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Scattering in α^6Li-system in the framework of the wave packet continuum discretization method

Abstract

There is a consideration of α-particles scattering from weakly bound cluster nucleus ^{6}Li in the framework of three cluster $\alpha+(\alpha+d)$ model in the present work. Continuum $\alpha-d$ states are projected on the finite basis of stationary wave packets what allows to lead the total problem of three bodies to a problem of bound channels. There are represented the calculations results of $\alpha\,^6$Li-scattering differential cross sections in the framework of three particle model and comparison with other authors.
Keywords: Elastic and inelastic scattering, free wave packets, spectrum equidistant discretization, cluster, Jacobi coordinates, α - particle, compound particle ^{6}Li, relative motion, channels coupling potential.

Introduction

The problem about low energy scattering of α-particles on the weakly bound ^{6}Li and ^{7}Li isotopes has a great fundamental and applied importance.

From the fundamental side the scattering in $\alpha+{}^6$Li system is interesting, first of all, owing to anomalous low binding energy of ^{6}Li by $\alpha+d$ channel (ε_0 = 1.47 MeV), i.e. proximity to disintegration threshold. This means the relative easiness of ^{6}Li excitation into the states of $\alpha-d$ continuum in the process of interaction with incident α-particle. In turn, such an excitation of $\alpha-d$ continuum leads to the great contribution of intermediate three cluster $\alpha+\alpha+d$ states in the scattering process at small energies. And the three cluster configurations are virtually the excited states of ^{10}B nucleus. That is why it is possible to predict the strong transitions (for example, radiation) of $\alpha-{}^6$Li continuum states into the ground state of ^{10}B.

From the other side such a process of a particle and weakly bound nucleus interaction is typical for interaction of non stable (radioactive) nuclei as ^{9,11}Li, ^{10,12}Be and etc. with stable targets. That is why we can think that it is better to understand the features of such interactions of the stable particle with non stable (weakly bound) nucleus by investigation of interaction with well investigated cluster nucleus such as ^{6}Li which has an anomalous low binding energy.

From the applied side, the nuclear reactions of nucleons and α-particles with ^{6}Li nucleus are important for the better understanding of elements evolution in the Big Bang and at super nova outburst. This is closely connected to the old problem of abundance of ^{6}Li isotopes in the Universe, observed in the "iron" stars, in comparison with the predictions of the standard astrophysical models. It is clear that for sure estimations of ^{6}Li isotopes abundance it is necessary to know as possible exact the probability of its synthesis and its disintegration in different nuclear reactions. And the channel $\alpha+{}^6$Li is one of the main channels.

In this work we have investigated the role of the intermediate states of $\alpha-d$-continuum of target nucleus $^6\mathrm{Li}\to\alpha+\mathrm{d}$ in the process of α-particles interaction with ^6Li nucleus. The simple general physical considerations show that these $\alpha-d$ states are to give the most contribution into the cross sections of elastic and inelastic $\alpha+{}^6Li$ scattering, and also into the cross sections of $\alpha+{}^6Li\to{}^{10}B+\gamma$ radiation capture.

Since the total solution of scattering in $\alpha+\alpha+d$ system is too tedious and time-consuming because of the big coulomb effects, we carried out the packet reduction

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[1] of the problem. The packet reduction means in our case that we projected the states of the three particle continuum on the discrete basis of wave packets and instead of the solution of the total three particlesSchrodinger equation moved to a problem for the coupled channel.

So, there is a consideration of the three particles problem of ${}^6\mathrm{Li}+\alpha$ scattering in the framework of the wave packet continuum discretization (WPCD) method, which was developed by authors and collaborators in [1], where the detailed description of formulation of the three particles scattering problem on the basis of wave packet technique is shown, and there are also specific examples of calculation of partial phase shifts and three particles elastic scattering differential cross sections, and also comparison with the results of other methods in quantum scattering theory (for example, continuum discretized coupled channel (CDCC) method, variation method, finite elements methods and other). It was found that [1] the agreement of calculations on the basis of packet approach with other methods is quite well, and this is a reason for our choice of this method for $\alpha\,{}^6\mathrm{Li}$ -scattering consideration. The WPCD-method, in contrast to the folding model, allows taking into account the excitations into the continuum of the projectile nucleus. In contrast to the well known CDCC-method of continuum discretization in the WPCD-method there is a continuous spectrum discretization of not only by the coordinate of relative motion of particles composing the projectile, but by the coordinate of relative motion of the free target particle and the centre of mass of compound projectile particle. Thus the particles excitations are taken into account with the help of Hamiltonian spectrum discretization of all three particles continuum being under consideration. It is worth to notice that in contrast to exact wave functions for the continuous spectrum the three particles wave packets are square-integrable functions, i.e. belong to Hilbert space for three bodies' problem. The discretization of the three particles continuum with the help of the wave packets and especial properties of the three particles wave packets [1, 2] allow to lead the difficult integral equations, describing the scattering in the three particles system, to purely algebraic ones owing to the fact that the three particles channel resolvents in the packet approach are in an explicit finite-dimensional diagonal representation.

The model formulation

Let's consider the $\alpha+{}^6\mathrm{Li}$ -scattering in the framework of the WPCD-method as a scattering of the compound particle ${}^6\mathrm{Li}\{\alpha d\}$ (projectile) on a structureless α -particle (target). The choice of the projectile as a subsystem of $\{\alpha d\}$ is reasoned by the dominating cluster αd -structure of the ${}^6\mathrm{Li}$ nucleus. In $\{\alpha d\}$ subsystem there is one s -wave state with an energy ε_0 = – 1.47 MeV. In calculations a magnitude of the kinematic parameter $\hbar^2/m$ = 41.47 MeV·fm^{-2} is used, where m – nucleon mass.

Jacobi coordinates

Let's introduce the Jacobi coordinates for the three cluster $\alpha+\alpha+d$ -system in the $\alpha+{}^6\mathrm{Li}$ channel being under consideration. Define the *T*-set of the Jacobi coordinates $\{\vec{\xi},\vec{\eta}\}$: $\vec{\xi}=\vec{r}_2-\vec{r}_3$ – a coordinate of the relative motion of α and d clusters in the ${}^6\mathrm{Li}$ nucleus; $\vec{\eta}=\vec{r}_1-\frac{1}{6}\left(4\vec{r}_2+2\vec{r}_3\right)$ – a coordinate of the relative motion of the centre of mass of $\alpha+d$ pair and the free α -particle, where $\vec{r}_i$ – particles radius-vectors. Define also the *V*-set $\{\vec{x},\vec{y}\}$: $\vec{x}$ – a coordinate of the relative motion of two α -particles, $\vec{y}$ – a coordinate of the relative motion of the target α -particles and deuteron. The connection between the *T*- and *V*-sets of coordinates is realized by the relations: $\vec{x}=\vec{\eta}+\frac{1}{3}\vec{\xi}$, $\vec{y}=\frac{3}{2}\vec{\eta}+\frac{3}{2}\vec{\xi}$. If one needs it is possible to do the reverse transition for the Jacobi coordinates.

The nuclear interaction of the clusters α and d in the nucleus ${}^6\mathrm{Li}$ is well described by the Gauss potential [3, 4]:

$$V_{\alpha d}(\xi)=-V_0\exp\left(-\gamma\xi^2\right),$$

where $V_0=75.8469155$ MeV, $\gamma=0.2$ fm^{-2}. Exactly the same potential is chosen for interaction between the projectile α -particle and deuteron cluster from ${}^6\mathrm{Li}$. The nuclear interaction between two α -particles is also taken in Gauss form (Buck potential):

$$V_{\alpha\alpha}(x)=-U_0\exp\left(-\beta x^2\right),$$

where $U_0=129$ MeV, $\beta=0.225$ fm^{-2} [5].

Let's remind that the orbital momentum of $\alpha-d$ bound state in ${}^6\mathrm{Li}$ nucleus is $l=0$, hence the total orbital momentum of the three particles system is defined by the orbital momentum of the relative motion of the free α-particle and the orbital momentum of the pair $\alpha-d$. Although the higher orbital momentum in $\alpha-d$ subsystem must play a certain role, the main contribution apparently will be made by the state with $l=0$ of the $\alpha-d$ continuum (it responses to the oscillating states in $\alpha-d$ system). That is why we are taking into account these dominating states with $l=0$ of the $\alpha-d$ continuum in this work. The contribution of the states with $l>0$ must be investigated separately.

Wave packets basis formation

As it was shown in [1], instead of the exact wave packets $\left|Z_i^l\right\rangle$ for discretization of the $\alpha-d$ continuum it is possible to use the wave functions of pseudo states $\left|\tilde{Z}_i^l\right\rangle$, which one can get in the result of the diagonalization of the matrix of two particles Hamiltonian $h_1^{\alpha d}(\xi)$ with the L_2-basis of gauss type:

$$\psi_k(\xi)=e^{-\alpha_k \xi^2}, \text{ where } k=1,...,K,$$

here α_k – scale parameter, defined on the base of the generalized Tchebyshev distribution

$$\alpha_k=\alpha_0\left[tg\left(\frac{2k-1}{4K}\pi\right)\right]^t,$$

where α_0 – the general scale parameter of the basis, K – basis dimension, t – sparseness parameter of the nonlinear parameters of the basis α_k. One can show that at $K\to\infty$ such a basis is complete.

It is important to take into account that all three particles in our case are charged, and that is why considering of a long-range Coulomb potential at the construction of three particles wave packets is very important.

In the result of diagonalization of the two particles Hamiltonian of the bound state $h_1^{\alpha d}(\xi)$ in the above mentioned gauss basis we get the discrete set of the wave functions of the pseudo states:

$$\left|\tilde{Z}_i^l\right\rangle=\sum_{k=1}^{K}D_k^i\left|\psi_{kl}\right\rangle.$$

The next step is a construction of channels coupling potential:

$$V_{ii'}(\eta)=\left\langle \tilde{Z}_i^l\right|V_{\alpha\alpha}(x)+V_{\alpha d}(y)\left|\tilde{Z}_{i'}^l\right\rangle,\ i,\,i'=\overline{1,K}.$$

The case $i=i'=0$ responds to an interaction of α-particles with ^{6}Li in the ground states, what answers to the folding model, the other channels answer to excitation of the compound target into the continuum. The coupling potentials have the following analytical representation:

$$V_{ii'}(\eta)=\frac{\sqrt{\pi}}{4}\sum_{ii'}D^iD^{i'}\left\{\frac{V_0}{a_1^{3/2}}e^{-q_1\eta^2}+\right.$$
$$\left.+\frac{U_0}{b_1^{3/2}}e^{-p_1\eta^2}\right\},$$

where $a_1\equiv f+\frac{9}{4}\gamma$, $f\equiv\left(\alpha_k^i+\alpha_k^{i'}\right)$, $q_1\equiv a_3-\frac{a_2^2}{4a_1}$, $a_2\equiv\frac{9}{2}\gamma$, $a_3\equiv\frac{9}{4}\gamma$, $b_1\equiv f+\frac{1}{9}\beta$, $p_1\equiv\beta-\frac{b_2^2}{4b_1}$, $b_2\equiv\frac{2}{3}\beta$.

For the free wave packets there were used the approximate expressions through the exact functions of free motion on the correspondent "j" energy interval:

$$X_j^L(\eta)\approx\sqrt{\frac{\Delta m}{\hbar^2}}\frac{J_{L+1/2}(K_j\eta)}{\sqrt{\eta}},$$

$$K_j=\sqrt{\frac{2m\varepsilon_j^*}{\hbar^2}},\ j=1,...,N,$$

where $J_{L+1/2}(\zeta)$ – the spherical Bessel function, $\Delta=E_{\max}/N$ – width of energy bins of the spectrum equidistant discretization, ε_j^* – the average points of discretization intervals.

Thus, we filled the matrix of external interaction potential in the three particles basis of the free wave packets. After this the elements of matrix of three particles channel resolvent are calculated. And at last, the integral Lippmann-Shwinger equations in linear purely matrix form are solved.

For further simplification it is possible to use the projection formalism of Feshbach [1]. At this stage the channels coupling potential obtained and the free wave packets $\left|X_j^L(\eta)\right\rangle$ are used for a construction of the effective interaction potential, depending on energy, for α-particles with ^{6}Li as the whole particle:

$$V_{ij,i'j'}(E)=\left\langle X_j^L(\eta)\middle|V_{ii'}(\eta)\middle|X_{j'}^L(\eta)\right\rangle.$$

Such a way the problem of the three particles scattering is led to the effective two particles problem with such an effective interaction potential, including all the internal excitations of the projectile particle.

Numerical calculations results

In the numerical calculations the following magnitudes of the parameters were used: the basis dimension K = 15, t = 2. The parameter α_0 was chosen to describe in the best way the energy of $\alpha-d$ bound state with $l=0$ ($J=1$). The dimension of the external basis of the free wave packets was chosen equal to N = 70. The maximal angular momentum by the coordinate of the $\alpha-{}^6$Li relative motion was chosen equal to L = 20. The maximal quantity of energy spectrum $E_{\max}$ = 200 MeV.

There are represented the calculated differential cross sections for the three different energies in the figure1.

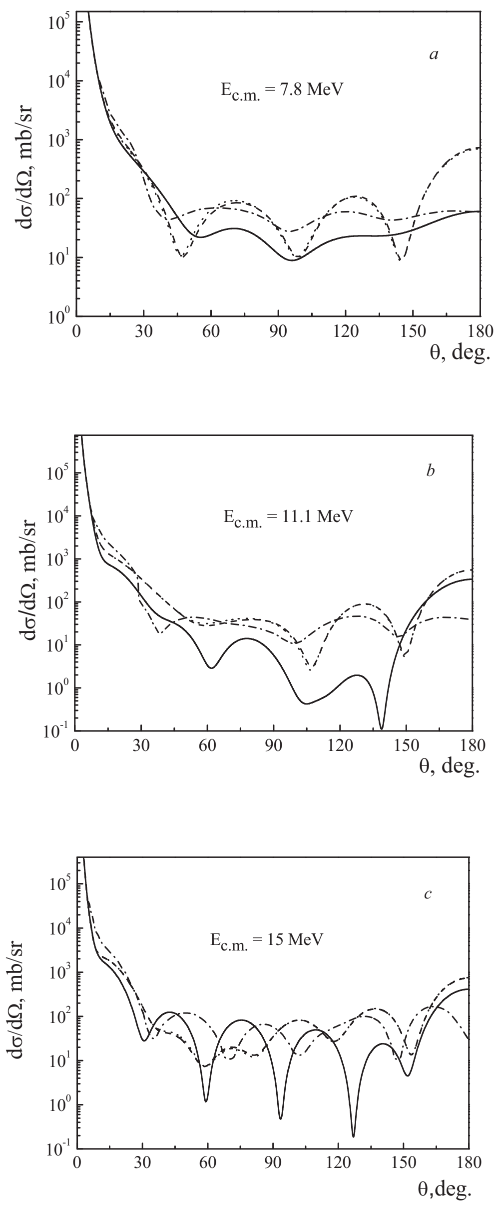

Figure 1– The differential cross sections of the elastic $\alpha\,^6\mathrm{Li}$ -scattering for the three energies: dashed curve – with the coupling potential accounting only the ground state of $^6\mathrm{Li}$, dotted – with the folding potential, the solid curve – with the potential, accounting all channels of excitation of $^6\mathrm{Li}$ in $\alpha-d$ continuum, the chain line – with the inverse potential [6]

In the work [6] the differential cross sections of $\alpha+{}^6\mathrm{Li}$ scattering were found for model of potential, obtained from the solution of the inverse scattering problem on the base of the empirical scattering data. In work [3] the wave functions of the nucleus $^6\mathrm{Li}$ in $\alpha-d$ model were found with the help of variation method, then these functions were used for a construction of the folding potential, responding to α-scattering on the nucleus $^6\mathrm{Li}$, considered as a whole particle, i.e. without accounting its internal excitations into the $\alpha-d$ continuum.

Conclusion

We considered the role of the states of the intermediate $\alpha-d$ continuum with $l=0$ in the elastic scattering of α-particles on the weakly bound nuclei $^6\mathrm{Li}$. For calculations in the framework of the $\alpha{+}\alpha{+}d$ model there were used the method of the stationary wave packets and packet reduction of the three particles scattering equations in the three cluster system $\alpha+(\alpha+d)$. We found, that the account of the states of the intermediate continuum noticeably smoothed the sharp oscillations of the scattering differential cross sections, typical for the folding model potential and made the cross sections closer to the empirical cross sections of scattering, obtained on the base of the potential of inverse problem, i.e. one can say that the account of the intermediate continuum substantially approaches the solution of the three particles model problem to the exact solution of scattering problem (extracted directly from the experimental data).

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