=0}$ where the coefficient σ varies in the interval [0.78, 0.75] in the range of central densities [10^5, 10^{11}] g cm^{-3} (see Boshkayev et. al. [1] for detail). It is important mentioning that the above range of σ remains approximately the same independently of the chemical composition of the WD. However, the actual numerical value of the critical angular velocity, $\Omega_K^{J\neq 0}$ is different for different compositions owing to the dependence on *(Z,A)* of mass-radius relation of non-rotating WDs.

Furthermore, as we have shown in Fig. 1, the evolution track followed by a SCWD depends strongly on the initial conditions of mass and angular momentum as well as on chemical composition, and evolution of the moment of inertia. It is clear that the assumption of fixed moment of inertia $I \sim 10^{49}$g cm^2, leads to a spin-down time scale depending only on the magnetic field strength. A detailed computation will lead to a strong dependence on the mass of the SCWD; resulting in a two-parameter family of delayed times *t(M,B)*.

To this end here we have performed analogous analyses to estimate the characteristic timescale for the realistic EoS of white dwarfs presented in [7] relaxing the constancy of the moment of inertia, radius and other parameters of WDs. Indeed, by equating the rotational energy loss and magnetic dipole radiation formulas, we find the characteristic timescale and show here that all parameters are the functions of the central density ρ_c and angular velocity (rotation period) Ω as follows

$$t = -\frac{3}{2}\frac{c^3}{B^2}\int_{J_{\max}}^{J_{\min}} \frac{dJ}{R^6\Omega^3}, \qquad (2)$$

where $R = R(\rho_c, J)$, $\Omega = \Omega(\rho_c, J)$, in turn $J = J(\rho_c, \Omega)$, c is the speed of light in vacuum, B is the magnetic field, J is the angular momentum, R is the mean radius of the WD. The inclination angle between the rotation axis and the magnetic field is adopted π/2. All parameters J, Ω and R are calculated along the specific constant rest mass sequences shown in Fig. 1. Hence we performed more refined analyses taking into consideration all the stability criteria, except for the pycnonuclear instabilities [20-22] for the sake of generality. The characteristic time t versus WD mass in units of $M_{\max}^{J=0}$ for ^{12}C RWDs is shown in Fig. 2 from where we can see that the higher the magnetic field the shorter the lifetime of rotating super-Chandrasekhar WDs. Correspondingly the more massive WDs will live the shorter lifespan and vice versa.

Conclusions

In this work, relaxing the constancy of the radii, moment of inertia, angular velocity and angular momentum of super-Chandrasekhar mass white dwarfs, we have computed the delay time of type SNe Ia and gravitational collapse.

A careful analysis has been performed in order to fulfill all the possible instability criteria during the evolution of the white dwarf at the pre supernova and gravitational collapse stages.

We have shown that the higher the magnetic field the shorter the lifetime of rotating super-Chandrasekhar WDs. Correspondingly the more massive WDs will live the shorter lifespan and vice versa.

Including the pycnonuclear reactions (instabilities) and the effects of finite temperatures will lead to the interesting consequences [20-22]. This problem will be studied in future and the results will be published in a forthcoming paper.

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