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## Convective heat transfer flow of MHD hybrid nanofluid over a stretching sheet with velocity and thermal slip

**Abstract.** In this study, we have investigated the convective heat transfer of a hybrid nanofluid with magnetohydrodynamic (MHD) properties over a permeable stretching sheet. The analysis takes into account the effects of velocity and thermal slip, and the hybrid nanofluid consists of two distinct nanoparticles—titanium dioxide ( $TiO_2$ ) and silver ( $Ag$ )—dispersed in water ( $H_2O$ ) as the base fluid. The use of  $Ag$  and  $TiO_2$  nano-composites is particularly significant in nano-biotechnology, especially in nano-medicine and cancer cell therapy, owing to their remarkable photocatalytic performances in handling pharmaceutical compounds and pollutants. The primary aim of this research is to explore how the behaviour of the MHD hybrid nanofluid over a permeable stretching surface is influenced by velocity and thermal slip conditions. By employing appropriate similarity transformations, the governing partial differential equations (PDEs) are transformed into dimensionless nonlinear ordinary differential equations (ODEs), which are then analytically solved using the Keller-box finite difference method. The results indicate that the velocity profile decreases with an increase in velocity slip, while the temperature and concentration fields decrease with thermal slip. The study further analyzes the significant impact of various parameters, including velocity and thermal slip parameter, porosity parameter, magnetic parameter, Prandtl number, and Eckert number, on the velocity, temperature, and concentration profiles. These findings are presented in graphical and tabular form, showing excellent agreement with previous literature.

**Key words:** Heat transfer, hybrid nanofluid, velocity slip, thermal slip, porous medium.

### 1. Introduction

High thermal conductivity is required for evolution of energy-efficient heat transfer. Initially, micro sized particles added to the base fluid by Maxwell. Choi introduced the term nanofluid, a fluid in which nanoparticles made of metals, carbides and oxide suspended in common fluids such as water, ethanol and ethylene glycol. These nanoparticles improve fluid properties such as thermal conductivity and specific heat etc. Nanofluid gained attention in many fields such as energy, industry, health with many applications as electronic cooling, automobile and heat exchangers. However, the applications of mono type nanofluid are limited. Single component nanofluid with metallic nanoparticles such as  $Ag$ ,  $Cu$ , and  $Al$  has very high thermal conductivity, but they are unstable due to high reactivity. While the non-metallic nanoparticles as  $CuO$ ,  $MgO$ ,  $Al_2O_3$  have more stability but low conductivity. Now days, the researchers oriented their study towards hybrid nanofluid, an advanced kind of nanofluid prepared

by suspending (i) two or more different type of nanoparticles in host fluid (ii) composite nanoparticles in regular single-phase liquid. In the last few years, the hybrid nanofluid gained abundant attention by researchers due to its energy storage applications such as heat exchanger in solar collectors, nuclear reactor cooling, photo electric devices, drug reduction, lubrication, optical fibre coating, cooling of electronic devices, coolant in machines, welding etc. Abu-Nada [1] investigated heat transfer enhancement in  $CuO$ -water nanofluid with the effect of thermal conductivity. Hejazian and Moravoji [2] analysed heat transfer for  $TiO_2$  nanofluid in a tube. Ramya et. al. [3] analysed viscous flow of nanofluid past a stretching sheet escorted by velocity and thermal slip.

Acharya et al. [4] compared the effect of variable thickness, magnetic field and variable temperature on  $TiO_2$  and  $Ag$  / water nanofluid over stretching sheet. Hamad et al. [5] studied heat transfer through non-linear stretching sheet. Yousefi [6] investigated stagnation point flow of nanofluid  $TiO_2$  and  $Cu/H_2O$  through a circular cylinder.

Benkhedda and Boufendi [7] investigated convective nanofluid flow of *Ag*/water in concentric annulus with constant heat flux on outer cylinder. Eiamsa-ard et al. [8] concluded that multiple twisted tapes and *TiO<sub>2</sub>* nanoparticle in water improves thermal performance in a heat exchanger tube. Ullah et al. [9] studied *Ag-TiO<sub>2</sub>*/water hybrid nanofluid with magnetic field over cylindrical surface. Benkhedda et al. [10] numerically analysed mixed convective flow of nanofluid *TiO<sub>2</sub>* and hybrid nanofluid *Ag-TiO<sub>2</sub>* water as base fluid in an annulus. Devi et al. [11] compared heat transfer rate of nanofluid *Cu/H<sub>2</sub>O* with hybrid nanofluid *Cu-Al<sub>2</sub>O<sub>3</sub>*/water. Rizk et al. [12] studied thermal heat transfer in hybrid nanofluid with the effect of heat source and magnetic field. Chahregh et al. [13] studied a mathematical model for *Ag-TiO<sub>2</sub>* hybrid nanofluid with pure blood as base fluid past the porous channel. Esfe et al. [14] compared the experimental data of thermal conductivity of *SWCNT-MgO* with *MgO* and *SWCNT* mono particle nanofluid in ethylene glycol as base fluid. Rostami et al. [15] studied hybrid nanofluid *SiO<sub>2</sub>-Al<sub>2</sub>O<sub>3</sub>* flow near the stagnation point through a permeable vertical plate. Waini [16] studied heat and mass transfer flow of *Al<sub>2</sub>O<sub>3</sub>-Cu*/water hybrid nanofluid over a moving surface. Suresh et al. [17] synthesized hybrid nanofluid *Al<sub>2</sub>O<sub>3</sub>/H<sub>2</sub>O* using two step thermo chemical method. Ghachem et al. [18] analysed enhanced heat transfer in hybrid nanofluid flow through a rectangular wavy channels. Suresh et al. [19] presented convective heat transfer flow of hybrid nanofluid *Al<sub>2</sub>O<sub>3</sub>-Cu*/water over a uniformly heated tube. Madhesh et al. [20] investigated heat transfer of *Cu-TiO<sub>2</sub>* using a tube counter flow. Sunder et al. [21] estimated friction factor and convective heat transfer coefficient for *MWCNT-Fe<sub>3</sub>O<sub>4</sub>*/water hybrid nanofluids through a circular tube. Saha [22] investigated water based nanofluid *Al<sub>2</sub>O<sub>3</sub>* and *TiO<sub>2</sub>* in a circular pipe with uniform heat flux. Nine et al. [23] investigated synthesis process of copper/cuprous oxide (*Cu/Cu<sub>2</sub>O*) nanoparticles with size below 30nm. Ghadikolaei et al. [24] investigated stagnation point flow of hybrid nanofluid *TiO<sub>2</sub>-Cu*/water toward a stretching sheet. Rashid et al. [25] studied the different shapes of *Ag* and *TiO<sub>2</sub>* hybrid nanofluid as sphere, blade and lamina with the effect of suction and injection over permeable stretching cylinder. Toghraie et al. [26]

presented impact on the thermal conductivity of hybrid nanofluid *ZnO-TiO<sub>2</sub>*/ ethylene glycol. Mohebbi et al. [27] studied thermal performance of *MWCNT-Fe<sub>3</sub>O<sub>4</sub>*/water hybrid nanofluid in a channel.

From an extensive review of existing literature, it can be stated that there has been no prior research conducted to investigate the influence of velocity and thermal slip conditions in a porous medium on the convective heat transfer of an MHD hybrid nanofluid over a permeable stretching surface. To the best of the authors' knowledge, this particular study has not been previously reported in the available literature. The governing partial differential equations PDEs are transformed into dimensionless nonlinear ordinary differential equations ODEs by using appropriate similarity transformations and solved analytically with Keller-box finite difference method. Notable effect of involved parameters such as velocity and thermal slip parameter, porosity parameter, magnetic parameter, Prandtl number, Eckert number on velocity, concentration and thermal profiles are analysed in graphical and tabular form with an excellent agreement with previous literature.

## 2. Flow Modelling

In this paper, we explore heat transfer analysis of steady incompressible hybrid nanofluid of silver and titanium dioxide (*Ag-TiO<sub>2</sub>/H<sub>2</sub>O*) over a nonlinear permeable stretching sheet with slip conditions in porous medium (Figure1). Let two-dimensional Cartesian coordinate system ( $x, y$ ), where  $x$ -axis is measured along the sheet and  $y$ -axis normal to it and the flow takes place at  $y \geq 0$ . The sheet is stretched with velocity  $u_w = ax^n$  keeping the origin fixed, where  $a$  is a constant and  $n$  is nonlinear stretching parameter. To study the effect of Lorentz force a uniform magnetic field  $B$  is applied along  $y$ -direction. The temperature and concentration are maintained constant, symbolized by  $T_w$  and  $C_w$  respectively at surface and far from the boundary they are symbolized by  $T_\infty$  and  $C_\infty$  respectively.

Under above suppositions the equations of continuity, momentum, energy and concentration that govern the flow are written as:

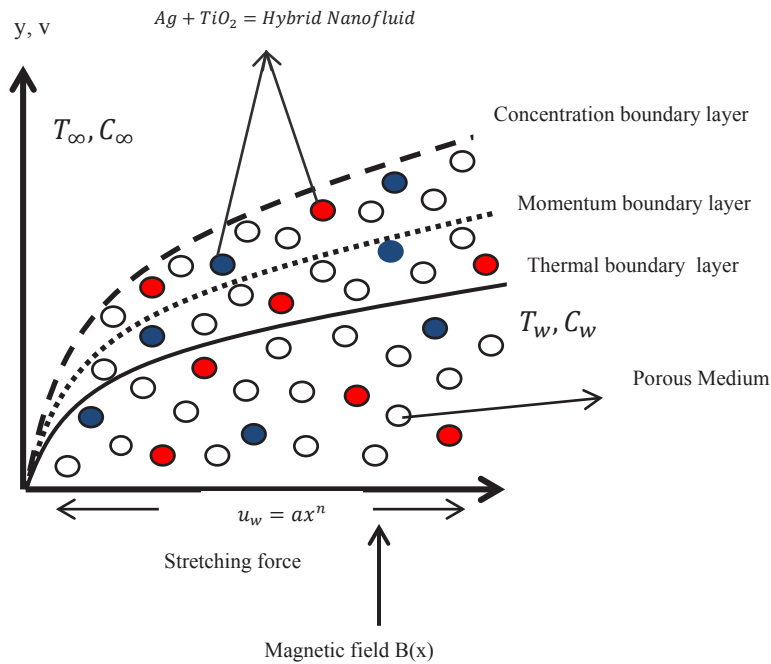


Figure 1 – Coordinate system and schematic diagram of the problem

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

Concentration equation

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \frac{\partial^2 T}{\partial y^2}. \tag{4}$$

Momentum equation

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu_{hnf} \frac{\partial^2 u}{\partial y^2} - \frac{\sigma_{hnf}}{\rho_{hnf}} B^2 u - \frac{\nu_{hnf}}{k} u + g \left[ \frac{(\rho\beta_T)_{hnf}}{\rho_{hnf}} (T - T_\infty) + \frac{(\rho\beta_C)_{hnf}}{\rho_{hnf}} (C - C_\infty) \right]. \tag{2}$$

The appropriate boundary conditions as per formulation are prescribed as:

$$\begin{aligned} \text{At } y=0, \quad u &= u_w + N\nu_f \frac{\partial u}{\partial y}, \\ v &= v_w, T = T_w + D \frac{\partial T}{\partial y}, C = C_w \\ \text{At } y \rightarrow \infty, \quad u &= 0, \quad v = 0, T = T_\infty, C = C_\infty \end{aligned} \tag{5}$$

Energy equation

$$\begin{aligned} u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} &= \frac{k_{hnf}}{(\rho c_p)_{hnf}} \frac{\partial^2 T}{\partial y^2} + \\ &+ \frac{\mu_{hnf}}{(\rho c_p)_{hnf}} \left( \frac{\partial u}{\partial y} \right)^2 + \\ &+ \tau \left[ \frac{D_T}{T_\infty} \left( \frac{\partial T}{\partial y} \right)^2 + D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right] \end{aligned} \tag{3}$$

Here  $T_w = T_\infty + bx^{2n}$ ,  $b$  is a constant. Velocity slip factor  $N = N_1 x^{\frac{-n-1}{2}}$  which proportional to  $x$  with  $N_1$  initial value of velocity slip. Thermal slip factor  $D = D_1 x^{\frac{-n-1}{2}}$  which proportional to  $x$  with  $D_1$  initial value of thermal slip. Similarity variables are introduced as follow:

$$\eta = y \left( \frac{a(n+1)}{2\nu_f} \right)^{\frac{1}{2}} x^{\left(\frac{n-1}{2}\right)}; u = ax^n f'(\eta);$$

$$v = - \left( \frac{(n+1)a\nu_f}{2} \right)^{\frac{1}{2}} x^{\left(\frac{n-1}{2}\right)} \left[ f(\eta) + \frac{n-1}{n+1} \eta f'(\eta) \right];$$

$$T = T_\infty + bx^{2n} \theta(\eta); \phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}. \quad (6)$$

Using equation (6), equations (1)-(5) are written as:

$$f''' - \frac{\nu_f}{\nu_{hmf}} \left( \frac{2n}{n+1} f'^2 - ff'' \right) -$$

$$- \frac{2}{n+1} \frac{\sigma_{hmf} \mu_f}{\sigma_f \mu_{hmf}} Mf' -$$

$$- \frac{2}{n+1} f' d + \frac{2}{n+1} \frac{\nu_f}{\nu_{hmf}} \frac{(\beta_T)_{hmf}}{(\beta_T)_f} \frac{Gr_x}{Re_x^2} \theta +$$

$$+ \frac{2}{n+1} \frac{\nu_f}{\nu_{hmf}} \frac{(\beta_C)_{hmf}}{(\beta_C)_f} \frac{Gc_x}{Re_x^2} \phi = 0 \quad (7)$$

$$\frac{\theta''}{Pr} + \frac{\mu_{hmf} k_f}{\mu_f k_{hmf}} Ec f'^2 + \frac{k_f}{k_{hmf}} \frac{(\rho c_p)_{hmf}}{(\rho c_p)_f} \times$$

$$\times \left[ N_t \theta'^2 + N_b \theta' \phi' + f \theta' - \frac{4n}{n+1} f' \theta \right] = 0 \quad (8)$$

$$\phi'' + Le f \phi' + \frac{N_t}{N_b} \theta'' = 0 \quad (9)$$

$$f(0) = S, f'(0) = 1 + \lambda_1 f''(0), \theta(0) =$$

$$= 1 + \lambda_2 \theta'(0), \phi(0) = 1 \text{ and}$$

$$f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0 \quad (10)$$

$$\text{where } M = \frac{2\sigma_f B_0^2}{a(n+1)\rho_f}, \quad S = -\frac{\nu_w}{\sqrt{a\nu_f}},$$

$$d = \frac{2\nu_f}{a(n+1)Kx^{n-1}}, \quad Gr_x = \frac{g\beta_T(T_w - T_\infty)x^3}{\nu^2},$$

$$Gc_x = \frac{g\beta_C(C_w - C_\infty)x^3}{\nu^2}, \quad Le = \frac{\alpha}{D_B}, \quad Pr = \frac{\nu}{\alpha},$$

$$N_b = \frac{\tau D_B}{\nu_f}(C_w - C_\infty), \quad N_t = \frac{\tau D_T}{T_\infty} \frac{(T_w - T_\infty)}{\nu_f}.$$

The skin-friction coefficient  $C_f$ , Nusselt number  $Nu$  and Sherwood number  $Sh$  that characterize this study are given by:

$$C_{fx} = \frac{\mu_f}{\rho_f u_w^2} \frac{\partial u}{\partial y} \Big|_{y=0}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)},$$

$$Sh_x = \frac{xq_m}{D_B(C_w - C_\infty)} \quad (11)$$

where  $q_w = \frac{\partial T}{\partial y} \Big|_{y=0}$  is heat flux and  $q_m = -D_B \frac{\partial C}{\partial y} \Big|_{y=0}$  is mass flux at surface with thermal conductivity  $k$ .

Using (6) equations (11) become

$$Re_x^2 C_{fx} = \sqrt{\frac{n+1}{2}} f''(0),$$

$$Re_x^{\frac{1}{2}} Nu_x = -\sqrt{\frac{n+1}{2}} \theta'(0),$$

$$Re_x^{\frac{1}{2}} Sh_x = -\sqrt{\frac{n+1}{2}} \phi'(0), \quad (12)$$

here local Reynolds number  $Re_x = \frac{u_w x}{\nu}$ .

**Hybrid nanofluid model**

Thermophysical properties of the hybrid nanofluid of silver-titanium dioxide/water ( $Ag-TiO_2/H_2O$ ) are given as:

Hybrid nanofluid dynamic viscosity

$$\frac{\mu_{hnf}}{\mu_f} = \frac{1}{(1 - \phi_{Ag} - \phi_{TiO_2})^{2.5}}$$

Hybrid nanofluid density

$$\frac{\rho_{hnf}}{\rho_f} = (1 - \phi_{Ag} - \phi_{TiO_2}) + \phi_{Ag} \frac{\rho_{Ag}}{\rho_f} + \phi_{TiO_2} \frac{\rho_{TiO_2}}{\rho_f}$$

Hybrid nanofluid specific heat capacity

$$\begin{aligned} \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f} &= (1 - \phi_{Ag} - \phi_{TiO_2}) + \\ &+ \phi_{Ag} \frac{(\rho c_p)_{Ag}}{(\rho c_p)_f} + \phi_{TiO_2} \frac{(\rho c_p)_{TiO_2}}{(\rho c_p)_f} \end{aligned}$$

Hybrid nanofluid thermal conductivity

$$\frac{k_{hnf}}{k_f} = \left[ \frac{2k_f + \left( \frac{\phi_{Ag} k_{Ag} + \phi_{TiO_2} k_{TiO_2}}{\phi_{Ag} + \phi_{TiO_2}} \right) + 2(\phi_{Ag} k_{Ag} + \phi_{TiO_2} k_{TiO_2}) - 2(\phi_{Ag} + \phi_{TiO_2}) k_f}{2k_f + \left( \frac{\phi_{Ag} k_{Ag} + \phi_{TiO_2} k_{TiO_2}}{\phi_{Ag} + \phi_{TiO_2}} \right) - (\phi_{Ag} k_{Ag} + \phi_{TiO_2} k_{TiO_2}) + (\phi_{Ag} + \phi_{TiO_2}) k_f} \right]$$

Thermal and solutal volumetric coefficient

$$\begin{aligned} (\rho \beta_T)_{hnf} &= \\ &= (1 - \phi_{TiO_2}) \left( (\rho \beta_T)_{Ag} \phi_{Ag} + (1 - \phi_{Ag}) (\rho \beta_T)_f \right) + \\ &+ \phi_{TiO_2} (\rho \beta_T)_{TiO_2} \\ (\rho \beta_C)_{hnf} &= \\ &= (1 - \phi_{TiO_2}) \left( (\rho \beta_C)_{Ag} \phi_{Ag} + (1 - \phi_{Ag}) (\rho \beta_C)_f \right) + \\ &+ \phi_{TiO_2} (\rho \beta_C)_{TiO_2} \end{aligned}$$

where  $\phi_{Ag}$  and  $\phi_{TiO_2}$  specify the solid volume fraction of  $Ag$  and  $TiO_2$ .

Thermophysical properties of base fluid water and nanofluid are:

Physical properties	Base fluid	Nanoparticles	
	$H_2O$	$Ag$	$TiO_2$
$\sigma(1/\Omega m)$	0.05	$2.6 \times 10^6$	$6.30 \times 10^7$
$\rho(kg m^{-3})$	997.10	10500	4250
$Cp(J kg^{-1} K^{-1})$	4179	235	686
$k(W m^{-1} K^{-1})$	0.620	429	8.9538
$\beta_T \times 10^5(1/K)$	21	1.89	0.9

**3. Solution Methodology**

Keller box finite difference method is used to solve the formulated problem. In the first step

Let  $f' = r, f'' = r' = s, \theta' = t, \phi' = w$

For, these assumptions the higher order ODEs (7)-(10) are altered to first order ODEs as follow:

$$\begin{aligned} s' - \frac{v_f}{v_{hnf}} \left( \frac{2n}{n+1} r^2 - fs \right) - \frac{2M}{n+1} \frac{\sigma_{hnf}}{\sigma_f} \frac{\mu_f}{\mu_{hnf}} r - \\ - \frac{2d}{n+1} r + \frac{2}{n+1} \frac{v_f}{v_{hnf}} \frac{(\beta_T)_{hnf}}{(\beta_T)_f} \frac{Gr_x}{Re_x^2} \theta + \\ + \frac{2}{n+1} \frac{v_f}{v_{hnf}} \frac{(\beta_C)_{hnf}}{(\beta_C)_f} \frac{Gc_x}{Re_x^2} \phi = 0 \end{aligned} \tag{13}$$

$$\frac{1}{Pr} t' + \frac{\mu_{hnf}}{\mu_f} \frac{k_f Ec}{k_{hnf}} s^2 + \frac{k_f}{k_{hnf}} \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f} \times \tag{14}$$

$$\times \left[ N_t t^2 + N_b t w + f t - \frac{4n}{n+1} r \theta \right] = 0$$

$$w' + Lefw + \frac{N_t}{N_b} t' = 0 \tag{15}$$

$$\begin{aligned}
 f(0) &= 0, r(0) = 1 + \lambda_1 s(0), \\
 \theta(0) &= 1 + \lambda_2 t(0), \phi(0) = 1 \text{ and} \\
 r(\infty) &= \theta(\infty) = \phi(\infty) = 0
 \end{aligned}
 \tag{16}$$

The finite-difference form of the above equations are

$$\left. \begin{aligned}
 (f_j - f_{j-1}) - \frac{h_j}{2}(r_j + r_{j-1}) &= 0 \\
 (r_j - r_{j-1}) - \frac{h_j}{2}(s_j + s_{j-1}) &= 0 \\
 (\theta_j - \theta_{j-1}) - \frac{h_j}{2}(t_j + t_{j-1}) &= 0 \\
 (\phi_j - \phi_{j-1}) - \frac{h_j}{2}(w_j + w_{j-1}) &= 0
 \end{aligned} \right\}
 \tag{17}$$

For these centralized forms equations (13-16) will take the following form:

$$\begin{aligned}
 s_j - s_{j-1} - \frac{v_f}{v_{hmf}} h_j \left( \frac{2n}{n+1} r_{j-1}^2 - (fs)_{j-\frac{1}{2}} \right) - \\
 - \frac{\sigma_{hmf}}{\sigma_f} \frac{\mu_f}{\mu_{hmf}} M h_j(r)_{j-\frac{1}{2}} - \frac{2h_j d}{n+1} (r)_{j-\frac{1}{2}} + \\
 + \frac{2h_j v_f (\beta_T)_{hmf}}{n+1 v_{hmf} (\beta_T)_f} \frac{Gr_x}{Re_x^2} \theta_{j-\frac{1}{2}} + \\
 + \frac{2h_j v_f (\beta_C)_{hmf}}{n+1 v_{hmf} (\beta_C)_f} \frac{Gc_x}{Re_x^2} \phi_{j-\frac{1}{2}} = 0
 \end{aligned}
 \tag{18}$$

$$\begin{aligned}
 \frac{1}{Pr} (t_j - t_{j-1}) + \frac{\mu_{hmf}}{\mu_f} \frac{k_f}{k_{hmf}} h_j Ec . s_{j-1}^2 + \frac{k_f}{k_{hmf}} \frac{(\rho c_p)_{hmf}}{(\rho c_p)_f} \times \\
 \times \left[ N_t t_{j-1}^2 + N_b (tw)_{j-\frac{1}{2}} + (ft)_{j-\frac{1}{2}} - \frac{4n}{n+1} (r\theta)_{j-\frac{1}{2}} \right] = 0
 \end{aligned}
 \tag{19}$$

$$w_j - w_{j-1} + h_j Le (fw)_{j-\frac{1}{2}} + \frac{N_t}{N_b} (t_j - t_{j-1}) = 0
 \tag{20}$$

$$\begin{aligned}
 f_0 = S, r_0 = 1 + \lambda_1 s(0), \theta_0 = 1 + \lambda_2 t(0), \\
 \phi_0 = 1, r_j = 0, \theta_j = 0, \phi_j = 0
 \end{aligned}
 \tag{21}$$

**Newton's Method**

By means of newton's method we linearized the non-linear system using following iterates:

$$\left. \begin{aligned}
 f_j^{(m+1)} &= f_j^{(m)} + \delta f_j^{(m)} \\
 r_j^{(m+1)} &= r_j^{(m)} + \delta r_j^{(m)} \\
 s_j^{(m+1)} &= s_j^{(m)} + \delta s_j^{(m)} \\
 \theta_j^{(m+1)} &= \theta_j^{(m)} + \delta \theta_j^{(m)} \\
 t_j^{(m+1)} &= t_j^{(m)} + \delta t_j^{(m)} \\
 \phi_j^{(m+1)} &= \phi_j^{(m)} + \delta \phi_j^{(m)} \\
 w_j^{(m+1)} &= w_j^{(m)} + \delta w_j^{(m)}
 \end{aligned} \right\}
 \tag{22}$$

$$m = 0, 1, 2, 3, \dots$$

Substituting Equation (22) in equations (18-21) we have the following tridiagonal system.

$$\delta f_j - \delta f_{j-1} - \frac{h_j}{2} (\delta r_j + \delta r_{j-1}) = (a_1)_{j-\frac{1}{2}}
 \tag{23}$$

$$\delta r_j - \delta r_{j-1} - \frac{h_j}{2} (\delta s_j + \delta s_{j-1}) = (a_2)_{j-\frac{1}{2}}
 \tag{24}$$

$$\delta \theta_j - \delta \theta_{j-1} - \frac{h_j}{2} (\delta t_j + \delta t_{j-1}) = (a_3)_{j-\frac{1}{2}}
 \tag{25}$$

$$\delta \phi_j - \delta \phi_{j-1} - \frac{h_j}{2} (\delta w_j + \delta w_{j-1}) = (a_4)_{j-\frac{1}{2}}
 \tag{26}$$

$$\begin{aligned}
 (b_1) \delta s_j + (b_2) \delta s_{j-1} + (b_3) \delta f_j + (b_4) \delta f_{j-1} + \\
 + (b_5) \delta r_j + (b_6) \delta r_{j-1} = (a_5)_{j-\frac{1}{2}}
 \end{aligned}
 \tag{27}$$

$$\begin{aligned}
 (c_1) \delta t_j + (c_2) \delta t_{j-1} + (c_3) \delta f_j + (c_4) \delta f_{j-1} + \\
 + (c_5) \delta r_j + (c_6) \delta r_{j-1} + (c_7) \delta \theta_j + \\
 (c_8) \delta \theta_{j-1} + (c_9) \delta w_j + (c_{10}) \delta w_{j-1} + \\
 + (c_{11}) \delta s_j + (c_{12}) \delta s_{j-1} = (a_6)_{j-\frac{1}{2}}
 \end{aligned}
 \tag{28}$$

$$\begin{aligned}
 (d_1) \delta w_j + (d_2) \delta w_{j-1} + (d_3) \delta f_j + \\
 + (d_4) \delta f_{j-1} + (d_5) \delta t_j + (d_6) \delta t_{j-1} = (a_7)_{j-\frac{1}{2}}
 \end{aligned}
 \tag{29}$$

where

$$\left. \begin{aligned}
 (b_1)_j &= 1 + \frac{D_2}{D_1} \frac{h_j}{2} f_{j-\frac{1}{2}} \\
 (b_2)_j &= (b_1)_j - 2 \\
 (b_3)_j &= \frac{D_2}{D_1} \frac{h_j}{2} s_{j-\frac{1}{2}} \\
 (b_4)_j &= (b_3)_j \\
 (b_5)_j &= -\frac{D_2}{D_1} \frac{2n}{n+1} h_j r_{j-\frac{1}{2}} - \frac{M}{2} \frac{D_7}{D_1} h_j - \frac{h_j}{n+1} d \\
 (b_6)_j &= (b_5)_j
 \end{aligned} \right\}, \quad \left. \begin{aligned}
 (d_1)_j &= 1 + Le \frac{h_j}{2} f_{j-\frac{1}{2}} \\
 (d_2)_j &= (d_1)_j - 2 \\
 (d_3)_j &= Le \frac{h_j}{2} w_{j-\frac{1}{2}} \\
 (d_4)_j &= (d_3)_j \\
 (d_5)_j &= \frac{N_t}{N_b} \\
 (d_6)_j &= -(d_5)_j
 \end{aligned} \right\}$$
  

$$\left. \begin{aligned}
 (c_1)_j &= \frac{1}{Pr} + \frac{h_j}{2} D_3 D_4 \left[ 2N_t t_{j-\frac{1}{2}} + N_b w_{j-\frac{1}{2}} + f_{j-\frac{1}{2}} \right] \\
 (c_2)_j &= (c_1)_j - \frac{2}{Pr} \\
 (c_3)_j &= D_3 D_4 \frac{h_j}{2} t_{j-\frac{1}{2}} \\
 (c_4)_j &= (c_3)_j \\
 (c_5)_j &= -D_3 D_4 \frac{h_j}{2} \frac{4n}{n+1} \theta_{j-\frac{1}{2}} \\
 (c_6)_j &= (c_5)_j \\
 (c_7)_j &= -D_3 D_4 \frac{h_j}{2} \frac{4n}{n+1} r_{j-\frac{1}{2}} \\
 (c_8)_j &= (c_7)_j \\
 (c_9)_j &= D_3 D_4 \frac{h_j N_b}{2} t_{j-\frac{1}{2}} \\
 (c_{10})_j &= (c_9)_j \\
 (c_{11})_j &= D_1 D_4 h_j Ecs_{j-\frac{1}{2}} \\
 (c_{12})_j &= (c_{11})_j
 \end{aligned} \right\};$$

$$\left. \begin{aligned}
 (a_1)_j &= f_{j-1} - f_j + h_j \left( r_{j-\frac{1}{2}} \right) \\
 (a_2)_j &= r_{j-1} - r_j + h_j \left( s_{j-\frac{1}{2}} \right) \\
 (a_3)_j &= \theta_{j-1} - \theta_j + h_j \left( t_{j-\frac{1}{2}} \right) \\
 (a_4)_j &= \phi_{j-1} - \phi_j + h_j \left( w_{j-\frac{1}{2}} \right) \\
 (a_5)_j &= s_{j-1} - s_j + \frac{D_2}{D_1} \frac{2n}{n+1} r_{j-\frac{1}{2}}^2 - \frac{D_2}{D_1} h_j (fs)_{j-\frac{1}{2}} + \frac{MD_7}{D_1} h_j r_{j-\frac{1}{2}} + \frac{2dh_j}{(n+1)} r_{j-\frac{1}{2}} \\
 (a_6)_j &= \frac{1}{Pr} (t_{j-1} - t_j) - \left( -h_j (ft)_{j-\frac{1}{2}} + h_j \left( \frac{4n}{n+1} \right) (r\theta)_{j-\frac{1}{2}} - h_j N_b (tw)_{j-\frac{1}{2}} - h_j N_t t_{j-\frac{1}{2}}^2 - h_j Ec s_{j-\frac{1}{2}}^2 \right) D_3 D_4 \\
 (a_7)_j &= w_j - w_{j-1} - h_j Le (fw)_{j-\frac{1}{2}} - \frac{N_t}{N_b} (t_j - t_{j-1})
 \end{aligned} \right\}$$

$$\delta f_0 = \delta r_0 = \delta s_0 = \delta \theta_0 = \delta \phi_0 = \delta r_j = \delta \theta_j = \delta \phi_j = 0$$

**Block-elimination method**

$$A\delta = \alpha \tag{30}$$

The linearized equations (23-29) solved by block tridiagonal structure having variables or constants. The matrix-vector form of equations (23-29) is

The matrices elements are defined as

$$A = \begin{bmatrix} [A_1] & [C_1] & & & & & & \\ [B_2] & [A_2] & [C_2] & & & & & \\ & & & \ddots & & & & \\ & & & & \ddots & & & \\ & & & & & \ddots & & \\ & & & & & & [B_{j-1}] & [A_{j-1}] & [C_{j-1}] \\ & & & & & & & [B_j] & [A_j] \end{bmatrix}; \delta = \begin{bmatrix} [\delta_1] \\ [\delta_2] \\ \vdots \\ [\delta_{j-1}] \\ [\delta_j] \end{bmatrix} \text{ and } \alpha = \begin{bmatrix} [a_1] \\ [a_2] \\ \vdots \\ [a_{j-1}] \\ [a_j] \end{bmatrix}$$

where

$$[A_1] = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ \frac{-h_j}{2} & 0 & 0 & 0 & \frac{-h_j}{2} & 0 & 0 \\ 0 & \frac{-h_j}{2} & 0 & 0 & 0 & \frac{-h_j}{2} & 0 \\ 0 & 0 & \frac{-h_j}{2} & 0 & 0 & 0 & \frac{-h_j}{2} \\ (b_2)_1 & 0 & 0 & (b_3)_1 & (b_1)_1 & 0 & 0 \\ (c_{12})_1 & (c_2)_1 & (c_{10})_1 & (c_3)_1 & (c_{11})_1 & (c_1)_1 & (c_9)_1 \\ 0 & (d_6)_1 & (d_2)_1 & (d_2)_1 & 0 & (d_5)_1 & (d_1)_1 \end{bmatrix}$$

$2 \leq j \leq J:$



$$[A_j] = \begin{bmatrix} \frac{-h_j}{2} & 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & \frac{-h_j}{2} & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & \frac{-h_j}{2} & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & \frac{-h_j}{2} \\ (b_6)_j & 0 & 0 & (b_3)_j & (b_1)_j & 0 & 0 \\ (c_6)_j & (c_8)_j & 0 & (c_3)_j & (c_{11})_j & (c_1)_j & (c_9)_j \\ 0 & 0 & 0 & (d_3)_j & 0 & (d_5)_j & (d_1)_j \end{bmatrix};$$

$$[B_j] = \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{-h_j}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{-h_j}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-h_j}{2} \\ 0 & 0 & 0 & (b_4)_j & (b_2)_j & 0 & 0 \\ 0 & 0 & 0 & (c_4)_j & (c_{12})_j & (c_2)_j & (c_{10})_j \\ 0 & 0 & 0 & (d_4)_j & 0 & (d_6)_j & (d_2)_j \end{bmatrix};$$

$$[C_j] = \begin{bmatrix} \frac{-h_j}{2} & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ (b_5)_j & 0 & 0 & 0 & 0 & 0 & 0 \\ (c_5)_j & (c_7)_j & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$L = \begin{bmatrix} [\alpha_1] & & & & & & \\ [\beta_2] & [\alpha_2] & [c_2] & & & & \\ & & & \ddots & & & \\ & & & & \ddots & & \\ & & & & & \ddots & \\ & & & & & & [\alpha_{j-1}] \\ & & & & & & [\beta_j] & [\alpha_j] \end{bmatrix},$$

$$[\delta_1] = \begin{bmatrix} \delta s_0 \\ \delta \theta_0 \\ \delta \phi_0 \\ \delta f_1 \\ \delta s_1 \\ \delta t_1 \\ \delta w_1 \end{bmatrix}, [\delta_j] = \begin{bmatrix} \delta s_{j-1} \\ \delta \theta_{j-1} \\ \delta \phi_{j-1} \\ \delta f_j \\ \delta s_j \\ \delta t_j \\ \delta w_j \end{bmatrix}, [a_j] = \begin{bmatrix} (r_1)_{j-\frac{1}{2}} \\ (r_2)_{j-\frac{1}{2}} \\ (r_3)_{j-\frac{1}{2}} \\ (r_4)_{j-\frac{1}{2}} \\ (r_5)_{j-\frac{1}{2}} \\ (r_6)_{j-\frac{1}{2}} \\ (r_7)_{j-\frac{1}{2}} \end{bmatrix}.$$

$$U = \begin{bmatrix} [I_1] & [\Gamma_{j-1}] & & & & & \\ & [I_1] & & & & & \\ & & \ddots & & & & \\ & & & \ddots & & & \\ & & & & \ddots & & \\ & & & & & [I_{j-1}] & [\Gamma_{j-1}] \\ & & & & & & [I] \end{bmatrix},$$

where [I] is identity matrix,  $[\alpha_i]$  and  $[\Gamma_i]$  are determined as follow:

$$[\alpha_i] = [A_1]$$

$$[A_1][\Gamma_1] = [C_1]$$

$$[\alpha_i] = [A_1] - [B_j][\Gamma_{j-1}], j = 2, 3, \dots, J$$

$$[\alpha_j][\Gamma_j] = [C_j], j = 2, 3, \dots, J - 1.$$

Now let the non-singular matrix A is factorized into

$$A = LU,$$

where

So we have

$$LU\delta = a, \text{ Let } U\delta = V \text{ and we get } LV = a$$

$$V = \begin{bmatrix} [V_1] \\ [V_2] \\ \vdots \\ [V_{j-1}] \\ [V_j] \end{bmatrix}$$

Where,  $[\alpha_1][V_1] = [a_1]$ ,  
 $[\alpha_i][V_j] = [a_j] - [B_i][V_{j-1}], 2 \leq j \leq J$   
 $[\delta_j] = [W_j], [\delta_i] = [V_j] - [\Gamma_j][\delta_{j+1}], 2 \leq j \leq J$

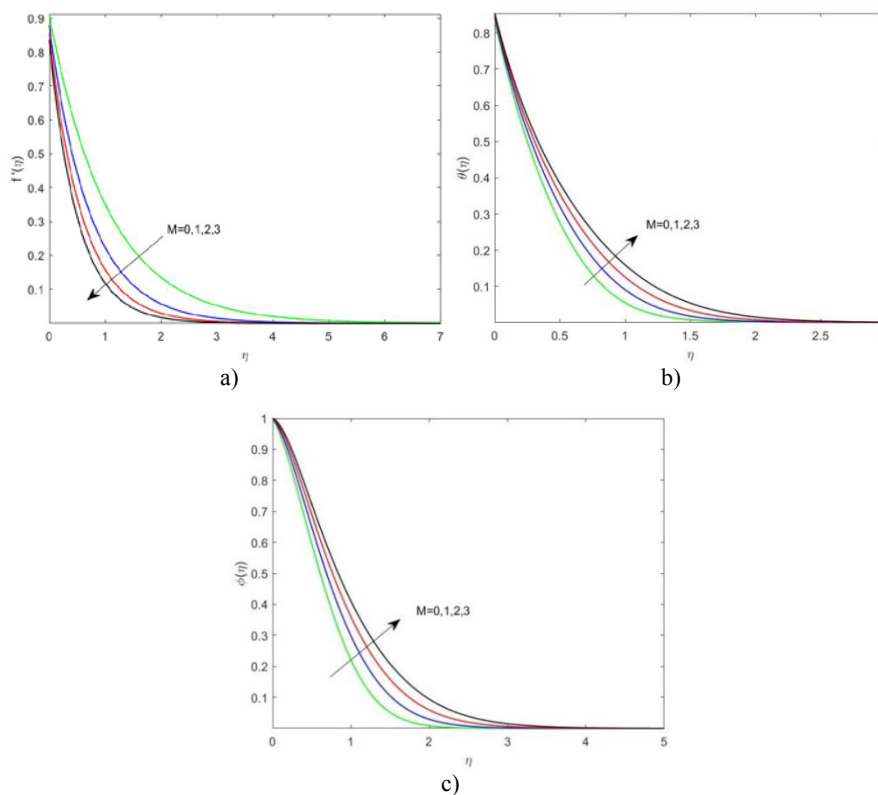
Using uniform grid of size  $\Delta \eta = 0.006$  the process is continued, until the convergence criteria is satisfied.

#### 4. Results and discussion

To analyse the physical aspects of the considered problem, a comprehensive study have been done. The graphs for fluid velocity, temperature and concentration for the different

values of physical parameters such as velocity and thermal slip parameter, Prandtl number, suction parameter, magnetic parameter, Eckert number are shown in figures (2-12). By setting parameter  $M=1, \lambda=0.1, \delta=0.1, S=0.5, Ec=1, Pr=6, Nb=Nt=0.1, Le=2$  the numerical results for local skin friction, Nusselt and Sherwood number are presented graphically and also in tabular form (Table 1-2) with a good agreement with published literature.

Figure 2(a)-2(c) illustrates the impact of magnetic field on velocity, temperature and concentration fields respectively. Magnetic field initiates a resistive force called Lorentz force. By upsurging the value of magnetic field parameter  $M$ , the Lorentz force increases and the thickness of boundary layer reduces, therefore velocity profile decreases with an increasing magnetic field, figure 2(a). Thermal and concentration boundary layers become thicker with enhancing Lorentz force because it is a body retarding force that transverses the motion of the fluid and heat is evolved. It is shown in figure (2(b), 2(c)) that the profiles of temperature and concentrations are enhances with increasing magnetic field.

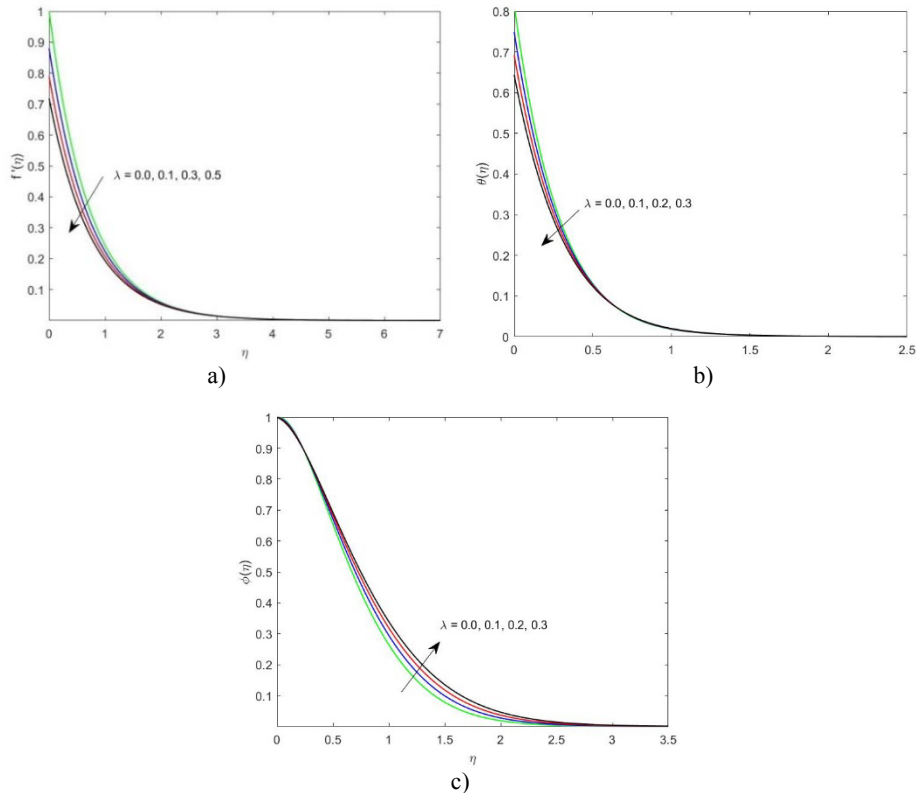


**Figure 2** – Effect of Magnetic parameter  $M$  on a) velocity profile, b) temperature profile, c) concentration profile

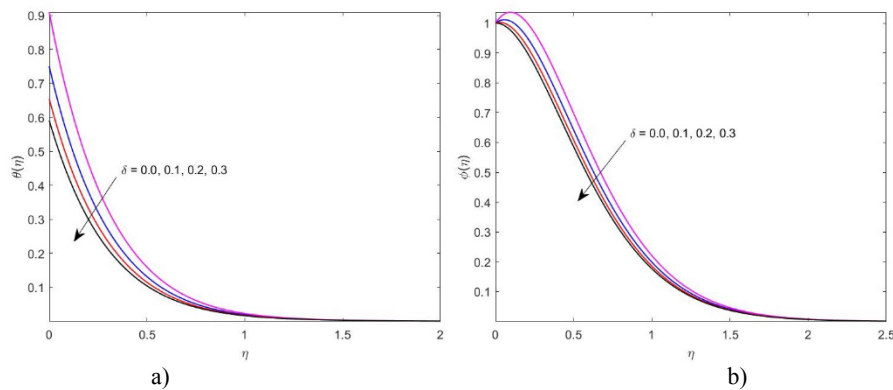
Figure 3(a)-3(c) represents the impact of the velocity slip parameter  $\lambda$ . As the slip condition occurs, the fluid velocity at surface is not same as the velocity of surface, therefore fluid velocity decreases as the velocity slip parameter increases, figure 3(a). Figure 3(b) and 3(c) illustrates that the temperature and concentration fields enhances with velocity slip parameter. As the thermal slip parameter  $\delta$  increases, heat is

transformed from the fluid to surface and the thickness of thermal boundary layer decreases.

As the thermal slip parameter  $\delta$  increases, heat is mutating from the fluid to surface and the thickness of thermal boundary layer decreases. It is shown in figure 4(a) and 4(b) that the thermal and concentration profile decreases with increasing thermal slip parameter.



**Figure 3** – Effect of velocity slip parameter  $\lambda$  on a) velocity profile, b) temperature profile, c) concentration profile

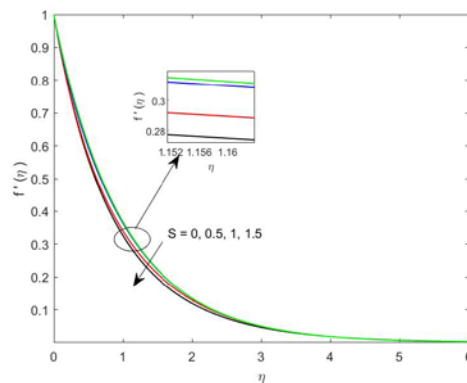


**Figure 4** – Effect of thermal slip parameter  $\delta$  on a) temperature profile, b) concentration profile

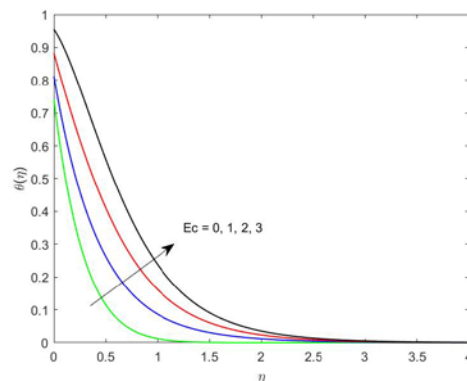
It is observed in figure (5) that as the suction parameter  $S$  increases, velocity profile decreases, since the suction effects retards the fluid motion. Figure (6) depicts the effects of Eckert number  $Ec$  on the thermal field  $\theta(\eta)$ . Since Eckert number (viscous dissipation parameter) is the ratio of advective heat transfer to heat dissipation potential i.e. the transformation of kinetic energy into internal energy. Due to this dissipative heat the thickness of thermal boundary layer increases and thus the

temperature profile increases with increasing Eckert number  $Ec$ .

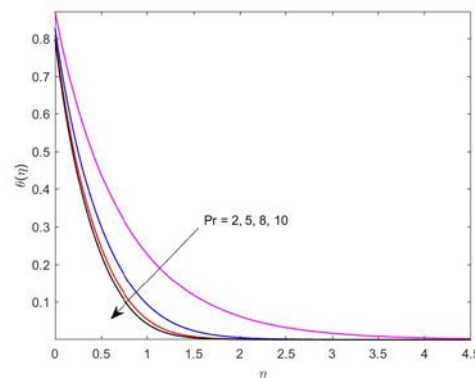
The effect of Prandtl number  $Pr$  on temperature profile is illustrates via Figure (7). Since Prandtl number is defined as the ratio between convective to conductive method of heat transfer. So heat conduction reduces due to low thermal conductivity by increasing Prandtl number. Thus the temperature profile drop off with higher Prandtl number.



**Figure 5** – Effect of suction parameter  $S$  on velocity profile



**Figure 6** – Effect of Eckert number  $Ec$  on temperature profile



**Figure 7** – Effect of Prandtl number  $Pr$  on temperature profile

Lewis number is the ratio of thermal diffusivity to species diffusivity. By enhancing the Lewis number tends to a flow separation close to the wall which results to a reduced concentration boundary layer thickness. It is observed in figure (8) that the concentration profile reduces with high Lewis number. The impacts of Brownian motion parameter  $Nb$  and thermophoresis parameter  $Nt$  are illustrate in figure 9(a)-9(c). Due to the temperature gradient,

thermophoresis force and Brownian motion higher the fluid speed away from the surface and fluid is heated further. This increases the thickness of thermal boundary layer and temperature profile increases with both thermophoresis parameter  $Nt$  and Brownian motion parameter  $Nb$ , the concentration field increases with thermophoresis parameter  $Nt$  and decreases with Brownian motion parameter  $Nb$ .

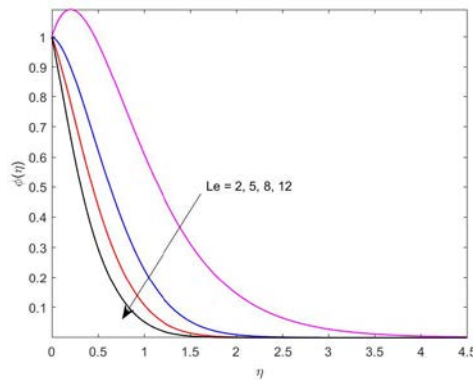


Figure 8 – Effect of Lewis number  $Le$  on concentration profile.

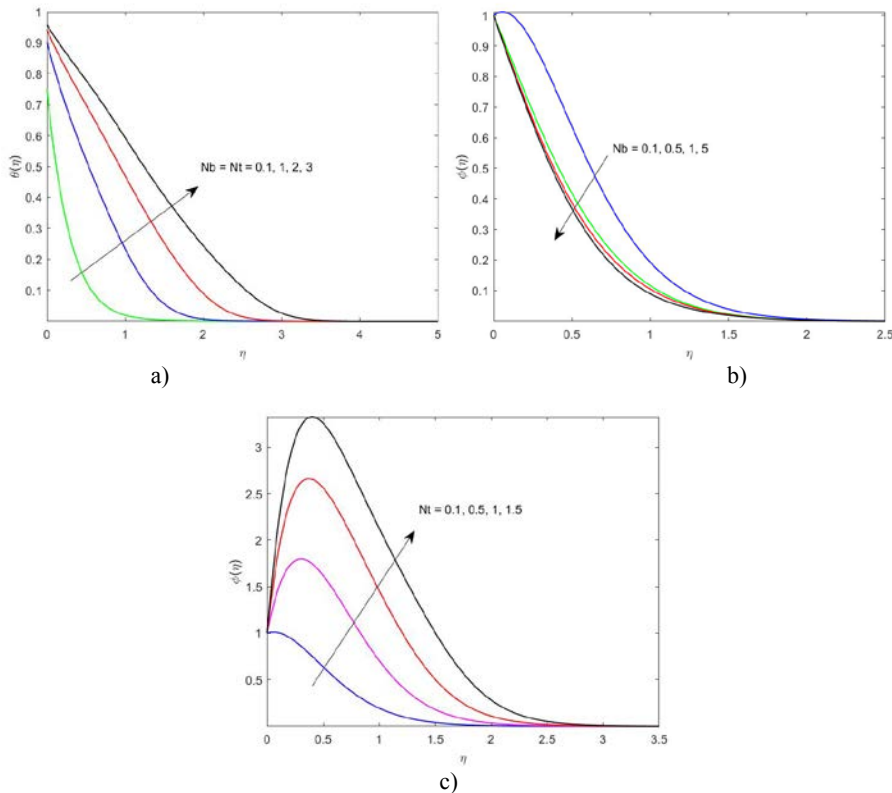
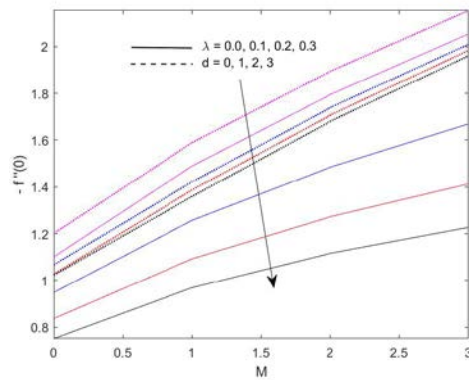


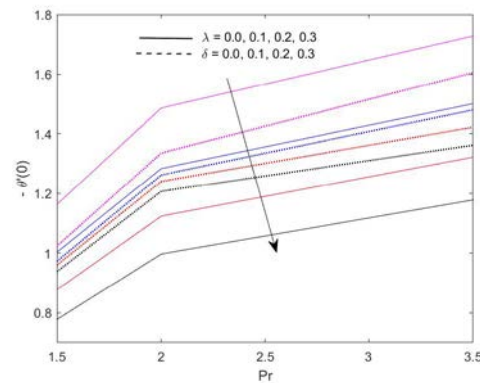
Figure 9 – Effect of Brownian motion parameter  $Nb$  and thermophoresis parameter  $Nt$  on a) temperature profile, b) concentration profile, c) concentration profile

The variation in coefficient of skin-friction, Nusselt and Sherwood number for involved parameters have been depicted in figures (10-12). It is observed that the coefficient of skin-friction depreciates with both velocity slip parameter and porosity parameter. The variation in local Nusselt number for velocity and thermal slip parameters

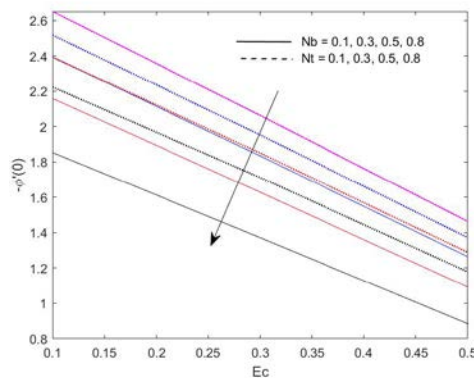
have been depicted in figures (11). It is observed that the rate of heat transfer decreases for both velocity and thermal slip, while increases for Prandtl number. Figure (12) displays the impact of Brownian motion parameter and thermophoresis number on Sherwood number, both depreciates the Sherwood number.



**Figure 10** – Effect of velocity slip parameter  $\lambda$  and porosity parameter  $d$  on skin-friction coefficient



**Figure 11** – Effect of velocity slip parameter  $\lambda$  and thermal slip parameter  $\delta$  on Nusselt number



**Figure 12** – Effect of Brownian motion parameter  $Nb$  and thermophoresis number  $Nt$  on Nusselt number

Table-(1) and (2) presents a comparison of the current findings with previous studies by Ramya et al.[3] and Hamad et. al.[5]. The alignment between the present results and earlier studies demonstrates a high level of consistency. Additionally, this agreement supports the reliability and accuracy of the current findings.

**Table 1** – Comparison for Nusselt number and Sherwood number Pr=10, n=10, Le=10, Nt=0.3, Ec=0, M=0

Nb	$-\theta'(0)$			$\phi'(0)$		
	Ramya et al.[3]	Hamad et. al.[5]	Present Study	Ramya et al.[3]	Hamad et. al.[5]	Present Study
0.1	3.7716	3.7716	3.7715	5.6212	5.6219	5.6211
0.2	3.2514	3.2515	3.2514	0.9977	0.9981	0.9976
0.3	2.8278	2.8279	3.8277	0.4521	0.4517	0.4522

**Table 2** – Comparison for skin-friction Le=1, Pr=6.8, Nb=0.1, Nt=0.1, Ec= $\lambda$ = $\delta$ =M=0

n	Ramya et al. [3]	Hamad et al. [5]	Present Study
0.2	0.7675	0.7674	0.7670
0.5	0.8901	0.8901	0.8897
1.0	1.0005	1.0004	1.0003
3.0	1.1490	1.1489	1.1485
10	1.2352	1.2352	1.2346
20	1.2578	1.2577	1.2571

### 5. Conclusion

A computational study is done to investigate convective heat transfer on MHD hybrid nanofluid with velocity and thermal slip over a nonlinear stretching sheet in porous medium. The governing PDE's are evolved and transformed into ODE's by using suitable similarity transformations. The governing ODE's are solved by Keller-box finite difference approach by using MATLAB software. Then the following results are made:

- Velocity slip, thermal slip, and the presence of a magnetic field result in a reduction of the velocity field. While, the velocity field is enhanced by the porosity parameter and the Lewis number.
- Velocity slip, thermal slip, and a high Prandtl number lead to a decrease in the temperature field. Conversely, the temperature field is enhanced by higher Eckert number, Lewis number, the presence of a magnetic field, the Brownian motion parameter, and the thermophoresis parameter.

- The concentration field diminishes as the Brownian motion parameter, porosity parameter, thermal slip, and Lewis number increase, while it improves with higher thermophoresis parameter, magnetic field, and velocity slip.
- Skin friction coefficient improves for magnetic parameter and depreciates for velocity slip parameter and porosity parameter.
- Nusselt number improves for Prandtl number, while depreciate for velocity and thermal slip parameter.
- Sherwood number depreciates for increasing Eckert number, Brownian motion parameter and thermophoresis number.

### Nomenclature:

- $u, v$  Velocity components in x and y axis, respectively (unit:  $m.s^{-1}$ ).
- $u_w$  Stretching velocity of the sheet (unit:  $ms^{-1}$ ).
- $n$  Non linear stretching parameter.
- $B$  Magnetic field strength (unit:  $A.m^{-1}$ ).
- $g$  Gravitational acceleration ( $m.s^{-2}$ ).
- $\beta_T$  Temperature convection coefficient.
- $\beta_c$  Concentration convection coefficient.
- $\mu$  Dynamic viscosity (unit:  $kg.m^{-1}.s^{-1}$ ).
- $\nu$  Kinematic viscosity (unit:  $m^2.s^{-1}$ ).
- $\rho$  Density of the fluid ( $kg.m^{-3}$ ).
- $\sigma$  Electrical conductivity ( $S.m^{-1}$ ).
- $k'$  Permeability of porous medium.
- $\phi$  Volume fraction.
- $\lambda$  Velocity slip parameter.
- $\delta$  Thermal slip parameter.
- $T$  Temperature.
- $C$  Concentration.
- $d$  Porosity parameter.
- $c_p$  Specific heat capacity ( $J.kg^{-1}.K$ )
- $Ag$  Silver.
- $TiO_2$  Titanium Dioxide.
- $Le$  Lewis number.
- $M$  Magnetic parameter.
- $Nb$  Brownian motion parameter.
- $Nt$  Thermophoresis parameter.
- $N_1$  Initial velocity slip.
- $D_1$  Initial thermal slip.
- $Pr$  Prandtl number.
- $S$  Suction parameter.
- $Gr$  Thermal Grashof number.
- $Gc$  Solutal Grashof number.
- $Ec$  Eckert number.
- $Re$  Reynolds number.

**Subscripts**

- $f$  Base fluid water.  
 $\infty$  Conditions far away from the surface.  
 $w$  Conditions at the surface.  
 $nf$  Nanofluid.  
 $hnf$  Hybrid nanofluid.

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