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Forced dynamics of oscillator ensembles with global nonlinear coupling

Abstract. We perform experiments with 72 electronic limit-cycle oscillators, globally coupled via a linear or nonlinear feedback loop. While in the linear case we observe standard Kuramoto-like synchronization transition, in the nonlinear case, with increase of the coupling strength, we first observe a transition to full synchrony and then a desynchronization transition to quasiperiodic state. In this state the ensemble remains, however, coherent so that the mean field is non-zero, but mean field frequency is large than frequencies of all oscillators. Next, we analyze common periodic forcing of the linearly or nonlinearly coupled ensemble and demonstrate regimes when the mean field is entrained by the force whereas the oscillators are not.
Keywords: Oscillator populations, external forcing, nonlinear coupling, electronic experiment

Introduction

Ensemble of many interacting oscillatory units is a popular model, widely used for description of collective dynamics of such various objects as lasers and Josephson junctions, spontaneously beating atrial cells and firing and/or busting neurons, pedestrians on the footbridges and handclapping individuals in a large audience, electrochemical oscillators, metronomes, and many others. Quite often the networks of such elements can be approximately considered as fully connected, with the same strength of interaction within each pair of elements. In this case one speaks of the global or mean field coupling. Analysis of collective behavior of globally coupled systems is not only important for applications, but also poses a number of problems which are highly nontrivial from the standpoint of nonlinear dynamics. Due to these reasons, this topic remains in the focus of interest within the last three decades. Basic theory and further references can be found in the following books, book chapters, and review articles [1–3].

The main effect of global coupling is emergence of a collective mode, or mean field, due to synchronization of some or all elements of the population. The degree of the collective synchrony is reflected in the amplitude of the collective mode; this amplitude is often called order parameter. Typically, the order parameter increases with the interaction strength, if the latter is larger than a certain threshold value. This effect is well-understood within the framework of the analytically solvable model of infinitely many sine-coupled phase oscillators [4]. The character of the Kuramoto transition from the incoherent state, where the order parameter is zero, to the partially or fully synchronous state with non-zero mean field depends on the distribution of the natural frequencies within the population; this transition can be either smooth [4, 5] or abrupt [6]. The described scenario is, however, not universal: consideration of more complicated oscillators and/or general coupling results in such effects as clustering [7] chaotization of the mean field [8, 9], and appearance of robust heteroclinic network attractors [7, 10]. Another subject of recent interest is partial synchrony in networks of pulse coupled identical integrate-and-fire units, globally coupled via an additional equation for the mean field [9, 11]. This model exhibits a collective mode that is not synchronized with individual units, while synchronous state is not stable. Similar regime was numerically observed for a model of active mechanical oscillators, coupled via an inertial load [12]. Coherent but not synchronous dynamics in ensembles of nonlinearly coupled Stuart-Landau oscillators was demonstrated numerically and analyzed theoretically in the framework of phase

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approximation in. The latter system demonstrates self-organized quasiperiodic dynamics (SOQ), when the frequency of the mean field differs from the frequency of oscillators, and non-monotonic dependence of the order parameter on the coupling strength. Experimental investigation of such regimes is a the primary goal of this paper.

In this paper we extend our experimental analysis of electronic oscillators, coupled via the common load, communicated in [13]. Using the new experimental setup with 72 units instead of 20, we systematically analyze the ensemble dynamics for the cases of linear and nonlinear coupling. The latter is organized as follows: the phase shift in the feedback loop depends on the voltage across the common load (mean field amplitude). We demonstrate that increase of the global coupling first results in the full synchrony and then in its destruction. After synchrony breaking, the system exhibits quasiperiodic state: frequency of the mean field is larger than frequencies of all individual oscillators. Next, we investigate the effect of the external forcing of the globally coupled system. Here the main result is a demonstration of the theoretical prediction, made in [14]. Namely, we show, that in case of nonlinear coupling, the weak external driving entrains only the mean field, but not individual oscillators. Thus, the forced global dynamics remains quasiperiodic.

Ensemble of electronic oscillators

In this Section we describe our setup with 72 globally coupled electronic generators. First present implementation of an individual unit. Next, we discuss organization of linear and nonlinear global coupling and of the common external forcing.

Scheme of an individual generator is given in Fig. 1; it represents a nonlinear amplifier with a positive frequency-dependent feedback via the Wien-bridge. The amplifier is implemented by the operational amplifier U_1, resistors R_4, R_5, R_6, R_7 and diodes D_1, D_2. The Wien-bridge consists of resistors R_1, R_2, R_3 and capacitors C_1, C_2. These elements determine the frequency of the oscillation. Fine frequency tuning is performed by the trimmer resistor R_3, so that all oscillators in the ensemble have close frequencies ≈ 1.1 kHz. With the help of the trimmer resistor R5 the amplitudes of all uncoupled oscillators were tuned to approximately same value V ≈ 1.5 V.

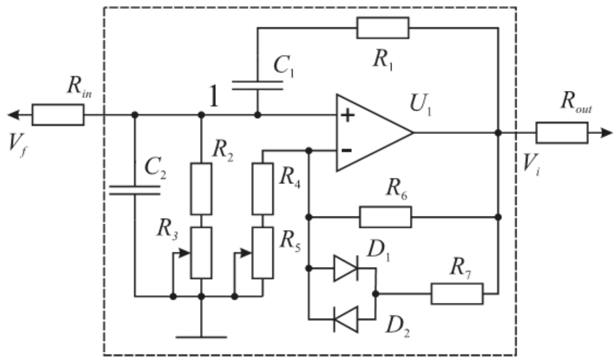

Figure 1 - Wien-bridge oscillator. Here V_i is the output voltage of the *i*-th oscillator and V_f is the output voltage of the global feedback loop (cf. Fig. 3).

Global coupling is organized via the common resistive load R_c, see Fig. 2. A fraction of the voltage V_1 across this potentiometer is fed back to the individual oscillators via the linear and nonlinear phase-shifting units and resistors R_1. The input to the feedback loop can be written as $V_c = \varepsilon V_L$, where parameter ε, $0 \le \varepsilon \le 1$, quantifies the strength of the global coupling. It is easy to show that

$$V_c = \varepsilon \frac{\sum_{i=1}^{N} V_i}{N + R_2 / R_c} \qquad (1)$$

where V_i is the output voltage of the *i*-th oscillator. Since $R_2 \ll NR_c$, we have $V_c \sim \sum V_i$, i.e. the coupling is of the mean field type

The voltage V_c from the common load is fed back to all oscillators via the feedback loop which includes either linear or both linear and nonlinear phase-shifting units; their schemes are depicted in Fig. 3. The linear subunit is an active all-pass filter which shifts the phase of the signal but keeps its amplitude, see Fig. 4a,b. The phase shift can be adjusted by the resistor R_{10}, as shown in Fig. 4c. The nonlinear subunit is implemented by a high-pass first order filter, where nonlinear properties of diodes provide a dependence of the phase shift between input and output on the amplitude of the input (Fig. 4a,b). In experiments with external forcing of the globally coupled ensemble, the sine-wave generated by NI ELVIS II Instrumentation, Design, and Prototyping Platform was supplied to the feedback loop via a summator.

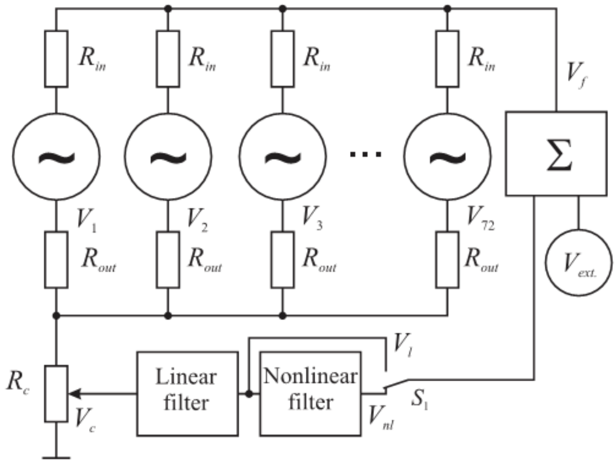

Figure 2 - Scheme of the globally coupled system. Individual generators are shown here by one symbol, there detailed scheme is given in Fig. 1, whereas the schemes of the linear and nonlinear phase-shifting filters are given in Fig. 3. With the help of the switch, the nonlinear unit can be excluded from the feedback loop. The strength of the feedback is governed by the potentiometer R_c. Common forcing by the external voltage V_{ext} is organized via the summator Σ.

Experimental results

First we perform the experiments with the linear phase-shifting filter only. Synchronization transition for the fixed phase shift, Δφ≈0.65π is illustrated in Fig. 5. As expected, we observe a monotonic growth of the order parameter R with the coupling strength ε. Due to the finite size of the ensemble, R is not small in the asynchronous state; the transition to synchrony is much better characterized by the minimal mean field amplitude A_{min} [13]. One can see that A_{min} is practically zero when frequencies of oscillators differ and starts to grow when some oscillators synchronize. Generally, we can understand A_{min} as a measure of coherence of the ensemble. Indeed, if the finite-size ensemble is in a coherent state (synchronous or partially synchronous), the mean field looks like a periodic process, corrupted by some noise, and its minimal amplitude essentially deviates from zero. Otherwise, if the ensemble is in an asynchronous state, the mean field fluctuates and looks like filtered noise; the amplitude then can be very small.

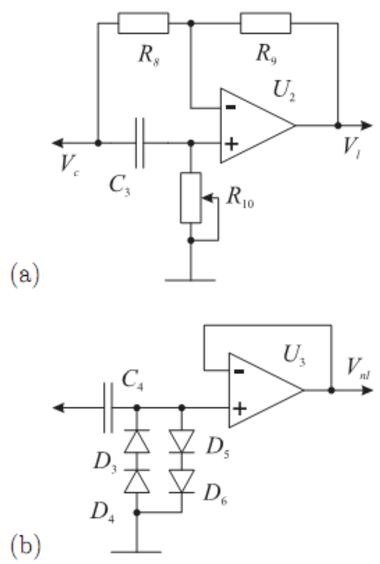

Figure 3 - Linear (a) and nonlinear (b) phase-shifting units.

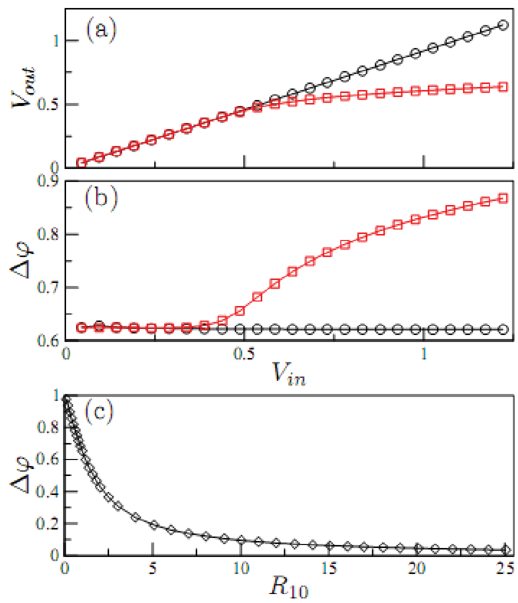

Figure 4 - (Color online) Characteristics of the linear (black circles) and nonlinear (red squares) phase-shifting units: (a) Output voltage and (b) phase shift $\Delta\varphi = (\phi_{out} - \phi_{in})/\pi$ vs. the input voltage. (c) Phase shift of the linear unit vs. R_{10}.

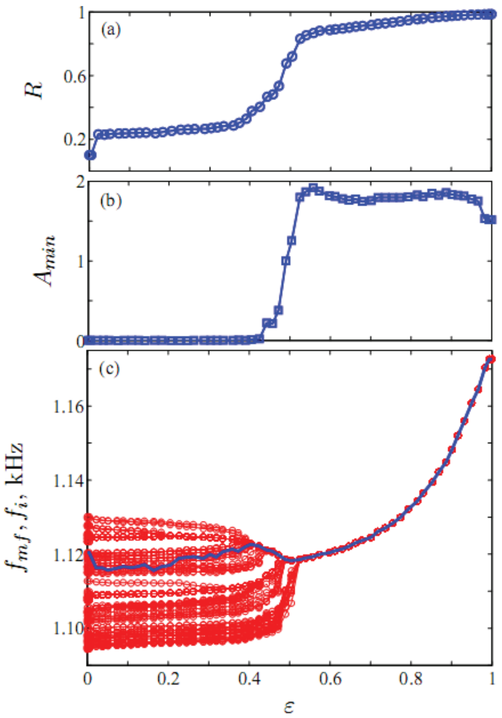

Figure 5 - (Color online) Transition to synchrony in ensemble of 72 oscillators with the linear phase-shifting unit in the global feedback loop. (a) Order parameter monotonically growth with ε. (b) Minimal amplitude of the mean field is a measure of the coherence of the ensemble; its deviation from zero reveals the transition to synchrony. (c) Frequencies of individual oscillators (red circles) and of the mean field (solid blue line).

Now we switch on the nonlinear phase-shifting filter in the global feedback loop and analyze the collective dynamics by increasing coupling strength ε. The transition for $\Delta\varphi = 0.65$ is shown in Fig. 6. We see that the oscillators synchronize for the coupling $\varepsilon \approx 0.5$ and then synchrony becomes unstable. The slow oscillator leaves the synchronous group and the order parameter decreases. For sufficiently large coupling all oscillators are not entrained by the mean field. However, the mean field has a non-zero amplitude and the SOQ state takes place. The picture quantitatively coincides with the theoretical and numerical result for phase oscillators in [15].

Now we present the results of experiments where nonlinearly coupled ensemble was forces by common periodic signal. The case of the nonlinearly coupled ensemble in the SOQ state was treated in [14]. Investigation of the common forcing of large ensembles is relevant, e.g. for neuroscience, where this model can be used for description of rhythms of a large neuronal population, influenced by rhythms from other brain regions. In the first approximation the ensemble exhibiting a collective mode can be considered as a macroscopic oscillator, and therefore can be entrained by an external forcing. However, if we go beyond this simplistic description and consider the dynamics of the level of individual units, we can expect different effects. So, in the case of the harmonically forced Kuramoto model one observes formation of synchronous subpopulations of oscillators with different frequencies. In the case of the nonlinearly coupled ensemble in the SOQ state the theory for identical oscillators [14] predicts that external force can lock the mean field without entraining individual oscillators. Here we verify this prediction. Results on forcing the nonlinearly coupled ensemble are presented in Fig.7. First, we see that the mean field is entrained by the external force, if the amplitude of the forcing exceeds a critical value. Such behavior is typical for noisy and chaotic oscillators. Since the mean field of a finite size ensemble is not exactly periodic, but fluctuates, it is natural that the response of the ensemble to an external forcing is similar to response of as macroscopic noisy oscillator.

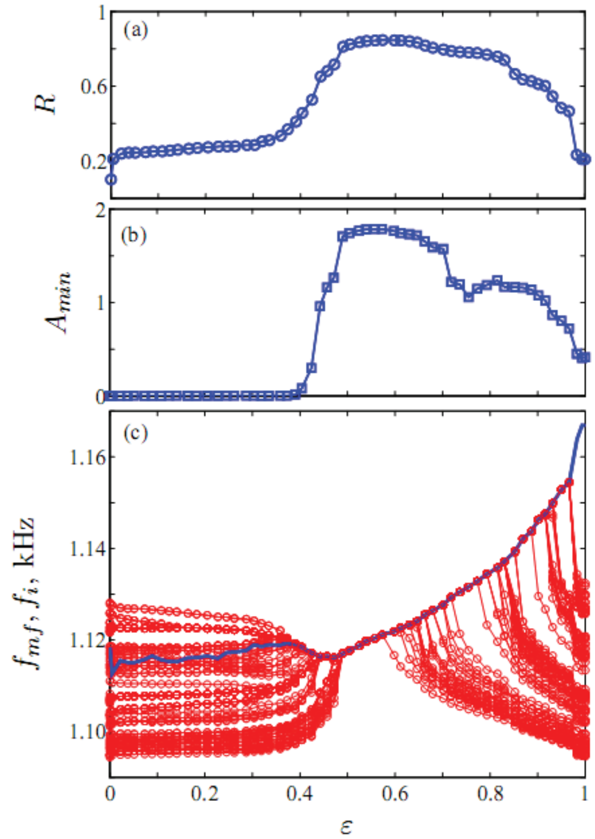

Figure 6 - (Color online) Nonlinearly coupled ensemble: Transitions from asynchronous to fully synchronous and from fully synchronous to SOQ state. (Notation are the same as in Fig. 5).

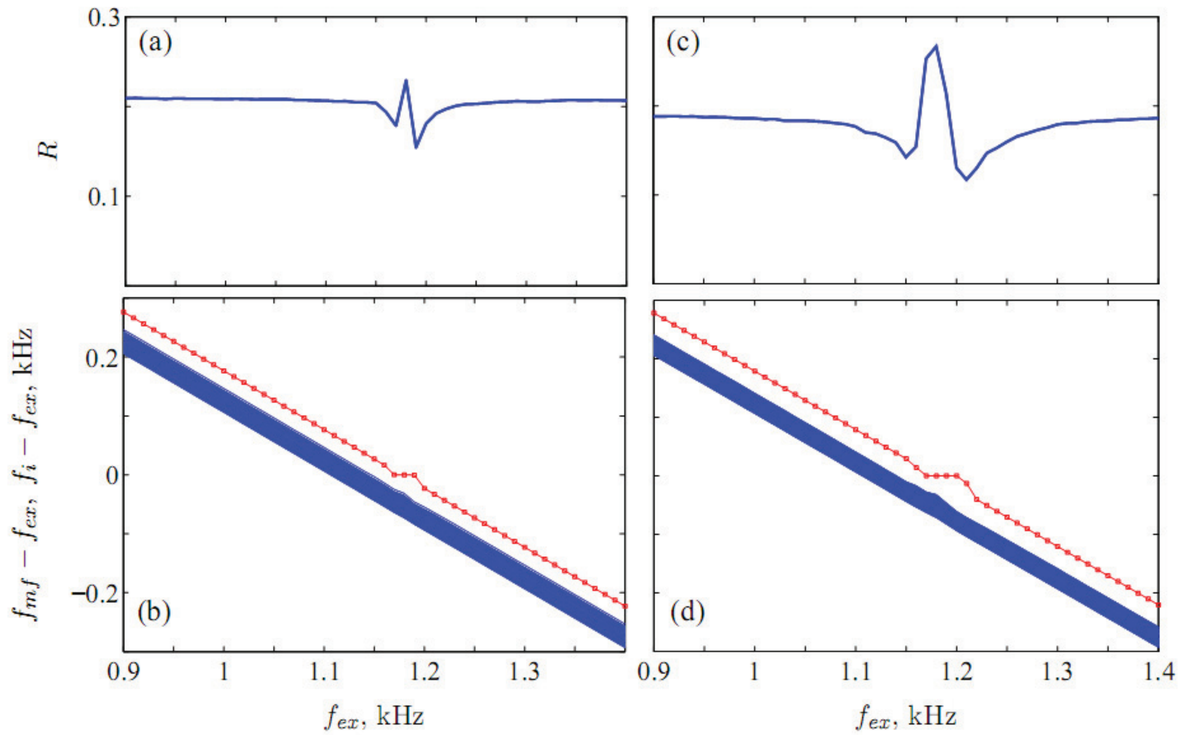

Figure 7 - (Color online) Results of harmonic forcing of the nonlinearly coupled ensemble, for two different amplitudes of the force: 0.08 (a,b) and 0.15 (c,d). Here f_{ex} is the frequency of the common external forcing. $f_{mf} - f_{ex}$ is shown with red dots, $f_i - f_{ex}$ is shown by blue lines.

In the case of the ensemble in the SOQ state, we observe regimes where the mean field is locked to the external force but the oscillators are not. Thus the system remains in the SOQ state. For stronger coupling we have both SOQ and fully synchronous states. It means that for some (relatively narrow) range of external frequencies, the force destroys the quasiperiodic dynamics and imposes full synchrony; this is accompanied by an essential (up to 2 times) increase of the order parameter.

Conclusion

Thus, we have experimentally demonstrated a state where oscillators are synchronized neither with each other nor with the mean field, but the amplitude of the latter is, nevertheless, nonzero. This peculiar coherent state is possible because phases of oscillators, though not locked, are coordinated in a way that their distribution is nonuniform. Our results correspond well to analytical results for phase oscillators [14, 15]. The SOQ regime we observe emerges when the system is brought, due to the phase shift, close to the point where attractive interaction becomes repulsive. Thus, we expect SOQ to be observed in other physical systems where the global coupling is characterized by an amplitude-dependent phase shift or time delay.

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