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Polarization effects in a pseudopotential model of dusty plasmas

Abstract
The main goal of this report is to derive the macropotential of dust particles interaction in plasmas taking into account both the collective events and polarization phenomena. This reveals a new possible attraction mechanism between grains which results in the crystalline like structure formation of dust particles. The consideration is made using the renormalization theory treating the screening phenomena at large distances. Dust particles are assumed to be immersed in a buffer plasma of electrons and ions which interact with each other via the Coulomb potential. The polarization phenomena are treated via the electrostatic image approximation under the assumption that dust particles are conducting spherical balls. The detailed comparison is provided with the Debye-Huckel theory corresponding to the interaction of pointlike charges immersed in a buffer plasma.
Key words: Particles interaction, Debye radius, microscopic force, Boltzmann distribution, Debye-Huckel theory, macropotential, polarization phenomenon, dusty plasma, pseudopotential.

Introduction

Herein we consider two isolated dust grains of radius R and electric charge $-Ze$ each immersed in a two-component hydrogen plasma which consists of electrons (with the electric charge $-e$ and the number density n_e) and ions (with charge e and the number density $n_i = n_e = n$). The strength of particles interaction is defined by the dimesionless parameter $\Gamma_R = e^2 / R k_B T$, where k_B is the Boltzmann constant, T refers to the plasma temperature. Another dimensionless parameter which relates the dusty component with the buffer plasma is the screening parameter $\kappa = R / \lambda_d$, where $\lambda_d = \sqrt{k_B T / 4\pi n e^2}$ stands for the well known Debye radius.

For the purpose of taking into account the collective events in pairwise interaction potentials, we consider the interaction of the two isolated particles in the presence of a third (see Fig. 1). The total force, called then the macroforce $\mathbf{F}_{ij}^{mac}$, exerted on the i-th particle from the whole system is written as:

$$\mathbf{F}_{ij}^{\ mac} = \mathbf{F}_{ij}^{\ mic} + \sum_k \int \mathbf{F}_{ik}^{\ mic} P(\mathbf{r}_{ik}, \mathbf{r}_{jk}) d\mathbf{r}_k \ . \qquad (1)$$

Here $\mathbf{F}_{ij}^{mic}$ is the microscopic force between i-th and j-th particles, $P(\mathbf{r}_{ik}, \mathbf{r}_{jk})$ stands for the probability density of finding the k-th particle at a certain distance from the i-th and j-th particles. The summation in Eq.(1) is implied over all the articles but the i-th and j-th and, then, the average is taken over the position of the k-th particle by means of the integration over $\mathbf{r}_k$.

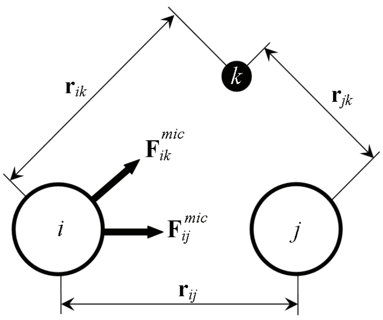

Figure 1 - The interaction diagram of two dust grains in the presence of a third particle

It is well known that a microscopic force $\mathbf{F}_{ij}^{mic}$ can be expressed through a microscopic potential φ_{ij} with the aid of the nabla operator as $\mathbf{F}_{ij}^{mic} = -\nabla_i \varphi_{ij}$. We assume that the same relation holds for the macroscopic force, i.e. with the macroscopic potential Φ_{ij} that takes into acoount collective events in plasmas. To close up our consideration it is also necessary to assume a certain form for the probability density $P(\mathbf{r}_{ik}, \mathbf{r}_{jk})$ which has to be derived from stricter theories. But the most natural way is just to make use of the Boltzmann distribution of the form

$$P(\mathbf{r}_{ik}, \mathbf{r}_{jk}) = \frac{1}{V} \exp\left(-\frac{\Phi_{ik} + \Phi_{jk}}{k_B T} \right), \qquad (2)$$

normalized by the system volume V. Substituting Eq.(2) in Eq.(1) and linearizing the exponential terms, we obtain the so-called generalized Boltzmann-Poisson equation [1,2]

$$\Delta_i \Phi_{ij}(\mathbf{r}_i^a, \mathbf{r}_j^b) = \Delta_i \varphi_{ij}(\mathbf{r}_i^a, \mathbf{r}_j^b) -$$

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$$-\sum_c \frac{n_c}{k_B T}\int \Delta_i \varphi_{ik}(\mathbf{r}_i^a,\mathbf{r}_k^c)\Phi_{jk}(\mathbf{r}_j^b,\mathbf{r}_k^c)d\mathbf{r}_k^c \qquad (3)$$

In Fourier space the set of equations (3) turns into the linear algebraic set of equation whose solution is easily derived to yield the following expression for the Fourier transform of interaction potential $\tilde{\Phi}_{dd}(k)$ between dust grains

$$\begin{aligned}\tilde{\Phi}_{dd}(k) &= \frac{1}{\Delta}\Big[\tilde{\varphi}_{dd}(k)+\\ &+A_e\left(\tilde{\varphi}_{dd}(k)\tilde{\varphi}_{ee}(k)-\tilde{\varphi}_{ed}^2(k)\right)\\ &+A_i\left(\tilde{\varphi}_{dd}(k)\tilde{\varphi}_{ii}(k)-\tilde{\varphi}_{id}^2(k)\right)\\ &+A_eA_i\big(2\tilde{\varphi}_{ed}(k)\tilde{\varphi}_{id}(k)\tilde{\varphi}_{ei}(k)\\ &+\tilde{\varphi}_{dd}(k)\tilde{\varphi}_{ee}(k)\tilde{\varphi}_{ii}(k)\\ &-\tilde{\varphi}_{dd}(k)\tilde{\varphi}_{ei}^2(k)-\tilde{\varphi}_{ee}(k)\tilde{\varphi}_{id}^2(k)\\ &-\tilde{\varphi}_{ii}(k)\tilde{\varphi}_{ed}^2(k)\big)\Big], \qquad (4)\end{aligned}$$

$$\begin{aligned}\Delta &= 1+A_e\tilde{\varphi}_{ee}(k)+A_i\tilde{\varphi}_{ii}(k)+.\\ &+A_eA_i\left(\tilde{\varphi}_{dd}(k)\tilde{\varphi}_{ee}(k)-\tilde{\varphi}_{ed}^2(k)\right) \qquad (5)\end{aligned}$$

The pure Coulomb potential is taken as a micropotential of interaction between the constituents of the charged component of buffer plasma

$$\varphi_{ee}(r)=\varphi_{ii}(r)=-\varphi_{ei}(r)=\frac{e^2}{r} \qquad (6)$$

The interaction of the charged component of buffer plasma, i.e. electrons and ions, with the conductive dust grain of radius R is found by the electrostatic image method [3,4] as

$$\varphi_{ed}(r)=\frac{Ze^2}{r}-\frac{e^2R^3}{2r^2(r^2-R^2)},$$

$$\varphi_{id}(r)=-\frac{Ze^2}{r}-\frac{e^2R^3}{2r^2(r^2-R^2)} \qquad (7)$$

and the conductive dusty particles interacts with each other through the following micropotential [3,4]

$$\varphi_{dd}(r)=\frac{Z^2e^2}{R}\left[\frac{1}{\sinh[\beta]\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sinh[n\beta]}}-1\right] \qquad (8)$$

where $\cosh[\beta]=r/2R$.

In Eqs.(7) and (8) the polarization effect of the conductive dust grains is accounted for by the electrostatic image method. That the dust grains are assumed to be conductive does not restrict the generality of the present consideration since the polarization effect will come to play independently of the material the dust grains are made of.

It is easily seen from Eq.(7) that the interaction potential of a charged particle with a charged conducting sphere (dust particle) contains, along with the pure Coulomb potential, an additional term standing for the electrostatic induction. The same is true for describing the interaction of two charged conducting spheres, in both cases the method of electrical images is employed.

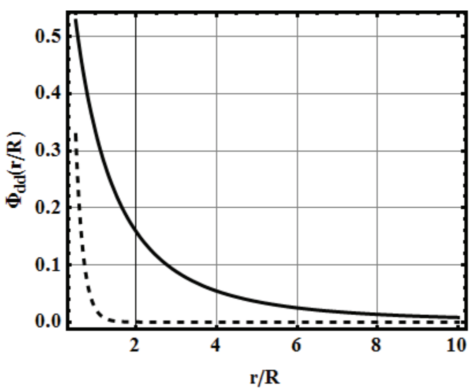
Strictly speaking the Fourier transforms of micropotentials Eqs.(7) and (8) do not exist because of diverging integrals. On the other hand, the integration in Eq.(1) and, correspondingly in Eq.(3), should obviously exclude the inner parts of dust grains due to the averaging procedure described above. The following transformations,

$$\varphi_{di}(r)\to\varphi_{di}(r+R),$$

$$\phi_{dd}(r)\to\phi_{dd}(r+2R),$$

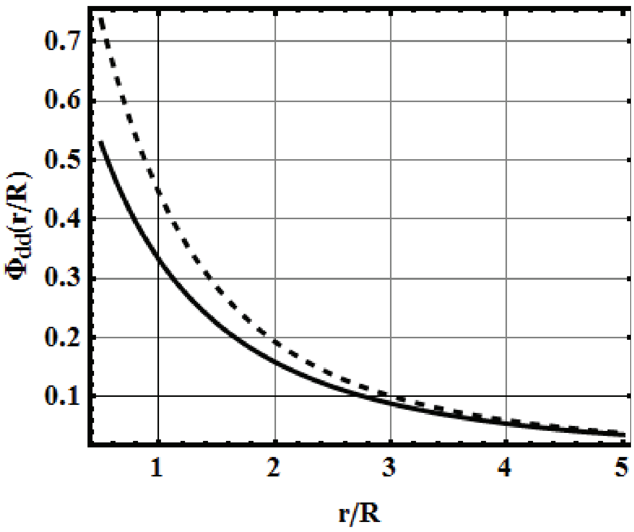
make again available the whole space for the integration and, thus, the solution in Eqs.(4), (5) of (3) still validates. After that the Fourier transforms of the macropotential is simply obtained and the numerical result is presented in Fig. 2. It is easily seen that the interaction potential between dust grains is less screened than the corresponding potential in the Debye-H.uckel theory. In Fig.3 comparison is made for the macropotential of intergrain interaction for two different values of the screeening parameter. It is observed that increasing the screening parameter results in more rapid vanishing of the intergrain interaction potential.

The nonmonotonic behaviour of the macropotential was witnessed in [5] indicating an attraction between grains for at least some values of plasma parameters. The attraction mechanism was assumed the same as, e.g., for the molecule formation. Namely, if the dust grains are negatively charged, then, according to Eq.(2), the probability to find protons has a maximum located right between the dust grains. Thus, a cloud of protons is formed to attract both dust grains. It has to be said here that the polarization phenomenon may play essential role to overcome screened electrostatic repulsion [3]. It was argued, however, in [6,7] that the attraction is impossible in the Poisson-Boltzmann model considered above. In this context it has to be mentioned here that the buffer plasma was treated in [6,7] as a liquid dielectric medium [4] and, then, the stress tensor was applied to calculate the force between the dust grains. In fact, the plasma is not a dielectric medium and its electrodynamics is completely different from the electrodynamics of simple dielectrics considered in the corresponding chapter of [4]. In simple dielectrics the electric displacement vector at a spatial point depends on the electric field strength vector at the same spatial point. On the contrary, the plasma medium reveals the so-called spatial dispersion which means that the electric displacement vector at a spatial point depends on the electric field strength vector at neighboring points as well. This usually results in that the dielectric constant of the simple dielectric has to be replaced by the dielectric function of the plasma with the corresponding wavenumber dependence.



Solid line: the intergrain macripotential; dashed line: the potential in the Debye-Huckel theory

Figure 2 – The intergrain potential in comparison with the Debye-Huckel theory at $Z=10$, $\Gamma_R=0.1$ and $\kappa=5$



Solid line: $\kappa = 5$; dashed line: $\kappa = 2$

Figure 3 – Comparison of the two intergrain potentials at $Z = 10$, $\Gamma_R = 0.1$ and $\kappa = 5$

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