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Dark energy model with anisotropic fluid and time-varying lambda in Kaluza-Klein metric

Abstract. In this paper, we have investigated the dark energy cosmological model in the presence of anisotropic fluid in Kaluza–Klein metric with generalized time-dependent lambda $\Lambda=\alpha H^2 + \beta S^2$ (α, β are free parameters; H is Hubble parameter and S is normal scale factor). Considering the equation of state (EOS) $p = \omega\rho$ for normal dimensions and $p_\psi= (\omega+\delta)\rho$ for the fifth dimension, exact solutions of Einstein field equations of the anisotropic model are obtained (where p - the pressure for normal dimensions, p_ψ – the pressure of the fifth dimension, ρ - density of the fluid, ω - EOS parameter, and δ -skewness parameter). It is concluded that the universe at its early stage shows anisotropic behavior due to its finite value δ. The variations of ω and δ demonstrate the evolution from radiation dominated early universe to a dark energy-dominated universe. We have also investigated dark energy density, pressure, and other physical parameters. The physical parameters are dependent on free parameters and power index factor n which relates the extra dimension scale factor to the normal scale factor.
Key words: Dark energy, Cosmological constant, Equation of state, Kaluza-Klein Cosmological model, Exact solution.

Introduction

The present arena of the universe showcases that it consists of 68% of dark energy (DE), 28% dark matter (DM), and 4% visible matter. The existence of dark energy has been confirmed through the accelerated expansion of the universe, illustrated by the High-Z supernova search team led by Reiss et al [1] and the supernova cosmology project headed by Perlmutter et al [2]. Recently, the expected data through Dark Energy Spectroscopic Instrument (DESI) survey explains the nature of dark energy [3-5] which is yet to be cracked. Another constitute of the universe, dark matter (DM) is also a mystery. In 1937, Zwicky [6] identified the discrepancy between observed and predicted galactic rotational curves and suggested that it could be due to the presence of DM. In the present scenario, the existence of DE and DM has been well established, but the mystery of their nature remains unsolved. This has prompted many cosmologists to explore the study of models with dark energy fluid. Cheng et al [7], and Jain [8] have studied dark energy–dark matter interactive models with varied perspectives. These interactive DE-DM models are studied extensively nowadays.

This paper explores a cosmological model with a generalized time-dependent cosmological constant (Λ) in the presence of anisotropic dark energy fluid in the Kaluza-Klein metric.

Cosmological studies have been revealed by assuming the universe consists of fluid modeled with the equation of state (EOS) $\omega = p/\rho$ (p - pressure, ρ - density of the fluid), the values of which have been utilized to study different phases of the universe [9]. In this regard, DE EOS ω is considered to be equal to -1. Jimenez, Usmani, et al, & Amendola [10-12] have suggested quintessence and phantom forms of dark energies models with ω >-1 & ω< -1 respectively. Gorbunova & Timoshkin [13], and Das et al [14] have proposed theoretical models with time-dependent ω which is yet to be confirmed experimentally. Melia [15] put forward supernova cosmological project results with the new constraints on ω given by $\approx$ -1.05 ±.09 for a flat universe. The experimental observations for luminosity distance, high redshift, and galaxy clustering conclude with values of -1.44 <ω< -0.92 at a 68% confidence level [16-17]. In this regard, Wellar & Lewis [18] have concluded with similar results. These constraints on depicting the existence of dark energy and the variation of pressure in different directions bring out the anisotropic nature of the universe. The presence of a small anisotropy in the current isotropic scenario has also been concluded through the study of minute temperature variations observed at the Cosmic Microwave Background (CMB) level [19]. This is supposed to contain information about early universe phenomena, phase transition, etc. Microwave Anisotropy Probe (MAP) [20] and COBRAS-SAMBA (Planck surveyor) [21], BAO-SDSS (Baryon Acoustic Oscillation –Sloan Digital Sky Survey) (Eisenstein et al, 2005) [22] experimental results led to a further study of anisotropy in CMBR radiations.

Various Bianchi-type anisotropic dark energy models, FRW isotropic & anisotropic models have been dealt with by studying the nature of dark energy in the current scenario [23-31]. These models have explained the dynamics of dark energy in the anisotropic universe, considering parameterized ω (t) and introducing skewness parameters (δ) in different directions.

Berman [32], Koivisto & Mota [33] investigated the models with variations of the Hubble parameter in the absence of a cosmological constant (Λ). Several other forms of dark energy models in the absence of cosmological constant have also been investigated in quintessence, braneworld, f(R) gravity, scalar field, etc. [34-37]

Recent studies and experimental observations in cosmology do consider the significance of the cosmological constant that has been first introduced by Einstein and is now known to be physically significant with dark energy [38]. An excellent review of the cosmological constant by Weinberg [39] and Sahni [40] has revealed that the cosmological constant has been plagued with a cosmological constant puzzle (CCP). The CCP has a discrepancy of about 120 orders between its cosmological observed value and the calculated one at the Planck level. Overduin & Cooperstock, [41]; Chen & Wu [42], and Carvalho et al [43] have tackled this problem by investigating 4D FRW models with time-dependent lambda varying with H^2, R^{-m}, qH^2 (q – deceleration parameter), or $\alpha H^2+\beta R^{-2}$ (generalized lambda). Here, the cosmological constant varies with the time-dependent scale factor. Considering time-dependent lambda in Einstein-Hilbert action, the diffeomorphism invariance implies the modification of Einstein field equations. But these modifications led to the explanation of the accelerated expansion of the Universe [44-45].

The dark energy model with a generalized lambda has successfully dealt with age, low-density problems, and the presence of anisotropy in the universe's cosmic background. However, shortfalls of 4D models in dealing with CCP and cosmological coincidence problems have led cosmologists to search for an alternative in the higher dimensional field.

With the upsurge of string theory, 5D models have gained popularity for their simplicity which can not only explain early universe phenomena but also can depict present universe scenarios. In this regard, Kaluza [46] first put forth five-dimensional models to unify gravitational and electromagnetic forces, and later Klein [47] employed gauge theory to explain the five-dimensional theory. In this regard, Wesson P [47] proposed the space-time-matter theory or induced matter theory which helps in the explanation for the unification of gravity and weak forces.

The original Kaluza-Klein theory placed two very strong constraints on the fifth dimension, namely, (i) that all partial derivatives concerning the fifth coordinate are zero (cylinder condition), and (ii) that the fifth dimension has a closed short-scale topology (compactification condition). The most important consequences of these conditions are that no change in 4-dimensional physical quantities can be ascribed to the presence of an extra spatial dimension and that such a fifth dimension is unobservable at low energies. Condition (ii) was also a vital ingredient in the attempt to explain the quantization of electric charge. It is interesting to note that condition (ii) prevents whatever microscopic object from spanning the fifth dimension [48]. The extra dimensions were thought to be lower than the Planck scale and so could not be tested experimentally but its effect can be experienced [49].

In this regard, the existence of the Kaluza-Klein particles can confirm the presence of an extra dimension. Considering the fifth dimension as a scalar function ϕ in the form of a circle of radius r and its Fourier expansion results in higher orders of the functions or towers of massive modes. These are termed Kaluza-Klein excitations or are also identified as K-K particles [50-51]. One of the constituents of dark matter has been supposed to be K-K particles since its constituents are still unknown to us.

A rich literature on Kaluza-Klein cosmological models is now available which has been studied in various contexts. The Kaluza-Klein models with different forms of matter in the presence of generalized lambda ($\Lambda = \alpha H^2+\beta R^{-2}$) have been investigated by Chodos & Detweiler [52], Singh et al [53], and Jain et al [54-57]. These models have demonstrated the effects of extra dimensions on the nature of dark energy and various physical parameters. Milton [58] & Radar [59] have concluded the correlation between dark energy and extra dimension. Some cosmologists [60-63] have studied Kaluza-Klein cosmology with anisotropic dark energy in the absence of lambda, explaining the universe in late times. These models have been inspirational for the present study of the anisotropic model.

With the above motivation, we have investigated the cosmological model in the Kaluza-Klein metric in the presence of time-varying lambda. We have examined DE cosmological model with anisotropic fluid by introducing skewness parameter δ in EOS of extra-dimension. This led to the study of directional dark energy fluid. This paper is organized into six sections. With the introduction in the first section; metric and field equations are discussed in section 2. Solutions of field equations and some physical parameters are obtained in sections 3 and 4 respectively, followed by discussion and conclusion in sections 5 and 6 respectively.

Metric and Field equations

To find the Einstein field equation, we consider the Kaluza-Klein metric [55] as given below

$$ds^2 = -dt^2 + \\ +S^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2\right] + \\ +A^2(t)d\psi^2. \quad (1)$$

Here $S(t)$, and $A(t)$ are fourth and fifth-dimension scale factors respectively, k is the curvature constant which is equal to 0,1 and -1 for flat, closed, and open universes respectively., and energy-momentum tensor for the anisotropic model [62] for the above metric is given as below,

$$T_j^i = diag(T_0^0, T_1^1, T_2^2, T_3^3, T_4^4) = \\ = diag\left(-\rho, p, p, p, p_\psi\right). \quad (2)$$

Whereρ, p and p_ψ are the DE fluid's density, pressure, and extra dimension pressure respectively. EOS for normal dimensions has been assumed to be $p = \omega\rho$,ω is the equation of state parameter (for normal dimensions), while $p_\psi = (\omega+\delta)\rho$ for extra-dimension, and is the skewness parameter introducing the deviation from isotropy. Hence,

$$T_j^i = diag\left(-\rho, \omega\rho, \omega\rho, \omega\rho, (\omega+\delta)\rho\right), \quad (3)$$

ω and δ may not necessarily be constants.

Einstein's field equations are arrived at by the following equation:

$$G_j^i = R_j^i - \frac{1}{2}R\delta_j^i = -8\pi G T_j^i + \Lambda\delta_j^i . \quad (4)$$

In deriving Einstein's field equation we assume $8\pi G = c = 1$ and using *ansatz* for metric potentials [56 and references therein] $A = R^n$, field equations derived from Eq.(4) are given below :

$$G_1^1 = (n+2)\frac{\ddot{S}}{S} + \\ +(n^2+n+1)\frac{\dot{S}^2}{S^2} + \frac{k}{S^2} = -p + \Lambda, \quad (5)$$

$$G_4^4 = 3\frac{\ddot{S}}{S} + 3\frac{\dot{S}^2}{S^2} + 3\frac{k}{S^2} = -p_\psi + \Lambda, \quad (6)$$

$$G_5^5 = 3(n+1)\frac{\dot{S}^2}{S^2} + 3\frac{k}{S} = \rho + \Lambda. \quad (7)$$

These field equations are rewritten as,

$$(n+2)\frac{\ddot{S}}{S} + (n^2+n+1)\frac{\dot{S}^2}{S^2} + \frac{k}{S^2} = \\ = -\omega\rho + \Lambda , \quad (8)$$

$$3\frac{\ddot{S}}{S} + 3\frac{\dot{S}^2}{S^2} + 3\frac{k}{S^2} = -(\omega+\delta)\rho + \Lambda. \quad (9)$$

$$3(n+1)\frac{\dot{S}^2}{S^2} + 3\frac{k}{S^2} = \rho + \Lambda . \quad (10)$$

Divergence of Einstein's tensor has been given by,

$$\left(R_j^i - \frac{1}{2}R\delta_j^i\right)_{;j} = \left(-T_j^i + \Lambda\delta_j^i\right)_{;j} = 0. \quad (11)$$

From the above equation, the energy conservation equation [62] is obtained as:

$$\dot{\rho} + (\rho+p)3\frac{\dot{S}}{S} + (\rho+p_\psi)n\frac{\dot{S}}{S} + \dot{\Lambda} = 0. \quad (12)$$

Substituting $p = \omega\rho$ and $p = (\omega+\delta)\rho$ in equation (11), it is further simplified as:

$$\dot{\rho} + (1+\omega)(3+n)\frac{\dot{S}}{S} + n\delta\frac{\dot{S}}{S} + \dot{\Lambda} = 0\,. \quad (13)$$

Eq. (13) can be separated into two equations. One equation contains a deviation-free parameter and the other has a skewness parameter so that the presence of anisotropic conditions in the present isotropic universe can be explained. The two equations are:

$$\dot{\rho} + (1+\omega)(3+n)\frac{\dot{S}}{S} = 0. \quad (14)$$

$$n\delta\frac{\dot{S}}{S} + \dot{\Lambda} = 0. \quad (15)$$

The solution of field equations is obtained in the next section by substituting $\Lambda = \alpha\frac{\dot{S}^2}{S^2} + \beta\frac{1}{S^2}$ in the above equation.

Solution of field equations

There are three independent equations (field equations) and S, ρ, ω, δ, and Λ are five independent variables. So the solution of field equations is obtained with the help of time-dependent lambda i.e. $\Lambda = \alpha\frac{\dot{S}^2}{S^2} + \beta\frac{1}{S^2}$, where α and β are free parameters.

Subtracting equation (8) from equation (9) we get,

$$-\delta\rho = (1-n)\frac{\ddot{S}}{S} + (2-n-n^2)\frac{\dot{S}^2}{S^2} + \frac{2k}{S^2}. \quad (16)$$

Equation (15) is rewritten as,

$$n\delta\rho\frac{\dot{S}}{S} = -\dot{\Lambda}\,. \quad (17)$$

Now consider $\Lambda = \alpha\frac{\dot{S}^2}{S^2} + \beta\frac{1}{S^2}$, we find, $\dot{\Lambda} = 2\alpha\frac{\dot{S}\ddot{S}}{S^2} - 2\alpha\frac{\dot{S}^3}{S^3} - 2\beta\frac{\dot{S}}{S^3}$, substituting this in Eq. (17)

Eq. (17) is rewritten as,

$$n\delta\rho\frac{\dot{S}}{S} = -2\alpha\frac{\dot{S}\ddot{S}}{S^2} + 2\alpha\frac{\dot{S}^3}{S^3} + 2\beta\frac{\dot{S}}{S^3}\,. \quad (18)$$

Rewriting the above equation as:

$$\delta\rho = -\frac{2\alpha}{n}\frac{\ddot{S}}{S} + \frac{2\alpha}{n}\frac{\dot{S}^2}{S^2} + \frac{2\beta}{n}\frac{1}{S^2}\,. \quad (19)$$

From Eq. (16) and (19) we get:

$$\left[(1-n) - \frac{2\alpha}{n}\right]\frac{\ddot{S}}{S} + \left[(2-n-n^2) + \frac{2\alpha}{n}\right]\frac{\dot{S}^2}{S^2} + +2\left(\frac{\beta}{n} + k\right)\frac{1}{S^2} = 0. \quad (20)$$

Simplifying the above Eq., we get:

$$\frac{\ddot{S}}{S} + \frac{\left[(n^2-n-2) - \frac{2\alpha}{n}\right]}{\left[(n-1) + \frac{2\alpha}{n}\right]}\frac{\dot{S}^2}{S^2} - \frac{2\left(\frac{\beta}{n}+k\right)}{\left[(n-1)+\frac{2\alpha}{n}\right]}\frac{1}{S^2} = 0\,. \quad (21)$$

Assuming $m = \frac{\left[(n^2-n-2)-\frac{2\alpha}{n}\right]}{\left[(n-1)+\frac{2\alpha}{n}\right]}$, $k_1 = \frac{2\left(\frac{\beta}{n}+k\right)}{\left[(n-1)+\frac{2\alpha}{n}\right]}$, Eq.(20) is simplified as,

$$\frac{\ddot{S}}{S} + m\frac{\dot{S}^2}{S^2} - k_1\frac{1}{S^2} = 0\,. \quad (22)$$

The above equation is a homogeneous second-order equation. The first-order integral equation is obtained by integrating the above equation and is given by,

$$\dot{S}^2 = A_1 S^{-2m} + \frac{k_1}{m}\,. \quad (23)$$

where A_1 is the constant of integration. We consider $m = -1/2$ to deal with present observational data. The solution of the above equation is obtained as:

$$S(t) = \frac{2k_1}{A_1} + \frac{A_1}{4}(t+c)^2\,. \quad (24)$$

In the above equation, c is the constant of integration. Constant A_1 and c can be determined by initial conditions. At t =0 let S(t) =0 then $c = -\frac{8k_1}{A_1^2}$, Let us assume c = -t_0, for simplicity, Thus the above equation is rewritten as:

$$S(t) = \frac{2k_1}{A_1} + \frac{A_1}{4}(t-t_0)^2\,. \quad (25)$$

Other physical parameters are obtained in the following section.

Determination of physical parameters

Other physical parameters i.e. H, q, ω and δ are determined using Eq. (25) as follows:

$$A(t) = S^n(t) = \left[\frac{2k_1}{A_1} + \frac{A_1}{4}(t-t_0)^2\right]^n\,. \quad (26)$$

$$H = \frac{A_1(t-t_0)}{2\left[\frac{2k_1}{A_1}+\frac{A_1}{4}(t-t_0)^2\right]}. \quad (27)$$

$$q = -\frac{S\ddot{S}}{\dot{S}^2} = -\left(\frac{1}{2}+\frac{4k_1}{A_1^2(t-t_0)^2}\right). \quad (28)$$

Substituting equation (10) and rewriting it we get,

$$\rho = [3(n+1)-\alpha]\frac{\dot{S}^2}{S^2}+\frac{(3k-\beta)}{S^2} \quad (29)$$

Using Eq. (25) in Eq.(28), we obtain,

$$\rho(t) = [3(n+1)-\alpha]\frac{A_1^2(t-t_0)^2}{4\left[\frac{2k_1}{A_1}+\frac{A_1}{4}(t-t_0)^2\right]^2}+ +(3k-\beta)\frac{1}{\left[\frac{2k_1}{A_1}+\frac{A_1}{4}(t-t_0)^2\right]^2} \quad (30)$$

$$\Lambda(t) = \alpha\frac{A_1^2(t-t_0)^2}{4\left[\frac{2k_1}{A_1}+\frac{A_1}{4}(t-t_0)^2\right]^2}+\beta\frac{1}{\left[\frac{2k_1}{A_1}+\frac{A_1}{4}(t-t_0)^2\right]^2}. \quad (31)$$

Substituting Eq. (8) and simplifying by using Eq.(29) in it, ω is calculated as,

$$\omega = -\frac{[(2n^2+3n+4)-2\alpha]A_1^2(t-t_0)^2+8k_1(n+2)+8(k-\beta)}{2[(3(n+1)-\alpha)A_1^2(t-t_0)^2+4(3k-\beta)]}. \quad (32)$$

δ is calculated from equation (16) which is given by,

$$\delta = \frac{[(2n^2+3n-5)A_1^2(t-t_0)^2+4k_1(n-1)-8k]}{2[(3(n+1)-\alpha)A_1^2(t-t_0)^2+4(3k-\beta)]}. \quad (33)$$

Expansion factor θ and Shear scalar σ^2 is determined as given below:

$$\theta = 3\frac{\dot{S}}{S}+\frac{\dot{A}}{A} = (n+3)\,H \text{ and,}$$

$$\sigma^2 = \frac{3}{8}\left(\frac{\dot{S}}{S}-\frac{\dot{A}}{A}\right)^2 = \frac{3}{8}(1-n)^2H^2.$$

Hence,

$$\frac{\sigma^2}{\theta} = \frac{3(1-n)^2}{8(n+3)}H = \frac{3(1-n)^2}{8(n+3)}\left[\frac{A_1(t-t_0)}{2\left[\frac{2k_1}{A_1}+\frac{A_1}{4}(t-t_0)^2\right]}\right]. \quad (34)$$

Discussion

The present model is investigated by assuming directional EOS for normal and extra dimensions as $p = \omega\rho$ and $p_\psi = (\omega+\delta)\rho$ respectively. The solution of the Einstein field equation is obtained by assuming $m = -1/2$ in equation (23). From equation (25) it is observed that $S\to\infty$ if $t\to\infty$, leads to continuous expansion.

Considering $m = -1/2$, it is found that $\alpha = [n(2n+3)(1-n)]/2$. Similarlyβ is calculated from k_1. For $k_1 = 1/2$, $\beta = n(n+1)(1-n)/2 - nk$. Here there are constraints on n i.e. $n \neq (0, -1, 1)$ to have a positive value of Λ. We have found $\alpha = 1/2$ and $\beta = -3/16$ for $n=-1/2$. From equation (26), it is observed that the extra dimension decreases rapidly with time. This results in the compactification of extra dimensions at present times. For $A_1^2/k_1 = 8$, equation (27) and equation (28) reveal that $H(t) \propto 1/t$ and $q(t) \to -1$, depict the acceleration of expanding universe which decreases with time (Fig.1 and Fig.2). From equation (30) it is observed that the density of the universe is proportional to $1/t^2$ and decreases with the advance of the time. For $n=-1/2$, and $t\to t_0$, $\omega \to -1$, δ has a small negative value, indicating the presence of anisotropic dark energy fluid. The following graphs are plotted for $n = -1/2$, $A_1 = 2$, and $k_1 = ½$ for $(t-t_0) > 0$ for the flat universe to reconcile with present observational data [64].

From Fig.3 and Fig. 4, we observe that there is zero-crossing of ω and δ. This indicates the transition from radiation dominated phase to a dark energy-dominated universe. The finite but small value of δ in Fig.4 points towards the presence of slight anisotropy in the present universe. It is also observed from Eq.(34) that the anisotropy factor $\sigma^2/\theta \to 0$ when $t \to t_0$, leads to the isotropic present universe. Fig.5 shows that the cosmological constant decreases at present times. The presence of negative lambda can have its significance at a very early stage of the universe [65-66]. In Fig. 6 the fall of density at present times indicates the expansion of the universe.

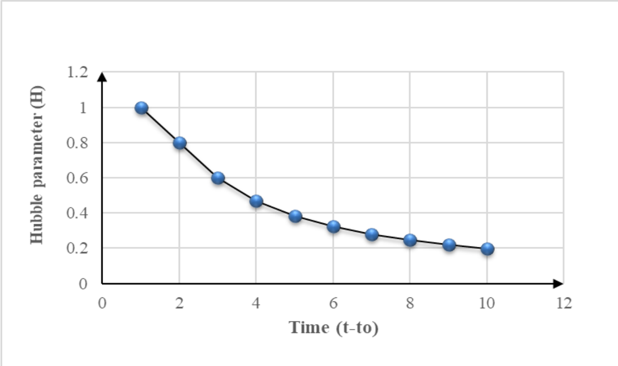

Figure 1 - The plot of H v/s (t-t_0)

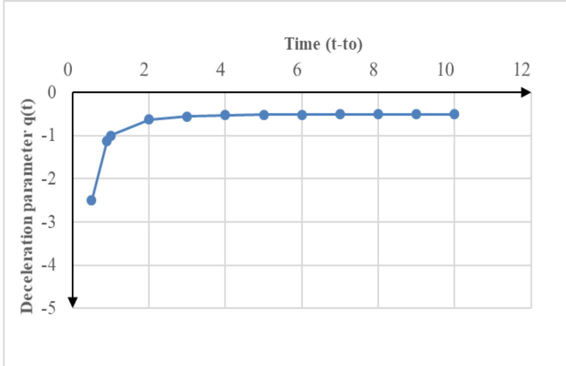

Figure 2 - Plot of q v/s (t-t_0)

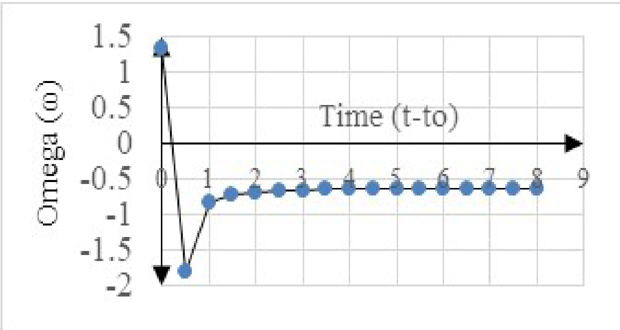

Figure 3 - The plot of ω v/s (t-t_0).

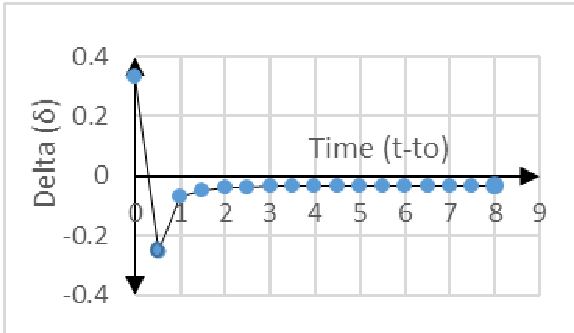

Figure 4 - Plot of δ v/s (t-t_0)

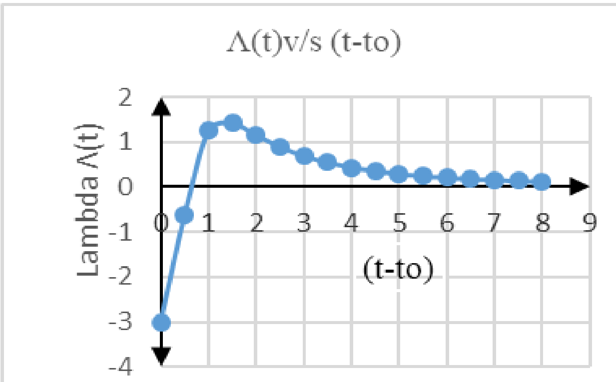

Figure 5 - Plot of lambda Λ(t) v/s (t-t_0)

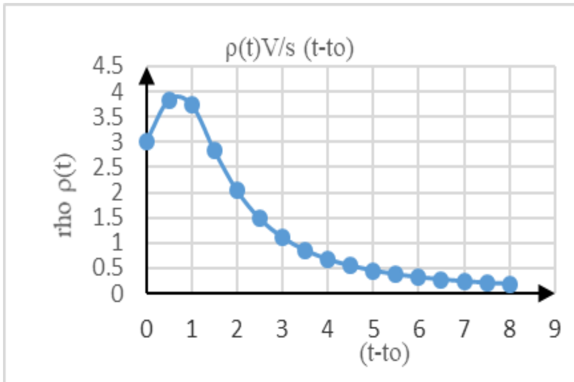

Figure 6 - Plot of rho ρ(t) v/s (t-t_0)

Conclusion

This paper has investigated the Kaluza - Klein anisotropic dark energy model with time-dependent lambda. It is found that the universe is expanding and accelerating but acceleration decreases with time. A small positive value of the cosmological constant is predicted in the present model. The joint effect of the cosmological constant and deviation parameter leads to an anisotropic early universe which later evolves as the isotropic universe. It is also observed that the present universe is dominated by dark energy. The investigation of our model of the universe also demonstrates the presence of anisotropy in the present era due to the finite value of δ. The model also explains the evolution of the universe from radiation dominated phase to a dark energy-dominated phase. Physical parameters are found to be dependent on n, free parameters β, demonstrating the impact of extra dimension and lambda on them at present times.

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References

1. Riess A. G., Strolger L.-G., and Tonry J., et al, “Type Ia Supernova discoveries at Z > 1 from Hubble telescope: evidence for past deceleration and constraints on dark energy evolution”, *The Astrophysical Journal*, (2004); 607, 665–687.
2. Perlmutter S., Gabi S., and Goldhaber G., et al, “Measurement of cosmological parameters and from the first seven supernovae at Z>=0.35”, *The Astrophysical Journal*, (1997); 483, 565-581.
3. Jaffe A. H, et al., “Cosmology from MAXIMA-1, BOOMERANG, and COBE DMR Cosmic Microwave Background Observations”, *Physical Review Letters*, (2001); 86, 3475-3479.
4. Pryke C. et al., “Cosmological parameter extraction from the first season of observations with DASI”, *Astrophys. J.*, (2002) 568, 46-51.
5. Seo Hee-Jong, and Eisenstein, D. J., “Probing Dark Energy with Baryonic Acoustic Oscillations from Future Large Galaxy Redshift Surveys”, *The Astrophysical Journal*,(2003); 598(2), 720–740.
6. Zwicky, F. “On the Masses of Nebulae and of Clusters of Nebulae”, *Astrophysical Journal*, (1937); 86, 217.
7. Cheng G., Yin-Zhe Ma, Fengquan, Jiajun Zhang and Xuelei Chen, “Testing interacting dark matter and dark energy model with cosmological data”, *Physics Review D*, (2020); 102, 043517-043529.
8. Jain N. I., “Dark energy–dark matter interactive model with time-varying lambda in Kaluza–Klein metric”, *Indian Journal of Physics*, (2020); 95,1015-1020.
9. Alcaniz J. S. “Dark Energy and Some Alternatives: a Brief Overview”, *Brazilian Journal of Physics*, (2006); 36,1109-1117.
10. Jimenez R., “The value of the equation of state”, *New Astron. Rev.*, (2003); 47, 761-767.
11. Usmani A. A. et al,. “The dark energy equation of state”, *Monthly Notices Royal Astronomical Society*, (2003);386, L92-L95.
12. Amendola L., “Acceleration at Z >1”, *Monthly Notices of the Royal Astronomical Society*, (2003); 342, 221-226.
13. Gorbunova O.G., Timoshkin A. V., “Dark energy with time-dependent equation of state, The Casimir effect and Cosmology”, *TSPU*, (2008);161-165. [arXiv:0903.1339[gr-qc](2009)]
14. Das A., Gupta S., Saini T.D., and Kar S., “Cosmology with decaying tachyon matter”, *Physics Review D*, (2005); 72, 043528-043542.
15. Melia F., “Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Observations: Implications for Cosmology”, *Astrophysical Journal Supplementary series*, (2007); 170(2), 377-408.
16. Knop R.A. et al, “New Constraints on Ω_M, Ω_Λ, and w from an Independent Set of Eleven High-Redshift Supernovae Observed with HST”, *Astrophys. J.* (2003); 598, 102-154.
17. Hinshaw G. L., et al. (WMAP Collaboration), “Five-Year Wilkinson Microwave Anisotropy Probe (WMAP1) Observations: Data Processing, Sky Maps, & Basic Results”, *Astrophysical Journal Supplements*, (2009);180,225-245.
18. Weller J., and Lewis A. M., “Large-scale cosmic microwave background anisotropies and dark energy”, *Monthly Notices Royal Astronomical Society (MNRAS*), (2003); 346, 987–993.
19. Spergel D. N., Bean R., and Dore O., et al, “Wilkinson Microwave Anisotropy Probe (WMAP) Three Year Observations: Implications for Cosmology”, *Astrophysical Journal Supplementary series*, (2007), 170(2), 377-408.

20. Hinshaw G. L., et al. (WMAP Collaboration), "Five-Year Wilkinson Microwave Anisotropy Probe (WMAP1) Observations: Data Processing, Sky Maps, & Basic Results", *Astrophysical Journal Supplements*, (2009); 180,225-245.

21. Bouchet F.R et al, "Perturbative Lagrangian approach to gravitational instability", *Astronomy, and Astrophysics*, (1995); 296, 575-608, [arXiv: Astro-ph/9507032 (1995)].

22. Eisenstein D. J., et al. "Detection of the baryon acoustic peak in the large-scale correlation function of SDSS luminous galaxies", *The Astronomical Journal*, (2005); 633, 560-575.

23. Aakarshu O., and Kilinc C. B., "Bianchi type III models with anisotropic dark energy", *General Relativity and Gravitation*, (2010); 42, 763-775.

24. Aakarshu O., and Kilinc C. B.,. "LRS Bianchi type I models with anisotropic dark energy and constant deceleration parameter", *General Relativity and Gravitation*, (2010); 42,119-139.

25. Middleton C.A., Stanley E., "Anisotropic evolution of 5D Friedmann-Robertson-Walker space-time", *Phys. Rev. D*, (2011); 84, 085013-085033

26. Pradhan A., Amirhashchi H., and Saha B., "Bianchi type-I anisotropic dark energy model with constant deceleration parameter", *Int. J. Theor. Phys*. (2011); 50(9), 2923-2938.

27. Saha B., "Isotropic and Anisotropic dark energy models", *Physics of Particles and nuclei*, (2014);45(2), 349-396.

28. Yadav A. K., Rahaman F., Ray S., "Dark energy models with the variable equation of state parameter", *Int. J. Theo. Phys.* (2011); 50, 871-881.

29. Mahanta K.L., "A dark energy model with variable EoS parameter in self-creation theory of gravitation", *Romanian Journal of Physics*, (2013); 58(3-4), 239-246.

30. Mukhopadhyay U., Ray S., & Duttachowdhary S.B., "Dark Energy Models with Variable Equation of State Parameter", *Int. J. Mod. Phys D*, (2007);17, 301-314.

31. Johri V. B., and Rath P. K., "Parametrization of dark energy equation of state", *Int. J. Mod. Phy. D*, (2007); 16(10), 1581-1591.

32. Berman M.S., "A special law of variation for Hubble's parameter", II *Nuovo Cimento B*, (1983); 74,182-186.

33. Koivisto T., Mota D.F., "Accelerating cosmologies with Anisotropic equation of state", *Astrophys. J.,* (2008); 679, 1-5. [arXiv: 0707.0279v3 [astro-ph] (2008)].

34. Adhav K. S. et al, "Kantowski-Sachs cosmological models with anisotropic dark energy", *Central European Journal of Physics*, (2011); 9(4), 919-925.

35. Mishra B., Sahoo P. K., and Ray P. P., "Accelerating Dark Energy Cosmological Model in Two-Fluid with Hybrid Scale Factor", *International Journal of Geometrical Methods in Modern Physics*, (2017); 14, 1750124-1750132.

36. Sahni V., and Starobinsky A. A., "Reconstructing dark energies", *Int. J. Mod. Phys. D*, (2006); 15, 2105-2132.

37. Ratra B., Peebles P.J.E., "Cosmological consequences of a rolling homogeneous scalar field", *Phys. Rev. D*, (1988); 37, 3406-3427.

38. Carroll, S. M., Press, W. H. and Turner, E. L., "The Cosmological Constant", *Annual Review of Astronomy and Astrophysics*, (1992); 30 (1), 499–542.

39. Weinberg S., "The cosmological constant problem", *Rev. Mod. Phys,* (1989);61,1-23.

40. Sahni V., "The Cosmological Constant Problem and Quintessence", *Class. Quant. Grav.* (2002), 19,3435-3448.

41. Overduin J.M., and Cooperstock F.I., "Evolution of the scale factor with a variable cosmological term", *Phys. Rev. D*, (1998); 58, 043506-043534.

42. Chen W., and Wu Y.S., "Implications of cosmological constant varying as R^{-2}", *Physics Review D*, (1990); 41, 695-698.

43. Carvalho J. C., Lima J. A. S., and Waga I., "Cosmological consequences of a time-dependent Λ term", *Physics Review D*, (1992); 46, 2404-2407.

44. Öztaş A.M, Dil E., and Smith M. L., "The varying cosmological constant: a new approximation to the Friedmann equations and universe model", *Monthly Notices of the Royal Astronomical Society*, (2018); 476, 1, 451-458.

45. Ray S., Mukhopadhyay U., Meng X.-H, "Accelerating Universe with a dynamical cosmological term", *Grav. Cos.* (2007); 13, 142-150.

46. Kaluza, Theodor, "Zum Unitätsproblem in der Physik" , *Sitzungsber. Preuss. Akad. Wiss. Berlin. (Math. Phys.)*, (1921); 966–972

47. Klein, O., "Quantentheorie und fünfdimensionale Relativitätstheorie", *Zeitschrift für Physik A.* (1926); 37 (12), 895–906.

48. Wesson P.S., *Space–Time–Matter, Modern Kaluza–Klein Theory*, *Singapore, World Scientific*. (1999); ISBN 978-981-02-3588-8.

49. Vassallo A., "General Covariance, Diffeomorphism Invariance, and Background Independence in 5 Dimensions", *Poznan Studies in the Philosophy of the Sciences and the Humanities,* (2015); 104,237-258 [https://doi.org/10.48550/arXiv.1410.1043]

50. Tanabashi M. et al., (Particle Data Group), “Extra dimension searches”, *Phys. Rev. D*, (2018); 98, 030001. (extradimrpp. DVI (lbl.gov))

51. Melb´eus H, “Particle phenomenology of compact extra dimensions”, Doctoral thesis, (2012); Chapter 4, 25-31, (ISBN 978-91-7501-305-3)

52. Bringmann T., “Cosmological aspects of universal extra dimensions”, Doctoral thesis, (2005); ISBN 91-7155-117-4. Stockholm University, dept. of physics.

53. Chodos A., Detweiler S., “Where did the fifth dimension go?”, *Physics Review D*, (1980); 21, 2167- 2170.

54. Singh G.P., Kotambkar S., and Pradhan A., “Cosmological models with a variable Λ term in higher dimensional space-time”, *Fizika B*, (2006); 15, 23-36.

55. Jain N. I., & Bhoga S. S., “Kaluza-Klein Bulk Viscous Cosmological Model with Time-dependent Gravitational Constant and Cosmological Constant”, *International Journal of Theoretical Physics*, (2015); 54, 2991-3003.

56. Jain N. I., Bhoga S. S., & Khadekar G.S., “Implications of Time-Varying Cosmological constant on Kaluza-Klein Cosmological Model”, *International Journal of Theoretical Physics*, (2013); 53, 4416-4426.

57. Jain N. I., Bhoga S. S., & Khadekar G.S., “Kaluza–Klein Cosmological Model, Strange Quark Matter, and Time-Varying Lambda”, *Z. Naturforsch*, (2014); 69a, 90-96.

58. Jain N. I., Bhoga S. S., and Khadekar G.S, “Kaluza Klein Cosmological Model with String, SQM, and Time-VaryingΛ”, *ARPN Journal of Science and Technology*, (2013); 3(6), 647-653.

59. Milton K.A., “Dark Energy as Evidence for Extra Dimensions”, *Grav. Cosmo.* (2003); 9, 66-70.

60. Rador T., “Extra dimensions, dilaton and dark energy”, *Phys. Lett. B*, (2011);703 (1), 20-24.

61. Demaret J., Hanquin J.-L., “Anisotropic Kaluza-Klein Cosmologies”, *Physics Review D*, (1985), 31, 258-261.

62. Katore S. D., Sancheti and M.M., Sarkate N.K., “Kaluza-Klein Anisotropic Magnetized Dark Energy Cosmological Model in Brans-Dicke Theory of Gravitation”, *Astrophys.* (2014); 57, 384-400.

63. Adhav K.S. et al, “Kaluza-Klein cosmological model with anisotropic dark energy”, *Modern Physics Letter A,* (2011); 26, 739-750.

64. Bean R., and Melchiorri A., “Current constraints on the dark energy equation of state”, *Physics Review D*, (2002); 65, 041302 [arXiv: Astro-ph/0110472/]

65. Biswas T., Koivisto T., and Mazumdar A., “Could our universe begun with –Λ?”, arXiv:1105.2636 [astro-ph.CO] (2011)

66. Calderon R. et al., “Negative cosmological constant in the dark sector?”, *Physics Review D,* (2021);103, 023526, [arXiv:2008.10237v3[astro-ph.CO]