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The Numerical Study Distribution of the Temperature Field in a Constructing Element of a Complex Form

Abstract. Many parts of internal combustion engines, gas turbine power plants, steam generators of nuclear power plants and manufacturing industries experience thermal effects of various forms. At the same time, a process of thermal expansion occurs on these parts and, as a result, a thermal stress-strain state arises on them with a value that in some cases can exceed the limit value. Therefore, knowledge of the stationary field of temperature distribution in the volume of partially thermally insulated parts of a complex configuration in the presence of a heat flux and heat exchange in parts of its surface is an urgent task. At the same time, it is very difficult to take into account all inhomogeneous boundary conditions when solving the problem of stationary heat conduction. Therefore, a new numerical method is proposed, based on the law of conservation of total thermal energy in combination with the finite element method. In this case, the procedure for minimizing the total thermal energy is used using quadrilateral bilinear finite elements. Partial thermal insulation, heat flux supplied to the local surface, and the process of heat exchange through the local surface area and ambient temperature are taken into account. Nodal temperature values are determined [1; 2].
Keywords: mathematical model, heat flow, functionality, heat exchange, thermal insulation.

Introduction

In the thermomechanical process, the main characteristic that has a significant impact on the strength of the load-bearing structural elements is an intensive temperature increase. Temperature is one of the most important characteristics of the growth process and affects the morphology and crystal structure of heat-resistant alloys. Depending on the parameters of the structure body, the distribution of the temperature field in its different parts is uneven. It should be noted that the simultaneous influence on the distribution of temperature over the volume of the body and such external factors as various forms of local thermal insulation, the property of heat transfer, and the temperature of the heat source. Consequently, during the thermomechanical process, in some parts of the structural elements, the temperature will be acceptable, and in some – critical, which leads to rapid wear of structural elements and to the loss of their physical qualities. In this regard, the exact calculation of the distribution of the temperature field at each nodal point of multidimensional bodies of complex shape is relevant [2; 3; 4].

Experimental: This article discusses a technique for constructing a mathematical model and the corresponding computational algorithm that allow solving problems of studying the patterns of the temperature distribution field in a structural element of a complex shape with the simultaneous presence of heat flow, heat transfer and partial thermal insulation on their local surfaces.

Literature review and problem statement

At present, in our country and abroad, there are many works devoted to the problem of the influence of a thermomechanical process on a change in the structure and composition of the material of any technical installation or design. This article takes into account the simultaneous influence of the heat flow on the body, partial thermal insulation and local heat transfer. A computational algorithm is presented for solving a problem obtained by discretizing bodies of complex shape made of heat-resistant alloys using quadratic finite elements [2; 7].

The aim and objectives of the study: The purpose of this article is to show the regularity of the distribution of the temperature field using a numerical study of heat transfer in the presence of heat flow, thermal insulation and heat transfer. The objectives of the study are to determine the temperature value at each nodal point of a multidimensional body to develop a computational algorithm based on minimizing the total thermal energy functional.

To illustrate the proposed numerical method and the corresponding computational algorithm, consider the following problem. Given a "channel-like" body of unlimited length $-\infty \leq z \leq \infty$ (Fig. 1). The outer side and inner surface of which are thermally insulated along the entire length. Through the areas of the upper surface $y = h, 0 \leq x \leq (r + 2l), -\infty \leq z \leq \infty$ heat exchange with its environment takes place. In this case, the heat transfer coefficient is h_{oc}, and the ambient temperature is T_{oc}. On the surface $y = 0, [(0 \leq x \leq l)\ and\ (r + l) \leq x \leq (r + 2l)]$, $-\infty \leq z \leq \infty$a heat flux of q - constant intensity is supplied. It is necessary to determine the steady temperature distribution field in the volume of the structural element under consideration. To do this, first, the initial cross section, which is shown in Fig. 1 is discretized by quadrangular finite elements. Within each finite element, we represent the temperature distribution field as [1; 2; 8]

$$T(x, y) = \alpha_1 + \alpha_2 x + \alpha_3 y + \alpha_4 y = \\ = \varphi_1(x, y) \cdot T_1 + \varphi_2(x, y) \cdot T_2 + \\ +\varphi_3(x, y) \cdot T_3 + \varphi_4(x, y) \cdot T_4 \quad (1)$$

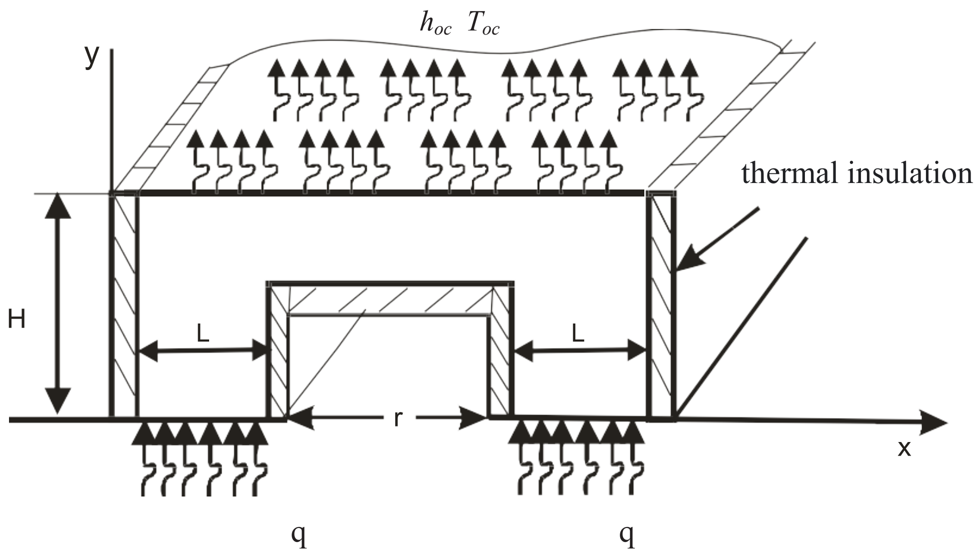

Figure 1 –The design scheme of the problem under consideration in the cross section of a structural element.

where $\varphi_i(x, y)$ are the shape function for a quadrangular finite element with four nodes [1]:

$$\varphi_1(x, y) = \frac{(b-x)(a-y)}{4ab}; \qquad \varphi_2(x, y) = \frac{(b+x)(a+y)}{4ab};$$

$$\varphi_3(x, y) = \frac{(b+x)(a+y)}{4ab}; \qquad \varphi_4(x, y) = \frac{(b-x)(a+y)}{4ab}. \quad (2)$$

where the size of the finite element along the direction of the coordinate axes x and y is $[2b;2a]$ (Fig. 2)

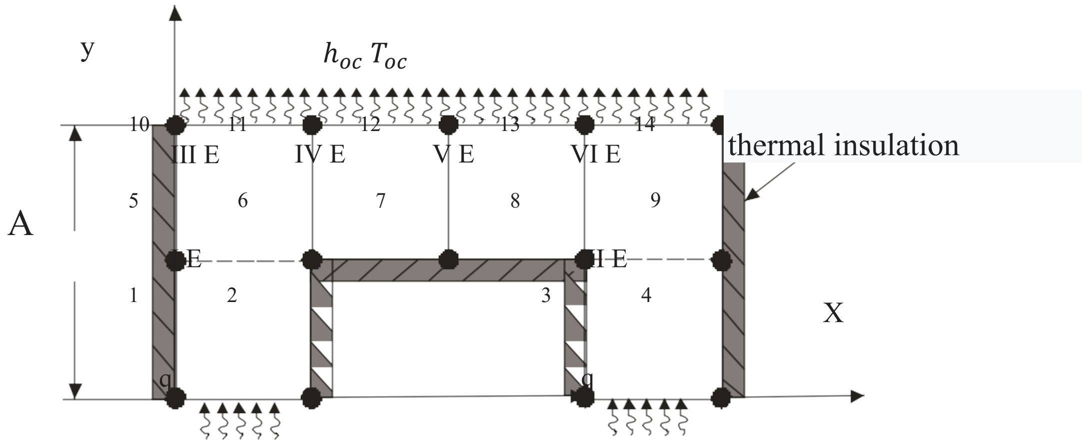

Figure 2 – Discretization of the computational domain in the context of a structural element.

$$J=\int_V \frac{1}{2}\left[K_{xx}\left(\frac{\partial T}{\partial x}\right)^2+K_{yy}\left(\frac{\partial T}{\partial y}\right)^2\right]dV+\int_{S(x=0)} qT dS+\int_{S(x=A)} \frac{h}{2}(T-T_{oc})^2 dS, \quad (3)$$

Where V is the volume of the timber in question; *S(x=0)* - the surface area of the beam (x=0), where the heat flow q ; S(x=A) - the surface area (x=A) of the beam through which heat is exchanged with the environment h; K_{xx}; K_{yy}, $(\frac{W}{cm\cdot{}^\circ C})$ - the coefficient of thermal conductivity of the timber under consideration, respectively, in the directions of the coordinate axes *ox* and *oy*.

The cross-sectional area of the timber in question (which has the shape of a rectangular quadrangle) is discretized using coordinate lines into quadrangular finite elements. The number of discrete finite elements will be $m \times n$ (respectively, on the axes *ox* and *oy*). For each element, we construct a local coordinate system oxy, so that the origin coincides with the geometric center of the element, as shown in Figure 2. The numbering of the element nodes is shown in this figure. The coordinates of the element nodes in the local coordinate system will be as follows $1(-a;-b); 2(a;-b); 3(a;b); 4(-a;b)$:

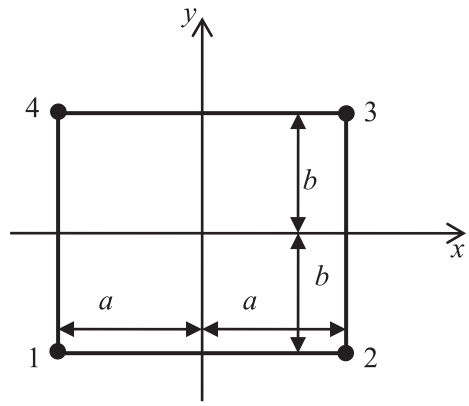

Figure 3 – Scheme of building a local coordinate system

The properties of these form functions will be as follows:

$$\left.\begin{array}{c} 1)\, In\ the\ first\ node, i.e.\, when\ x=-a; y=-b \\ \varphi_1(-a;-b)=1;\ \varphi_2(-a;-b)=\ \varphi_3(-a;-b)=\ \varphi_4(-a;-b)=0. \\ 2)\, In\ the\ second\ node\ \ i.e.\, when\ x=a; y=-b \\ \varphi_1(a;-b)=0;\ \varphi_2(a;-b)=1;\ \varphi_3(a;-b)=\ \varphi_4(a;-b)=0. \\ 3)\, In\ the\ third\ node, i.e.\, when\ x=a; y=b \\ \varphi_1(a;b)=0;\ \varphi_2(a;b)=0;\ \varphi_3(a;b)=1;\ \varphi_4(a;b)=0. \\ 4)\, In\ the\ fourth\ node, i.e.\, when\ x=-a; y=b \\ \varphi_1(-a;b)=\ \varphi_2(-a;b)=\ \varphi_3(-a;b)=0;\ \varphi_4(-a;b)=1. \end{array}\right\} \quad (4)$$

$$5)\sum_{i=1}^{4} \varphi_i = 1, \qquad (5)$$

$$6)\sum_{i=1}^{4} \frac{\partial \varphi_i}{\partial x} = 0, \qquad (6)$$

at any point in a discrete finite element.

In addition, from (1), (2) the values of temperature gradients at any point of a discrete element are easily determined:

$$\frac{\partial T}{\partial x} = \sum_{i=1}^{4} \frac{\partial \varphi_i}{\partial x} T_i;$$

$$\frac{\partial T}{\partial y} = \sum_{i=1}^{4} \frac{\partial \varphi_i}{\partial y} T_i; \qquad (7)$$

The expression is also defined:

$$\left(\frac{\partial T}{\partial x}\right)^2 = \left(\sum_{i=1}^{4} \frac{\partial \varphi_i}{\partial x} T_i;\right)^2 = \left[-\frac{b-y}{4ab}T_1 + \frac{b-y}{4ab}T_2 + \frac{b+y}{4ab}T_3 - \frac{b+y}{4ab}T_4\right]^2 =$$

$$= \frac{b^2-2by+y^2}{16a^2b^2} \cdot T_1{}^2 - 2 \cdot \frac{b^2-2by+y^2}{16a^2b^2} \cdot T_1T_2 - 2 \cdot \frac{b^2-y^2}{16a^2b^2} \cdot T_1T_3 + 2 \cdot \frac{b^2-y^2}{16a^2b^2} \cdot T_1T_4 +$$

$$+ \frac{b^2-2by+y^2}{16a^2b^2} \cdot T_2{}^2 + 2 \cdot \frac{b^2-y^2}{16a^2b^2} \cdot T_2T_3 - 2 \cdot \frac{b^2-y^2}{16a^2b^2} \cdot T_2T_4 +$$

$$+ \frac{b^2+2by+y^2}{16a^2b^2} \cdot T_3{}^2 - 2 \cdot \frac{b^2+2by+y^2}{16a^2b^2} \cdot T_3T_4 + \frac{b^2+2by+y^2}{16a^2b^2} \cdot T_4{}^2; \qquad (8)$$

$$\left(\frac{\partial T}{\partial y}\right)^2 = \left(\sum_{i=1}^{4} \frac{\partial \varphi_i}{\partial y} T_i;\right)^2 = \left[-\frac{a-x}{4ab}T_1 - \frac{a+x}{4ab}T_2 + \frac{a+x}{4ab}T_3 + \frac{a-x}{4ab}T_4\right]^2 =$$

$$= \frac{a^2-2ax+x^2}{16a^2b^2} \cdot T_1{}^2 - 2 \cdot \frac{a^2-x^2}{16a^2b^2} \cdot T_1T_2 - 2 \cdot \frac{a^2-x^2}{16a^2b^2} \cdot T_1T_3 + 2 \cdot \frac{a^2-2ax+x^2}{16a^2b^2} \cdot T_1T_4 +$$

$$+ \frac{a^2+2ax+x^2}{16a^2b^2} \cdot T_2{}^2 - 2 \cdot \frac{a^2+2ax+x^2}{16a^2b^2} \cdot T_2T_3 - 2 \cdot \frac{a^2-x^2}{16a^2b^2} \cdot T_2T_4 +$$

$$+ \frac{a^2+2ax+x^2}{16a^2b^2} \cdot T_3{}^2 + 2 \cdot \frac{a^2-x^2}{16a^2b^2} \cdot T_3T_4 + \frac{a^2-2ax+x^2}{16a^2b^2} \cdot T_4{}^2; \qquad (9)$$

For clarity of the proposed computational algorithm, we consider the cross section of the timber under consideration as one discrete quadrangular element, as shown in Figure 4.

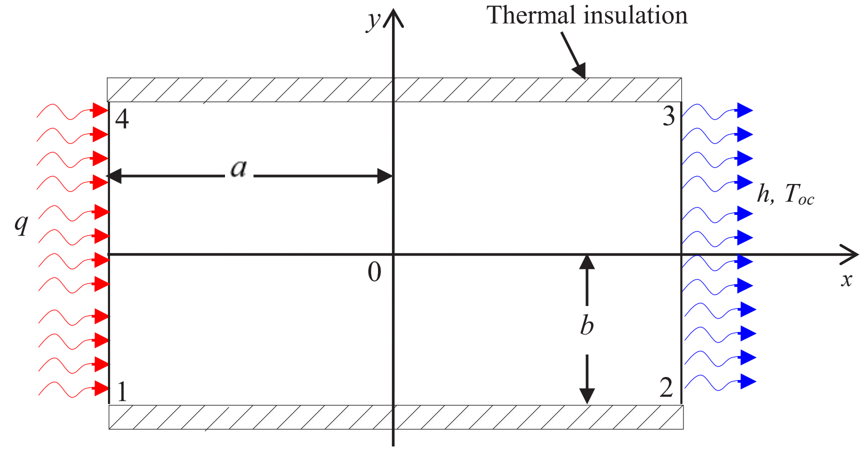

Figure 4 – The calculation scheme of the problem

Now for one discrete element, we calculate the integral over the volume. Here we use the following formula:

$$\int_V f(x,y)dV = L\int_{-a}^{a}\int_{-b}^{b} f(x,y)dxdy. \tag{10}$$

Using (10) we calculate the integral:

$$J_{11}=\int_V \frac{1}{2}\left[K_{xx}\left(\frac{\partial T}{\partial x}\right)^2\right]dV \tag{11}$$

In calculating this integral, we use the expression (8). As a result, we have:

$$1)\int_{-a}^{a}\int_{-b}^{b}\frac{b^2-2by+y^2}{16a^2b^2}T_1^{\,2}dxdy = \frac{2aT_1^{\,2}}{16a^2b^2}\int_{-b}^{b}(b^2-2by+y^2)dy =$$

$$= \frac{2aT_1^{\,2}}{16a^2b^2}\left[2b^3 - 0 + \frac{2b^3}{3}\right] = \frac{T_1^{\,2}}{8ab^2}\cdot\frac{8b^3}{3} = \frac{b}{3a}T_1^{\,2}; \tag{12}$$

$$2)\int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{b^2-2by+y^2}{16a^2b^2}\cdot T_1T_2\right)dxdy = \frac{-T_1T_2}{8a^2b^2}\cdot 2a\int_{-b}^{b}(b^2-2by+y^2)dy =$$

$$= \frac{-T_1T_2}{4a^2b^2}\left[2b^3 + \frac{2b^3}{3}\right] = -\frac{2b}{3a}T_1T_2; \tag{13}$$

$$3)\int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{b^2-y^2}{16a^2b^2}\cdot T_1T_3\right)dxdy = \frac{-T_1T_3}{8a^2b^2}\cdot 2a\left[2b^3 - \frac{2b^3}{3}\right] = -\frac{b}{3a}T_1T_3; \tag{14}$$

$$4)\int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{b^2-y^2}{16a^2b^2}\cdot T_1T_4\right)dxdy = \frac{b}{3a}T_1T_4; \tag{15}$$

$$5)\int_{-a}^{a}\int_{-b}^{b}\left(\frac{b^2-2by+y^2}{16a^2b^2}\cdot T_2^{\,2}\right)dxdy = \frac{2aT_2^{\,2}}{16a^2b^2}\left[2b^3 + \frac{2b^3}{3}\right] = \frac{b}{3a}T_2^{\,2}; \tag{16}$$

$$6)\int_{-a}^{a}\int_{-b}^{b}\left(2\cdot\frac{b^2-y^2}{16a^2b^2}\cdot T_2T_3\right)dxdy = \frac{T_2T_3}{8a^2b^2}\cdot 2a\cdot\frac{4b^3}{3} = \frac{b}{3a}T_2T_3; \tag{17}$$

$$7)\int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{b^2-y^2}{16a^2b^2}\cdot T_2T_4\right)dxdy = -\frac{b}{3a}T_2T_4; \tag{18}$$

$$8)\int_{-a}^{a}\int_{-b}^{b}\frac{b^2+2by+y^2}{16a^2b^2}T_3^{\,2}dxdy = \frac{T_3^{\,2}}{16a^2b^2}\cdot 2a\cdot\frac{8b^3}{3} = \frac{b}{3a}T_3^{\,2}; \tag{19}$$

$$9)\ \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{b^2+2by+y^2}{16a^2b^2}T_3T_4\right)dxdy = -\frac{2b}{3a}T_3T_4; \tag{20}$$

$$10)\ \int_{-a}^{a}\int_{-b}^{b}\frac{b^2+2by+y^2}{16a^2b^2}{T_4}^2dxdy = \frac{b}{3a}{T_4}^2; \tag{21}$$

Substituting (12) - (21) into (11), we find the integrated form J_{11}.

$$J_{11} = \int\frac{1}{2}\left[K_{xx}\left(\frac{\partial T}{\partial x}\right)^2\right]dV = \frac{LK_{xx}}{2}\int_{-a}^{a}\int_{-b}^{b}\left(\frac{\partial T}{\partial x}\right)^2dxdy =$$

$$= \frac{LK_{xx}}{2}\int_{-a}^{a}\int_{-b}^{b}\Big[\frac{b^2-2by+y^2}{16a^2b^2}\cdot{T_1}^2 - 2\frac{b^2-2by+y^2}{16a^2b^2}T_1T_2 - 2\frac{b^2-y^2}{16a^2b^2}\cdot T_1T_3 + 2\frac{b^2-y^2}{16a^2b^2}\cdot T_1T_4 +$$

$$+\frac{b^2-2by+y^2}{16a^2b^2}\cdot{T_2}^2 + 2\frac{b^2-y^2}{16a^2b^2}\cdot T_2T_3 - 2\frac{b^2-y^2}{16a^2b^2}\cdot T_2T_4 +$$

$$+\frac{b^2+2by+y^2}{16a^2b^2}{T_3}^2 - 2\frac{b^2+2by+y^2}{16a^2b^2}T_3T_4 + \frac{b^2+2by+y^2}{16a^2b^2}{T_4}^2\Big]dxdy$$

$$= \frac{bLK_{xx}}{6a}\left[{T_1}^2 - 2T_1T_2 - T_1T_3 + T_1T_4 + {T_2}^2 + T_2T_3 - T_2T_4 + {T_3}^2 - 2T_3T_4 + {T_4}^2\right]; \tag{22}$$

Examining the last expression, we find that the sum of the coefficients in front of the nodal temperature values will be zero. Indeed, from (22) we find that (1-2-1 + 1 + 1 + 1-1 + 1-2 + 1) = 0.

Next, similarly, we find the integrated form expression

$$J_{22} = \int_V \frac{1}{2}\left[K_{yy}\left(\frac{\partial T}{\partial y}\right)^2\right]dV \tag{23}$$

Using (9) we find that

$$1)\ \int_{-a}^{a}\int_{-b}^{b}\frac{a^2-2ax+x^2}{16a^2b^2}{T_1}^2dxdy = \frac{{T_1}^2}{16a^2b^2}\cdot 2b\cdot\frac{8a^3}{3} = \frac{a}{3b}{T_1}^2; \tag{24}$$

$$2)\ \int_{-a}^{a}\int_{-b}^{b}2\cdot\frac{a^2-x^2}{16a^2b^2}\cdot T_1T_2dxdy = \frac{2T_1T_2}{16b^2}\cdot 2b\cdot\frac{4a^3}{3} = \frac{a}{3b}T_1T_2; \tag{25}$$

$$3)\ \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{a^2-x^2}{16a^2b^2}\cdot T_1T_3\right)dxdy = -\frac{2a}{6b}T_1T_3 = -\frac{a}{3b}T_1T_3; \tag{26}$$

$$4)\ \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{a^2-2ax+x^2}{16a^2b^2}T_1T_4\right)dxdy = -\frac{2a}{3b}T_1T_4; \tag{27}$$

$$5)\ \int_{-a}^{a}\int_{-b}^{b}\left(\frac{a^2+2ax+x^2}{16a^2b^2}{T_2}^2\right)dxdy = \frac{a}{3b}{T_2}^2; \tag{28}$$

$$6)\ \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{a^2+2ax+x^2}{16a^2b^2}T_2T_3\right)dxdy = -\frac{2a}{3b}T_2T_3; \tag{29}$$

$$7)\ \int_{-a}^{a}\int_{-b}^{b}\left(-2\cdot\frac{a^2-x^2}{16a^2b^2}T_2T_4\right)dxdy = -\frac{a}{3b}T_2T_4; \tag{30}$$

$$8)\ \int_{-a}^{a}\int_{-b}^{b}\frac{a^2+2ax+x^2}{16a^2b^2}{T_3}^2dxdy = \frac{a}{3b}{T_3}^2; \tag{31}$$

$$9)\ \int_{-a}^{a}\int_{-b}^{b} 2\frac{a^2-x^2}{16a^2b^2}T_3T_4dxdy = \frac{a}{3b}T_3T_4; \quad (32)$$

$$10)\ \int_{-a}^{a}\int_{-b}^{b} \frac{a^2-2ax+x^2}{16a^2b^2}{T_4}^2dxdy = \frac{a}{3b}{T_4}^2; \quad (33)$$

Substituting (24) - (33) into (23) we define the integrated form J_{22}.

$$J_{22} = \int \frac{1}{2}\left[K_{yy}\left(\frac{\partial T}{\partial x}\right)^2\right]dV = \frac{LK_{yy}}{2}\int_{-a}^{a}\int_{-b}^{b}\left(\frac{\partial T}{\partial y}\right)^2 dxdy =$$

$$= \frac{aLK_{yy}}{6a}[{T_1}^2 + T_1T_2 - T_1T_3 - 2T_1T_4 + {T_2}^2 - 2T_2T_3 - T_2T_4 + {T_3}^2 + T_3T_4 + {T_4}^2] \quad (34)$$

In this expression, the sum of the coefficients in front of the nodal temperature values will be equal to zero [2; 9; 14].

Now we find the expression for J_1:

$$J_1 = J_{11} + J_{22} = \frac{bLK_{xx}}{6a}\left[{T_1}^2 - 2T_1T_2 - T_1T_3 + T_1T_4 + {T_2}^2 + T_2T_3 - T_2T_4 + {T_3}^2 - 2T_3T_4 + {T_4}^2\right] +$$

$$+ \frac{aLK_{yy}}{6a}[{T_1}^2 + T_1T_2 - T_1T_3 - 2T_1T_4 + {T_2}^2 - 2T_2T_3 - T_2T_4 + {T_3}^2 + T_3T_4 + {T_4}^2]; \quad (35)$$

Now from (3) we find:

$$J_2 = \int_{S(x=-a)} qTds = Lq\int_{-b}^{b}[\varphi_1(x,y)T_1 + \varphi_2(x,y)T_2 + \varphi_3(x,y)T_3 + \varphi_4(x,y)T_4]|_{x=-a}\,dy =$$

$$= \frac{Lq}{2b}\int_{-b}^{b}[(b-y)T_1 + (b+y)T_4]dy = \frac{Lq}{2b}[2b^2T_1 + 2b^2T_4] = Lqb[T_1 + T_4]; \quad (36)$$

From (3) we calculate:

$$J_3 = \int_{S(x=a)} \frac{h}{2}(T - T_{oc})^2 ds = \frac{hL}{2}\int_{-b}^{b}[\sum_{i=1}^{4}\varphi_i(x,y)T_i - T_{oc}]_{x=a}^2 dy =$$

$$= \frac{hL}{2}\int_{-b}^{b}[\varphi_1(x,y)T_1 + \varphi_2(x,y)T_2 + +\varphi_3(x,y)T_3 + \varphi_4(x,y)T_4 - T_{oc}]^2|_{x=a}dy =$$

$$= \frac{hL}{2}\int_{-b}^{b}[\frac{b^2-2by+y^2}{4b^2}{T_2}^2 + 2\frac{b^2-y^2}{4b^2}T_2T_3 - -2\frac{(b-y)}{2b}T_2T_{oc} + \frac{b^2+2by+y^2}{4b^2}{T_3}^2 - 2\frac{(b+y)}{2b}T_3T_{oc} + {T_{oc}}^2]dy; \quad (37)$$

Now in (37) we calculate each integral separately:

$$1)\ \int_{-b}^{b}\left[\frac{b^2-2by+y^2}{4b^2}{T_2}^2\right]dy = \frac{1}{4b^2}(2b^3 + \frac{2b^3}{3})\,{T_2}^2;$$

$$2)\ \int_{-b}^{b}\left[2\frac{b^2-y^2}{4b^2}T_2T_3\right]dy = \frac{1}{2b^2}(2b^3 - \frac{2b^3}{3})\,T_2T_3;$$

$$3)\ \int_{-b}^{b}\left[2\frac{b-y}{4b^2}T_2T_{oc}\right]dy = \frac{1}{b}(2b^2-0)\,T_2T_{oc};$$

$$4)\ \int_{-b}^{b}\left[\frac{b^2+2by+y^2}{4b^2}{T_3}^2\right]dy = \frac{1}{4b^2}(2b^3+\frac{2b^3}{3})\,{T_3}^2;$$

$$5)\ \int_{-b}^{b}\left[2\frac{b+y}{2b}T_3T_{oc}\right]dy = 2b\,T_3T_{oc};$$

$$6)\int_{-b}^{b}{T_{oc}}^2dy = 2b{T_{oc}}^2; \qquad (38)$$

Substituting (38) into (37) we find the integrated form J_3:

$$J_3 = \int_{S(x=a)}\frac{h}{2}(T-T_{oc})^2ds = \frac{hL}{2}[\frac{1}{4b^2}\left(2b^3+\frac{2b^3}{3}\right){T_2}^2 + \frac{1}{2b^2}\left(2b^3-\frac{2b^3}{3}\right)T_2T_3 - \frac{1}{b}(2b^2-0)T_2T_{oc} +$$

$$+\frac{1}{4b^2}\left(2b^3+\frac{2b^3}{3}\right){T_3}^2 - 2bT_3T_{oc} + 2b{T_{oc}}^2] = \frac{hL}{2}\left[\frac{2b}{3}{T_2}^2 + \frac{2b}{3}T_2T_3 - 2bT_2T_{oc} + \frac{2b}{3}{T_3}^2 - 2bT_3T_{oc}\right] =$$

$$= \frac{Lhb}{3}\left[{T_2}^2 + T_2T_3 - 3T_2T_{oc} + {T_3}^2 - 3T_3T_{oc} + 3{T_{oc}}^2\right]; \qquad (39)$$

It should also be noted here that in the expression in the bracket the sum of the coefficients will be equal to zero.

Given the expressions $J_1; J_2$ and J_3 from (3) we find the final integrated form of the functional J, that characterizes the total thermal energy of the timber in question, taking into account the simultaneous presence of heat flux, thermal insulation and heat transfer:

$$J = J_1 + J_2 + J_3 = \frac{bLK_{xx}}{6a}\left[{T_1}^2 - 2T_1T_2 - T_1T_3 + T_1T_4 + {T_2}^2 + T_2T_3 - T_2T_4 + {T_3}^2 - 2T_3T_4 + {T_4}^2\right] +$$

$$+\frac{aLK_{yy}}{6a}\left[{T_1}^2 + T_1T_2 - T_1T_3 - 2T_1T_4 + {T_2}^2 - 2T_2T_3 - T_2T_4 + {T_3}^2 + T_3T_4 + {T_4}^2\right] +$$

$$+bLq[T_1+T_4] + \frac{bLh}{3}[{T_2}^2 + T_2T_3 - 3T_2T_{oc} + {T_3}^2 - 3T_3T_{oc} + 3{T_{oc}}^2]; \qquad (40)$$

Further, minimizing the functional J with respect to the nodal values of the temperature T_1, T_2, T_3 and T_4 we find the resolving system of linear algebraic equations

$$1)\frac{\partial J}{\partial T_1} = 0; \Rightarrow \frac{bLK_{xx}}{6a}(2T_1 - 2T_2 - T_3 + T_4) + \frac{aLK_{yy}}{6b}(2T_1 + T_2 - T_3 - 2T_4) + bLq = 0;$$

$$2)\frac{\partial J}{\partial T_2} = 0; \Rightarrow \frac{bLK_{xx}}{6a}(-2T_1 + 2T_2 + T_3 - T_4) + \frac{aLK_{yy}}{6b}(T_1 + 2T_2 - 2T_3 - T_4) +$$

$$+\frac{bLq}{3}(2T_2 + T_3 - 3T_{oc}) = 0;$$

$$2)\frac{\partial J}{\partial T_3}=0;\Rightarrow \frac{bLK_{xx}}{6a}(-T_1+T_2+2T_3-2T_4)+\frac{aLK_{yy}}{6b}(-T_1-2T_2+2T_3+T_4)+$$

$$+\frac{bLq}{3}(T_2+2T_3-3T_{oc})=0;$$

$$4)\frac{\partial J}{\partial T_4}=0;\Rightarrow \frac{bLK_{xx}}{6a}(T_1-T_2-2T_3+2T_4)+\frac{aLK_{yy}}{6b}(-2T_1-T_2+T_3+2T_4)+bLq=0. \quad (41)$$

For convenience, we discretize with 6 elements. The global numbering of elements and nodes is shown in (Fig. 2). Now, for all finite elements, we write the expression for the functional J, which characterizes its total thermal energy, taking into account the existing boundary conditions [2; 7-19].

The integrated form of this functional for all discrete elements is as follows:

$$\mathrm{J}=\left(\frac{\mathrm{alK_{xx}}}{6\mathrm{b}}\right)_{\mathrm{IE}}\left(\mathrm{T}_1^2-2\mathrm{T}_1\mathrm{T}_2-\mathrm{T}_1\mathrm{T}_6+\mathrm{T}_1\mathrm{T}_5+\mathrm{T}_2^2+\mathrm{T}_2\mathrm{T}_6-\mathrm{T}_2\mathrm{T}_5+\mathrm{T}_6^2-2\mathrm{T}_6\mathrm{T}_5+\mathrm{T}_5^2\right)+$$

$$+\left(\frac{\mathrm{blK_{yy}}}{6\mathrm{a}}\right)_{\mathrm{IE}}\left(\mathrm{T}_1^2+\mathrm{T}_1\mathrm{T}_2-\mathrm{T}_1\mathrm{T}_6-2\mathrm{T}_1\mathrm{T}_5+\mathrm{T}_2^2-2\mathrm{T}_2\mathrm{T}_6-\mathrm{T}_2\mathrm{T}_5+\mathrm{T}_6^2+\mathrm{T}_6\mathrm{T}_5+\mathrm{T}_5^2\right)+(\mathrm{alq})_{\mathrm{IE}}(\mathrm{T}_1+\mathrm{T}_2)+$$

$$+\left(\frac{\mathrm{alK_{xx}}}{6\mathrm{b}}\right)_{\mathrm{IIE}}(\mathrm{T}_3^2-2\mathrm{T}_3\mathrm{T}_4-\mathrm{T}_3\mathrm{T}_9+\mathrm{T}_3\mathrm{T}_8+\mathrm{T}_4^2+\mathrm{T}_4\mathrm{T}_9-\mathrm{T}_4\mathrm{T}_8+\mathrm{T}_9^2-2\mathrm{T}_9\mathrm{T}_8+\mathrm{T}_8^2)+$$

$$+\left(\frac{\mathrm{blK_{yy}}}{6\mathrm{a}}\right)_{\mathrm{IIE}}(\mathrm{T}_3^2+\mathrm{T}_3\mathrm{T}_4-\mathrm{T}_3\mathrm{T}_9-2\mathrm{T}_3\mathrm{T}_8+\mathrm{T}_4^2-2\mathrm{T}_4\mathrm{T}_9-\mathrm{T}_4\mathrm{T}_8+\mathrm{T}_9^2+\mathrm{T}_9\mathrm{T}_8+\mathrm{T}_8^2)+$$

$$+(\mathrm{alq})_{\mathrm{IIЭ}}(\mathrm{T}_3+\mathrm{T}_4)+$$

$$+\left(\frac{\mathrm{alK_{xx}}}{6\mathrm{b}}\right)_{\mathrm{IIIE}}\left(\mathrm{T}_5^2-2\mathrm{T}_5\mathrm{T}_6-\mathrm{T}_5\mathrm{T}_{11}+\mathrm{T}_5\mathrm{T}_{10}+\mathrm{T}_6^2+\mathrm{T}_6\mathrm{T}_{11}-\mathrm{T}_6\mathrm{T}_{10}+\mathrm{T}_{11}^2-2\mathrm{T}_{11}\mathrm{T}_{10}+\mathrm{T}_{10}^2\right)+$$

$$+\left(\frac{\mathrm{blK_{yy}}}{6\mathrm{a}}\right)_{\mathrm{IIIE}}\left(\mathrm{T}_5^2+\mathrm{T}_5\mathrm{T}_6-\mathrm{T}_5\mathrm{T}_{11}-2\mathrm{T}_5\mathrm{T}_{10}+\mathrm{T}_6^2-2\mathrm{T}_6\mathrm{T}_{11}-\mathrm{T}_6\mathrm{T}_{10}+\mathrm{T}_{11}^2+\mathrm{T}_{11}\mathrm{T}_{10}+\mathrm{T}_{10}^2\right)+$$

$$+\left(\frac{\mathrm{blh}}{3}\right)_{\mathrm{IIIE}}(\mathrm{T}_{11}^2+\mathrm{T}_{11}\mathrm{T}_{10}-3\mathrm{T}_{11}\mathrm{T}_e+\mathrm{T}_{10}^2-3\mathrm{T}_{10}\mathrm{T}_e+3\mathrm{T}_e^2)+$$

$$+\left(\frac{\mathrm{alK_{xx}}}{6\mathrm{b}}\right)_{\mathrm{IVE}}(\mathrm{T}_6^2-2\mathrm{T}_6\mathrm{T}_7-\mathrm{T}_6\mathrm{T}_{12}+\mathrm{T}_6\mathrm{T}_{11}+\mathrm{T}_7^2+\mathrm{T}_7\mathrm{T}_{12}-\mathrm{T}_7\mathrm{T}_{11}+\mathrm{T}_{12}^2-2\mathrm{T}_{12}\mathrm{T}_{11}+\mathrm{T}_{11}^2)+$$

$$+\left(\frac{\mathrm{blK_{yy}}}{6\mathrm{a}}\right)_{\mathrm{IVE}}(\mathrm{T}_6^2+\mathrm{T}_6\mathrm{T}_7-\mathrm{T}_6\mathrm{T}_{12}-2\mathrm{T}_6\mathrm{T}_{11}+\mathrm{T}_7^2-2\mathrm{T}_7\mathrm{T}_{12}-\mathrm{T}_7\mathrm{T}_{11}+\mathrm{T}_{12}^2+\mathrm{T}_{12}\mathrm{T}_{11}+\mathrm{T}_{11}^2)+$$

$$+\left(\frac{\mathrm{blh}}{3}\right)_{\mathrm{IVE}}(\mathrm{T}_{12}^2+\mathrm{T}_{12}\mathrm{T}_{11}-3\mathrm{T}_{12}\mathrm{T}_e+\mathrm{T}_{11}^2-3\mathrm{T}_{11}\mathrm{T}_e+3\mathrm{T}_e^2)+$$

$$+\left(\frac{\mathrm{alK_{xx}}}{6\mathrm{b}}\right)_{\mathrm{VE}}(\mathrm{T}_7^2-2\mathrm{T}_7\mathrm{T}_8-\mathrm{T}_7\mathrm{T}_{13}+\mathrm{T}_7\mathrm{T}_{12}+\mathrm{T}_8^2+\mathrm{T}_8\mathrm{T}_{13}-\mathrm{T}_8\mathrm{T}_{12}+\mathrm{T}_{13}^2-2\mathrm{T}_{13}\mathrm{T}_{12}+\mathrm{T}_{12}^2)+$$

$$+\left(\frac{blK_{yy}}{6a}\right)_{VE}(T_7^2+T_7T_8-T_7T_{13}-2T_7T_{12}+T_8^2-2T_8T_{13}-T_8T_{12}+T_{13}^2+T_{13}T_{12}+T_{12}^2)+$$

$$+\left(\frac{blh}{3}\right)_{VE}(T_{13}^2+T_{13}T_{12}-3T_{13}T_e+T_{12}^2-3T_{12}T_e+3T_e^2)+$$

$$+\left(\frac{alK_{xx}}{6b}\right)_{VIE}(T_8^2-2T_8T_9-T_8T_{14}+T_8T_{13}+T_9^2+T_9T_{14}-T_9T_{13}+T_{14}^2-2T_{14}T_{13}+T_{13}^2)+$$

$$+\left(\frac{blK_{yy}}{6a}\right)_{VIE}(T_8^2+T_8T_9-T_8T_{14}-2T_8T_{13}+T_9^2-2T_9T_{14}-T_9T_{13}+T_{14}^2+T_{14}T_{13}+T_{13}^2)+$$

$$+\left(\frac{alh}{3}\right)_{VIE}(T_{14}^2+T_{14}T_{13}-3T_{14}T_e+T_{13}^2-3T_{13}T_e+3T_e^2).$$

Further, minimizing the last functional over nodal values, we obtain the following system of linear algebraic equations with respect to T_i:

$$\frac{\partial J}{\partial T_i}=0,(i=1\div 14). \qquad (42)$$

Solving the last system by the Gaussian method, we determine the nodal values of temperatures, and according to them, according to (1), the temperature value at any point of each finite element. In particular, with the following initial [1; 2]:

$$K_{xx}=K_{yy}=72\left[\frac{W}{cm\cdot{}^\circ C}\right];$$

$$a=b=1cm;\; q=-100\left[\frac{W}{cm^2}\right];$$

$$h_e=6\left[\frac{W}{cm^2\cdot{}^\circ C}\right];$$

$$T_e=40^\circ C;\quad r=2\text{ см},\quad l=1\text{см}$$

we find that

$$T_1=T_4=52{,}895^\circ C;\quad T_2=T_3=53{,}017^\circ C;$$

$$T_5=T_9=50{,}482^\circ C;\quad T_6=T_8=49{,}874^\circ C;$$

$$T_7=48{,}658^\circ C;\quad T_{10}=T_{14}=48{,}573^\circ C;$$

$$T_{11}=T_{13}=48{,}304^\circ C;\quad T_{12}=48{,}152^\circ C.$$

It can be seen from the obtained results that due to the symmetrical formulation of the problem under consideration, the process of the steady distribution of the temperature field in the section of the beam will also be symmetrical.

Results and Discussion

The proposed mathematical model, based on the law of conservation and change of thermal energy, allows us to solve a class of multidimensional problems of steady thermal conductivity for structural elements of any configuration, in the presence of partial thermal insulation, heat transfer, heat flow and temperature.

In this paper, because of the symmetry of the nodal points of the problem under consideration, the results of the numerical solution are symmetrical, i.e., the same temperature values.

Conclusion

The exact calculation of the distribution of the temperature field at each nodal point is determined by formula (1). In this paper, based on the energy principle combined with the finite element method, the steady-state temperature distribution field in the volume of a partially thermally insulated beam in the presence of heat flow and heat exchange is investigated numerically. A numerical solution is given for specific initial data. A numerical study of the convergence and accuracy of the obtained numerical solutions is carried out.

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