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An analytical approach for solving fractional financial risk system

Abstract. This article introduces an innovative analytical method tailored to address the complexities of non-linear FFR (”fractional financial risk”) models. The LRPS (”Laplace residual power se- ries”) approach non-linear FFR models empowers risk analysts to more accurately assess portfolios and predict potential losses span- ning diverse risk categories. These encompass credit risk, market risk, model risk, liquidity risk, and operational risk. By expand- ing the array of techniques for risk modeling, this study offers a valuable asset for refining risk assessment and management strate- gies across these distinct risk domains. Through the utilization of the LRPS approach, this methodology rapidly generates accu- rate solutions, providing an efficient pathway for approximating the intricate non-linear FFR models intrinsic to risk modeling. By means of numerical simulations and graphical representations, the article effectively demonstrates the efficacy of the LRPS technique. This study not only offers a practical and time-efficient tool for fi- nancial risk analysis but also contributes valuable insights to the advancement of novel techniques within the realm of financial risk management.
Key words: Fractional Financial Risk model, Risk assessment, Risk management strategies, Complex financial landscapes

Introduction

In the economic field, managing financial risk is crucial for economic entities to ensure their financial stability and avoid potential losses. Financial risk refers to the potential of suffering losses due to unpre- dictable changes in endogenous factors in financial or investment activ- ities, which can result in erratic fluctuations. It is, therefore, essential for businesses and financial institutions to understand financial risk and develop effective strategies to manage it [3, 1]. Financial risk can arise from various sources, including market risk, credit risk, operational risk, and liquidity risk. Market risk arises from changes in market conditions, such as fluctuations in interest rates, currency exchange rates, or commodity prices. Credit risk arises from the possibility of borrowers defaulting on their loans, and operational risk refers to the risk of loss resulting from inadequate or failed internal processes, people, and systems. Finally, liquidity risk arises when an entity is unable to meet its financial obligations as they become due [8]. To manage financial risk effectively, economic entities must understand the sources of risk, assess their risk exposure, and implement appropri- ate risk management strategies. This involves using financial instru- ments, models, and algorithms to measure and manage risk exposure, as well as monitoring and reporting tools to track and communicate risk metrics to stakeholders. Effective risk management helps busi- nesses and financial institutions avoid potential losses, enhance their financial performance, and maintain the trust and confidence of their investors and customers[4]. In conclusion, understanding and managing financial risk is crucial for economic entities to ensure their financial stability and avoid potential losses. By developing effective risk management strategies, businesses and financial institutions can mitigate financial risks and maintain their competitiveness in the dynamic and interconnected global economy[5].

Preliminaries In this section, we revisit important definitions and results pertaining to Caputo’s fractional derivatives and fractional Laplace transform. This includes recalling essential

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theories such as the fractional Taylor's formula as noted in previous works by [15].

2.1. **Definition.** The CFD [2] of function $h(t)$ as:

$$D_t^{\rho} h(t) = \frac{1}{\Gamma(n-\rho)} \int_0^t \frac{h^{(n)}(\delta)}{(t-\delta)^{\rho+1-n}} d\delta, t > 0. \quad (1)$$

The LRPS approach offers a solution for linear and nonlinear FDEs by determining the coefficients of the fractional power series approximate solution. The system of FDE in the context of Caputo's definition can be expressed as follows:

$$D_t^{\rho} h(t) = g_i\big(t, h_1(t), h_2(t), \dots, h_n(t)\big),$$
$$t \geq 0, \rho \in (0,1]. \quad (2)$$

Let h(t) represent the set of smooth functions, gi denote the linear as well as non-linear functions in the context described.

Initially, we commence the process by applying the Laplace transform to both sides of the system in the following manner:

$$L[D_t^{\rho} h(t)] =$$
$$= L[g_i\big(t, h_1(t), h_2(t), \dots, h_n(t)\big)] \quad (3)$$

By using the formula

$$L[D_t^{\rho} h(t)] = s^{\rho} H(s) - s^{(\rho-1)} h(0) \quad (4)$$

by setting $G(s) = L[g_i(t, h_1(t), h_2(t), \dots, h_n(t))]$ and using the initial conditions, system can be found as:

$$H(s) = \frac{a_n}{s} + \frac{1}{s^p} G(s), \quad (5)$$

where $n = 1, 2, 3\ldots$ Subsequently, the Laplace solution is presented as follows:

$$H(s) = \sum_{n=0}^{\infty} \frac{a_n}{s^{n\rho+1}}, s > 0 \quad (6)$$

Since $a_n = \lim_{s\to 0} sH(0)$. The k-th Laplace series solution is given as:

$$H^k(s) = \frac{a_0}{s} + \sum_{n=1}^{k} \frac{a_n}{s^{n\rho+1}}, s > 0 \quad (7)$$

And the k-th Laplace residual functions is defined in the following man ner:

$$L\big[Res_k\big(H(s)\big)\big] = H^k(s) - \frac{a_0}{s} - \frac{1}{s^{\rho}} G(s) \quad (8)$$

The coefficients a_n can be determined by solving

$$\lim_{s\to\infty} s^{k\rho+1} L[Res_k(H(s))] = 0,$$
for $k = 1, 2, 3, \dots$ and $0 < \rho < 1$.

Finally, in the concluding step, we proceed with the inverse Laplace to determine the k-th LRPS approximate solution.

3. Fraction Financial Risk Model

We are presented the fractional financial risk model with Caputo's approach as follows:

$$D_t^{\gamma} R^*(t) = -\varsigma(U^* - R^*) + U^* V^*,$$
$$D_t^{\gamma} U^*(t) = rR^* - U^* - R^* V^* \quad (9)$$
$$D_t^{\gamma} V^*(t) = R^* U^* - b\, V^*$$

subject to initial conditions,

$$R^*(0) = R_0^*, U^*(0) = U_0^*, V^*(0) = V_0^* \quad (10)$$

In this model, R^*, U^*, and V^* represent the occurrence value, analy- sis value risk, and control value risk in the current market, respectively. Here, ζ, r, and $k = 1 - b$ denote the analysis risk efficiency, transmission rate of previous risk, and distortion coefficient of risk control, respectively.

4. Laplace RPS solution of Fraction Financial Risk Model

We will now use the LRPS approach to find the approximate solution for the Fractional Financial Risk Model (9).

$$R(s) = \frac{R_0^*}{s} - \frac{\zeta}{s^{\gamma}}[U(s) - R(s)] +$$
$$+ \frac{1}{s^{\gamma}} L[L^{-1}U(s) L^{-1}V(s)]$$
$$U(s) = \frac{U_0^*}{s} - \frac{r}{s^{\gamma}} R(s) - \frac{U(s)}{s^{\gamma}} -$$
$$- \frac{1}{s^{\gamma}} L[L^{-1}R(s) L^{-1}V(s)] \quad (11)$$

$$V(s) = \frac{V_0^*}{s} - \frac{b}{s^{\gamma}} V(s) + \frac{1}{s^{\gamma}} L[L^{-1}R(s) L^{-1}V(s)]$$

Now, we are using the assumption of k-th series Laplace solutions of model (11) is as follows:

$$R^k(s) = \frac{R_0^*}{s} + \sum_{n=1}^{k} \frac{a_n}{s^{n\gamma+1}}, s > 0$$

$$U^k(s) = \frac{U_0^*}{s} + \sum_{n=1}^{k} \frac{b_n}{s^{n\gamma+1}}, s > 0 \quad (12)$$
$$V^k(s) = \frac{V_0^*}{s} + \sum_{n=1}^{k} \frac{c_n}{s^{n\gamma+1}}, s > 0$$

The convergence of the system (12) is similar as discussed in [7]. The coefficients a_n, b_n and c_n can be computed as:

$$L[Res_k R(s)] = R^k(s) - \frac{R_0^*}{s} - \frac{\zeta}{s^\gamma}[U^k(s) - R^k(s)] + + \frac{1}{s^\gamma} L[L^{-1}U^k(s) L^{-1}V^k(s)]$$

$$L[Res_k U(s)] = U^k(s) - \frac{U_0^*}{s} - \frac{r}{s^\gamma} R^k(s) + + \frac{U^k(s)}{s^\gamma} - \frac{1}{s^\gamma} L[L^{-1}R^k(s) L^{-1}V^k(s)]$$

$$L[Res_k V(s)] = V^k(s) - \frac{V_0^*}{s} - \frac{b}{s^\gamma} V^k(s) - - \frac{1}{s^\gamma} L[L^{-1}R^k(s) L^{-1}V^k(s)] \quad (13)$$

Here for the value of $k = 1, 2, 3...$, we can solve the system (13). The value of coefficients a_n, b_n and c_n can be computed by using these formula's:

$$\lim_{s\to\infty} s^{k\gamma+1} L[Res_k(R(s))] = = 0; \lim_{s\to\infty} s^{k\gamma+1} L[Res_k(U(s))] = = 0; \lim_{s\to\infty} s^{k\gamma+1} L[Res_k(V(s))] = 0;$$

$$a_1 = \zeta U_0^* + U_0^* V_0^* - \zeta R_0^*, b_1 = = rR_0^* - U_0^* - U_0^* V_0^*, c_1 = R_0^* V_0^* - bV_0^*$$

$$a_2 = \zeta b_1 + c_1 U_0^* - \zeta a_1, b_2 = = ra_1 - b_1 + c_1 U_0^* - b_1 V_0, c_2 = = c_1 R_0^* + a_1 V_0^* - bc_1$$

$$a_3 = \zeta b_2 - \zeta a_2 + c_2 U_0^* + \frac{\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)} c_1 b_1 + +b_2 V_0^*, b_3 = ra_2 - b_2 - c_2 U_0^* - b_1 c_1 - b_2 V_0^*, c_3 = = c_2 R_0^* + a_2 V_0^* + \frac{\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)} a_1 c_1 - bc_2$$, and so on.

The Laplace series solution of model (11) is given by:

$$(14)\ R(s) = \frac{R_0^*}{s} + \frac{\zeta U_0^* + U_0^* V_0^* - \zeta R_0^*}{s^{\gamma+1}} + + \frac{\zeta b_1 + c_1 U_0^* - \zeta a_1}{s^{2\gamma+1}} + + \left(\zeta b_2 - \zeta a_2 + c_2 U_0^* + \frac{\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)} c_1 b_1 + b_2 V_0^*\right) \frac{1}{s^{3\gamma+1}} + \ldots$$

$$U(s) = \frac{U_0^*}{s} + \frac{rR_0^* - U_0^* - U_0^* V_0^*}{s^{\gamma+1}} + + \frac{ra_1 - b_1 + c_1 U_0^* - b_1 V_0}{s^{2\gamma+1}} + +(ra_2 - b_2 - c_2 U_0^* - b_1 c_1 - b_2 V_0^*) \frac{1}{s^{3\gamma+1}} + \ldots$$

$$V(s) = \frac{V_0^*}{s} + \frac{R_0^* V_0^* - bV_0^*}{s^{\gamma+1}} + \frac{c_1 R_0^* + a_1 V_0^* - bc_1}{s^{2\gamma+1}} + + \left(c_2 R_0^* + a_2 V_0^* + \frac{\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)} a_1 c_1 - bc_2\right) \frac{1}{s^{3\gamma+1}} + \ldots$$

Taking inverse Laplace transform on both sides of model (14), we have the final solution:

$$(15)\ R^*(t) = R_0^* + \frac{(\zeta U_0^* + U_0^* V_0^* - \zeta R_0^*) t^\gamma}{\Gamma(\gamma+1)} + + \frac{(\zeta b_1 + c_1 U_0^* - \zeta a_1) t^{2\gamma}}{\Gamma(2\gamma+1)} + + \left(\zeta b_2 - \zeta a_2 + c_2 U_0^* + \frac{\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)} c_1 b_1 + b_2 V_0^*\right) \frac{t^{3\gamma}}{\Gamma(3\gamma+1)} + \ldots$$

$$U^*(t) = U_0^* + \frac{(rR_0^* - U_0^* - U_0^* V_0^*) t^\gamma}{\Gamma(\gamma+1)} + + \frac{(ra_1 - b_1 + c_1 U_0^* - b_1 V_0) t^{2\gamma}}{\Gamma(2\gamma+1)} + +(ra_2 - b_2 - c_2 U_0^* - b_1 c_1 - b_2 V_0^*) \frac{t^{3\gamma}}{\Gamma(3\gamma+1)} + \ldots$$

$$V^*(t) = V_0^* + \frac{(R_0^* V_0^* - bV_0^*) t^\gamma}{\Gamma(\gamma+1)} + + \frac{(c_1 R_0^* + a_1 V_0^* - bc_1) t^{2\gamma}}{\Gamma(2\gamma+1)} + + \left(c_2 R_0^* + a_2 V_0^* + \frac{\Gamma(2\gamma+1)}{\Gamma^2(\gamma+1)} a_1 c_1 - bc_2\right) \frac{t^{3\gamma}}{\Gamma(3\gamma+1)} + \ldots$$

Numerical Results

In this section, we give mathematical outcomes for the solution of the fractional order FFRS (9) to exhibit the presentation and the efficiency of the LRPS technique in taking care of such models. Utilizing this proposed technique, we get an approximate solution for the FFRS (9) as a fast convergent series. The surmised solutions are introduced in illustrations as a graphs and tables arranged upsides of $R^*(t)$, $U^*(t)$ and $V^*(t)$. The FFRS (9) for particular values of $\zeta = 10$, $r = 28$ and $b = 8/3$ as follows:

$$D_t^{\gamma} R^*(t) = -10(U^* - R^*) + U^*R^*, \quad (16)$$

$$D_t^{\gamma} U^*(t) = 28R^* - U^* - R^*V^*,$$

$$D_t^{\gamma} V^*(t) = R^*U^* - 8V^*/3$$

we are using IC's are $R^*(0) = 20$, $U^*(0) = 20$ and $V^*(0) = 20$. Here, γ indicates the fractional derivative order and $0 < \gamma \leq 1$. We are using the LRPS technique in FFR (16) and calculated all series solutions $R^*(t)$, $U^*(t)$ and $V^*(t)$.

In the Figures 1, 2, 3 and Table 1 shows the behavior of LRPS-solution at the fractional order $\gamma = 1$ over the interval [0, 0.5]. From these graphical outcomes, obviously the approximations got by the LRPS strategy are very efficient and the effectiveness can accomplished use moderately tiny number of terms in our example. However, The en- hancement of efficiency can be significantly magnified through the aug- mentation of terms within the power series. These graphs also shows that the presented method can predict the nature of compartments $R^*(t)$, $U^*(t)$ and $V^*(t)$ accurately for the region under consideration, where the behavior of such approximations are in good agreement with each other.

Table1 – Approximate solution of $R^*(t_i), U^*(t_i) and\ V^*(t_i)$V ∗(t) for different values of t using LRPS.

t_i	$R^*(t_i)$	$U^*(t_i)$	$V^*(t_i)$
0	20	20	20
0.1	213.285	22.227	225.848
0.2	1239.61	-350.05	1178.61
0.3	3888.7	-1623.27	3485.09
0.4	8950.25	-4323.87	7752.13
0.5	17214	-8978.29	14586.5

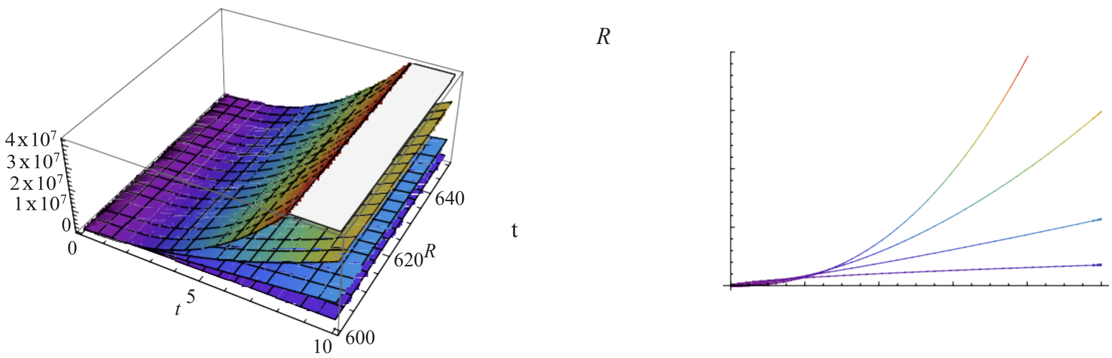

Figure 1 – Graphical comparison between LRPS Approximation solution of $R^*(t)$ for different values of $\gamma = 0.2, 0.4, 0.6, 0.8$ in 3D and 2D view shown in (a) and (b) respectively.

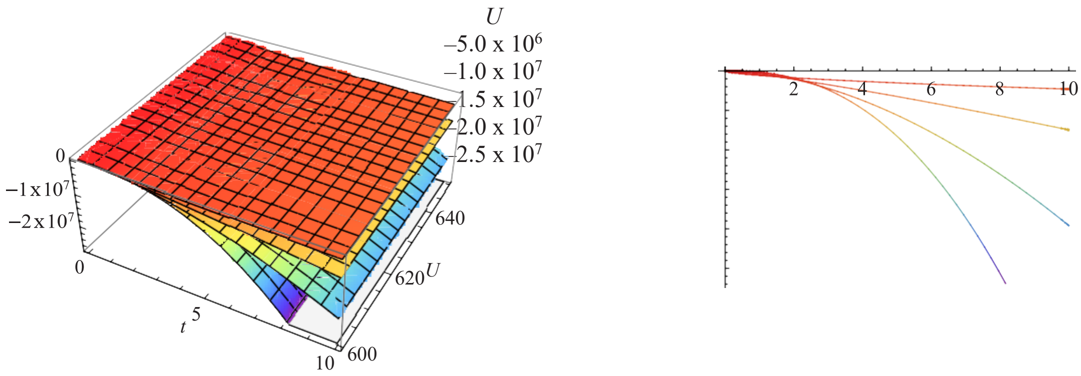

Figure 2 – Graphical comparison between LRPS Approximation solution of $U^*(t)$ for different values of $\gamma = 0.2, 0.4, 0.6, 0.8$ in 3D and 2D view shown in (a) and (b) respectively.

Conclusion

This study introduces an innovative analytical method, employing the LRPS approach, to efficiently address non-linear FFR models. The method's demonstrated effectiveness, supported by numerical simula- tions and graphical representations, underscores its practical value. By expanding the toolkit for risk modeling, the research enhances risk assessment and management strategies across diverse risk categories, contributing to more accurate and resilient financial systems. By en- hancing the repertoire of techniques available for risk modeling, this research contributes to the refinement of risk assessment and man- agement strategies across various risk categories, including credit risk, market risk, model risk, liquidity risk, and operational risk. This inno- vative approach holds implications not only for the financial sector but also for broader applications within risk analysis in diverse fields. Fur- thermore, this study's forward-looking insights highlight the potential of advanced methodologies to navigate complex financial landscapes and promote effective risk management practices.

Conflict of Interest The corresponding author states that there is no conflict of interest.

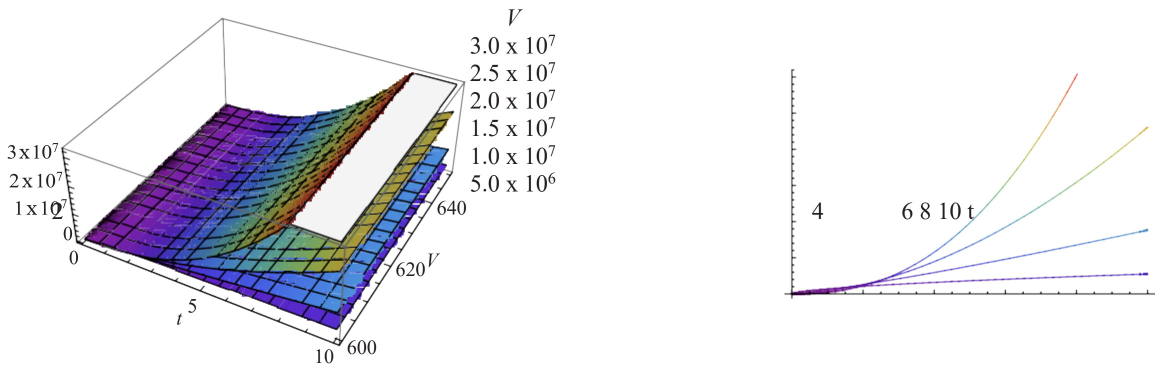

Figure 3 – Graphical comparison between LRPS Approximation solution of $V^*(t)$ for different values of $\gamma = 0.2, 0.4, 0.6, 0.8$ in 3D and 2D view shown in (a) and (b) respectively.

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