IRSTI 29.27.01, 29.27.31 https://doi.org/10.26577/ijmph.2023.v14.i1.09

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(Received 10 April 2023; accepted 29 May 2023)

Theory of energy loss of charged projectiles in magnetized one-component plasmas

Abstract. The polarizational stopping power of an electron fluid is studied within the quantum random-phase approximation using the canonical solutions of the Hamburger moment problem for the loss function. The loss function is not an even function here of the frequency as in, for example, non-magnetized one-component plasma. Since the loss function is proportional to the inverse longitudinal dielectric function we can deduce that it is a response function possessing consequtive properties. The moments are calculated using RPA longitudinal dielectric function and the asymptotic expansion of the polarization functions. Polarization function is written in terms of generalized Laguerre polynomial, Landau energy level and Fermi-Dirac distribution. The chemical potential in the Fermi-Dirac distribution obtained from the normalization condition. The final expression for the stopping power contains only one integral of the square of the Bessel function of some integer order and only two summations, one of which is a finite sum.
Key words: polarizational stopping power, Hamburger moment problem, loss function, method of moments, magnetized one-component plasmas.

Introduction

A number of authors have reduced the polarizational stopping power of a one-component plasma in a homogeneous constant magnetic field $\mathbf{B}(0,0,B)$ to the imaginary part of the longitudinal (along $\mathbf{k}$) component of its dielectric tensor. If we introduce cylindrical variables along the magnetic induction $\mathbf{B}$ and consider the stopping of a heavy projectile of a charge Ze penetrating the system with a velocity $\mathbf{w}$ so that $(\mathbf{w}\cdot\mathbf{B}) = Bw\cos\vartheta$, we can write for the stopping power the following expression, see [1-5] and references therein:

$$-\frac{dE}{dx} = \frac{2(Ze)^2}{\pi}\sum_{\nu=-\infty}^{\infty}\int_0^{\infty} k_\perp J_\nu^2\left(\frac{k_\perp w\sin\vartheta}{\omega_c}\right) W_\nu(\mathbf{w},\mathbf{k},\mathbf{B})\,dk_\perp\,, \quad (1)$$

$$W_\nu(\mathbf{w},\mathbf{k},\mathbf{B}) = \int_0^{\infty}\frac{k_z w\cos\vartheta+\nu\omega_c}{k_\perp^2+k_z^2}\,\mathrm{Im}\left(\frac{-1}{\varepsilon(\mathbf{k},k_z w\cos\vartheta+\nu\omega_c)}\right)dk_z = \quad (2)$$

$$= \int_0^{\infty}\frac{(k_z w\cos\vartheta+\nu\omega_c)^2}{k_\perp^2+k_z^2} L(\mathbf{k},k_z w\cos\vartheta+\nu\omega_c)\,dk_z\,,$$

where we have introduced the (first kind) Bessel function of integer order ν and the loss function

$$L(\mathbf{k},\omega) = -\operatorname{Im}\varepsilon^{-1}(\mathbf{k},\omega)/\omega \tag{3}$$

with

$$\mathbf{k} = \mathbf{k}_\perp + \mathbf{k}_z \quad \text{and} \quad \omega = k_z w \cos\vartheta + \nu\omega_c$$

which we calculate here in the random-phase approximation (RPA). Notice that

$$J_{-\nu}(y) = (-1)^\nu J_\nu(y), \quad \nu \in \mathbb{Z},$$

and that $k_\perp = \sqrt{k_x^2 + k_y^2}$ and $k_z = \sqrt{\mathbf{k}^2 - k_\perp^2}$ are the wavevector $\mathbf{k}$ components orthogonal and parallel to the vector $\mathbf{B}$; we limit our consideration here to the stopping power of a Fermi electron liquid, so that ω_c is the electronic cyclotron frequency.

The idea of the present work is to apply for the loss function the canonical solution of the corresponding Hamburger moment problem and to simplify the theoretical expressions. The canonical solution we refer to here does not take into account the processes of energy dissipation in the electron fluid, which is consistent with the traditional polarizational (loss-function) approach to the evaluation of the plasma stopping power [6]. We will choose this canonical solution using the power frequency moments of the loss function (3),

$$C_s(\mathbf{k}) = \frac{1}{\pi}\int_{-\infty}^{\infty} \omega^s L(\mathbf{k},\omega)\,d\omega, \quad s = 0,1,2,\ldots \tag{4}$$

They can be obtained from the asymptotic expansion of the (inverse) dielectric function obtained using the Kramers-Kronig relations. Notice that the loss function in the present setting is no longer an even function of the frequency.

Indeed, since the inverse longitudinal dielectric function $\varepsilon^{-1}(\mathbf{k}, z = \omega + i\delta)$ is a response function (it is analytical in the upper half-plane of the complex frequency, $\delta > 0$, and possesses there a positive imaginary part), we have that

$$\begin{aligned}
\varepsilon^{-1}(\mathbf{k},z) &= 1 + \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\operatorname{Im}\varepsilon^{-1} d\omega}{\omega - z} = \\
&= 1 - \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\omega L(\mathbf{k},\omega)\,d\omega}{\omega - z} = \\
&= 1 - C_0(\mathbf{k}) - \frac{z}{\pi}\int_{-\infty}^{\infty} \frac{L(\mathbf{k},\omega)\,d\omega}{\omega - z} = \\
&= 1 - C_0(\mathbf{k}) + \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{L(\mathbf{k},\omega)\,d\omega}{1 - \omega/z}.
\end{aligned} \tag{5}$$

Then, along any ray in the half-plane $\operatorname{Im} z > 0$,

$$\varepsilon^{-1}(\mathbf{k}, z \to \infty) \simeq$$

$$\simeq 1 - C_0(\mathbf{k}) + \frac{1}{\pi}\int \left(1 + \frac{\omega}{z} + \left(\frac{\omega}{z}\right)^2 + \left(\frac{\omega}{z}\right)^3 + \cdots\right) L(\mathbf{k},\omega)\,d\omega = \tag{6}$$

$$= 1 + \frac{C_1(\mathbf{k})}{z} + \frac{C_2(\mathbf{k})}{z^2} + \frac{C_3(\mathbf{k})}{z^3} + \cdots. \tag{7}$$

The value of the zero-order moment stems directly from (5):

$$\begin{aligned}
\varepsilon^{-1}(\mathbf{k},0) &= \lim_{z\downarrow 0}\left(1 - \frac{1}{\pi}\int \frac{\omega L(\mathbf{k},\omega)\,d\omega}{\omega - z}\right) = \\
&= 1 - C_0(\mathbf{k}).
\end{aligned} \tag{8}$$

In the present work we employ for the dielectric function the following.

Quantum random-phase approximation for a magnetized Fermi fluid of charged particles

It was shown by Akhiezer, Eleonsky and others [7, 8] that the RPA longitudinal dielectric function has the following form

$$\varepsilon_{\text{RPA}}\left(\mathbf{k},\omega\right)=1+\frac{4\pi e^2}{k^2}\Pi\left(\mathbf{k},\omega\right), \quad (9)$$

where the polarization function should be calculated using, instead of free waves, the Landau functions [9]. As a result, the following expression is obtained [10, 11]:

$$\Pi\left(\mathbf{k},\omega\right)= =\frac{2}{\left(2\pi\rho\right)^2}\sum_{\nu_1,\nu_2=0}^{\infty}H_{\nu_1,\nu_2}\left(\frac{\rho^2 k_\perp^2}{2}\right)J_{\nu_1,\nu_2}\left(\mathbf{k},\omega;\omega_c\right), \quad (10)$$

$$J_{\nu_1,\nu_2}\left(\mathbf{k},\omega;\omega_c\right)= =\int_{-\infty}^{\infty}\frac{\left[f\left(\nu_1,p+k_z\right)-f\left(\nu_2,p\right)\right]dp}{E\left(\nu_1,p+k_z\right)-E\left(\nu_2,p\right)-\hbar\omega-i0^+}, \quad (11)$$

where

$$\rho^2=\frac{\hbar c}{|e|B}=\frac{\hbar}{m\omega_c},$$

$$H_{\nu_1,\nu_2}\left(\lambda\right)= =\frac{\left(\mu!\right)^2\exp\left(-\lambda\right)}{\nu_1!\,\nu_2!}\lambda^{|\nu_1-\nu_2|}\left[L_\mu^{|\nu_1-\nu_2|}\left(\lambda\right)\right]^2, \quad (12)$$

$\mu=\min\left(\nu_1,\nu_2\right)$ and $L_\mu^\alpha\left(\lambda\right)$ is the generalized Laguerre polynomial, while

$$E\left(\nu,p\right)=\hbar\omega_c\left(\nu+\frac{1}{2}\right)+\frac{\hbar^2 p^2}{2m} \quad (13)$$

is the Landau energy level [9], and

$$f\left(\nu,p\right)=\left\{1+\exp\left[\beta E\left(\nu,p\right)-\eta\right]\right\}^{-1} \quad (14)$$

is, of course, the Fermi-Dirac distribution.

Notice that the dimensionless chemical potential η in the distribution (14) is to be determined from the normalization condition:

$$\frac{2}{\left(2\pi\rho\right)^2}\sum_{\nu=0}^{\infty}\int_{-\infty}^{\infty}f\left(\nu,p\right)dp=n. \quad (15)$$

The integral in (15) can be conveniently expressed in terms of the Fermi integral

$$I_\gamma\left(\eta\right)=\int_0^\infty\frac{x^{\gamma-1}dx}{\exp\left(x-\eta\right)+1}, \quad \gamma=\frac{1}{2},\frac{3}{2},\cdots \quad (16)$$

with $\gamma=1/2$:

$$\sum_{\nu=0}^{\infty}I_{1/2}\left(\tilde{\eta}_\nu\right)=\pi^2\rho^2 bn, \quad (17)$$

where

$$\tilde{\eta}_\nu=\eta-u\left(\nu+\frac{1}{2}\right), \quad b=\sqrt{\frac{\beta\hbar^2}{2m}}.$$

To begin with, let us calculate the static dielectric function and the zero-order moment (8):

$$C_0\left(\mathbf{k}\right)=1-\varepsilon_{\text{RPA}}^{-1}\left(\mathbf{k},0\right)=\frac{\phi\left(k\right)\Pi\left(\mathbf{k},0\right)}{1+\phi\left(k\right)\Pi\left(\mathbf{k},0\right)},$$

where

$$\Pi\left(\mathbf{k},0\right)= =\frac{2}{\left(2\pi\rho\right)^2}\sum_{\nu_1,\nu_2=0}^{\infty}H_{\nu_1,\nu_2}\left(\frac{\rho^2 k_\perp^2}{2}\right)J_{\nu_1,\nu_2}\left(\mathbf{k},0;\omega_c\right),$$

$$J_{\nu_1,\nu_2}\left(\mathbf{k},0;\omega_c\right)= =\frac{m}{\hbar^2 k_z}\mathcal{P}\int_{-\infty}^{\infty}\frac{\left[f\left(\nu_1,p+k_z\right)-f\left(\nu_2,p\right)\right]dp}{p-P_0};$$

here $\mathcal{P}$ stands for the principal value of the integral,

$$P_0\left(k_z;\nu_1,\nu_2\right)=-\frac{k_z}{2}-\frac{m\omega_c}{\hbar k_z}\left(\nu_1-\nu_2\right),$$

$$\phi\left(k\right)=4\pi e^2/k^2.$$

Then, let us consider the asymptotic expansion of the polarization function (10),

$$\Pi\left(\mathbf{k},z\to\infty\right)\simeq iM\left(\mathbf{k},z\right)- -\frac{M_0\left(\mathbf{k}\right)}{z}+\frac{M_1\left(\mathbf{k}\right)}{z^2}+\cdots, \quad (18)$$

where we keep the imaginary part of the polarization function,

$$M(\mathbf{k},z)=\frac{2}{(2\pi\rho)^2}\frac{\pi m}{\hbar^2 k_z}\sum_{\nu_1,\nu_2=0}^{\infty}H_{\nu_1,\nu_2}\left(\frac{\rho^2 k_\perp^2}{2}\right)\left[f(\nu_1,P+k_z)-f(\nu_2,P)\right],$$

though its asymptotic expansion vanishes, but its presence guarantees the existence of other characteristics, see below;

$$P(k_z,z;\nu_1,\nu_2)=P_0(k_z;\nu_1,\nu_2)+\frac{mz}{\hbar k_z}.$$

Due to (9) and by comparing the expansions (7) and (18) we obtain that

$$C_1(\mathbf{k})=\phi(k)M_0(\mathbf{k}),$$
$$C_2(\mathbf{k})=\phi(k)\left(M_1(\mathbf{k})+\phi M_0^2(\mathbf{k})\right),\ldots. \quad (19)$$

The moments

In order to find the moments (19), let us use the above definitions and let us study the asymptotic form of the auxiliary function

$$J_{\nu_1,\nu_2}(\mathbf{k},z;\omega_c)=$$
$$=\int_{-\infty}^{\infty}\frac{\left[f(\nu_1,p+k_z)-f(\nu_2,p)\right]dp}{E(\nu_1,p+k_z)-E(\nu_2,p)-\hbar z}, \quad (20)$$

as $z\to\infty$ in the half-plane $\operatorname{Im} z>0$:

$$J_{\nu_1,\nu_2}(\mathbf{k},z;\omega_c)=\int_{-\infty}^{\infty}\frac{\left[f(\nu_1,p+k_z)-f(\nu_2,p)\right]dp}{E(\nu_1,p+k_z)-E(\nu_2,p)-\hbar z-i0^+}\underset{z\to\infty}{\simeq}$$
$$\underset{z\to\infty}{\simeq}-\frac{1}{\hbar z}\int_{-\infty}^{\infty}\left(1+\frac{\Omega}{z}+\frac{\Omega^2}{z^2}+\ldots\right)\left[f(\nu_1,p+k_z)-f(\nu_2,p)\right]dp=-\frac{1}{\hbar z}\left(\mu_0+\frac{\mu_1}{z}+\ldots\right).$$

The moments $\left\{\mu_j(k_z;\nu_1,\nu_2;\omega_c)\right\}_{j=0,1,2,\ldots}$ can be easily expressed in terms of the Fermi integrals (16):

$$\mu_0(k_z;\nu_1,\nu_2;\omega_c)=\frac{1}{b}\left[I_{1/2}(\tilde\eta_{\nu_1})-I_{1/2}(\tilde\eta_{\nu_2})\right],$$

$$\mu_1(k_z;\nu_1,\nu_2;\omega_c)=-\left(\frac{\hbar k_z^2}{mb}\right)I_{1/2}(\tilde\eta_{\nu_1})+$$
$$+\left(\frac{\hbar k_z^2}{2m}+\omega_c(\nu_1-\nu_2)\right)\frac{1}{b}\left[I_{1/2}(\tilde\eta_{\nu_1})-I_{1/2}(\tilde\eta_{\nu 2})\right].$$

We use here the following notation:

$$\Omega=\frac{\hbar k_z p}{m}+\frac{\hbar k_z^2}{2m}+\omega_c(\nu_1-\nu_2),$$
$$\mu_j(k_z;\nu_1,\nu_2;\omega_c)=$$
$$=\int_{-\infty}^{\infty}\Omega^j\left[f(\nu_1,p+k_z)-f(\nu_2,p)\right]dp. \quad (21)$$

Now we are in a position to calculate the moments

$$M_j(\mathbf{k})=$$
$$=\frac{2}{(2\pi\rho)^2\hbar}\sum_{\nu_1,\nu_2=0}^{\infty}H_{\nu_1,\nu_2}\left(\frac{\rho^2k_\perp^2}{2}\right)\mu_j(k_z;\nu_1,\nu_2;\omega_c), \quad (22)$$
$$j=0,1,2$$

and the moments (19). Then we can follow the lines of [12].

The canonical solution approximation

As it was said in the Introduction, let us employ for the loss function (3) a simplified three-moment canonical solution [13]:

$$L(\mathbf{k},\omega)=\sum_{j=0}^{2}m_j(\mathbf{k})\delta\left(\omega-\Omega_j(\mathbf{k})\right) \quad (23)$$

with three characteristic frequencies

$$\Omega_0(\mathbf{k})=0,\quad \Omega_1(\mathbf{k})=\frac{C_1(\mathbf{k})}{C_0(\mathbf{k})},\quad \Omega_2(\mathbf{k})=\frac{C_2(\mathbf{k})}{C_1(\mathbf{k})}.$$

It is important that the stopping power is proportional to the imaginary part of the polarization operator, $M(\mathbf{k})$, which depends exponentially on the frequency substituted by $k_z w\cos\vartheta + \nu\omega_c$. Hence, the frequencies Ω_j can be determined explicitly as it is pointed out above.

The weights m_j in (23) can be found from the moment conditions (4):

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & \Omega_1 & \Omega_2 \\ 0 & \Omega_1^2 & \Omega_2^2 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} C_0 \\ C_0\Omega_1 \\ C_0\Omega_1\Omega_2 \end{bmatrix} \tag{24}$$

and, hence,

$$\begin{bmatrix} m_0 \\ m_1 \\ m_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & \Omega_1 & \Omega_2 \\ 0 & \Omega_1^2 & \Omega_2^2 \end{bmatrix}^{-1} \begin{bmatrix} C_0 \\ C_0\Omega_1 \\ C_0\Omega_1\Omega_2 \end{bmatrix} = = C_0 \begin{bmatrix} 1-\dfrac{\Omega_1}{\Omega_2} \\ 0 \\ \dfrac{\Omega_1}{\Omega_2} \end{bmatrix}. \tag{25}$$

In this case, the main factor in the stopping power simplifies significantly:

$$W_\nu(w,\mathbf{k},\mathbf{B}) = \int_0^\infty \frac{\left(k_z w\cos\vartheta+\nu\omega_c\right)^2}{k_\perp^2+k_z^2} L\left(\mathbf{k}, k_z w\cos\vartheta+\nu\omega_c\right) dk_z =$$

$$= \sum_{j=0}^{2}\int_0^\infty \frac{\left(k_z w\cos\vartheta+\nu\omega_c\right)^2}{k_\perp^2+k_z^2} m_j\left(k_z,k_\perp\right)\delta\left(k_z w\cos\vartheta+\nu\omega_c-\Omega_j\right) dk_z =$$

$$= \int_0^\infty \frac{\Omega_2^2(\mathbf{k})}{k_\perp^2+k_z^2} m_2\left(k_z,k_\perp\right)\delta\left(k_z w\cos\vartheta+\nu\omega_c-\Omega_2\right) dk_z .$$

Notice that for $j=0$ $\left(k_z w\cos\vartheta+\nu\omega_c\right)=\Omega_0=0$ and $m_1\left(k_z,k_\perp\right)=0$ as well. Consider then the function

$$G\left(k_z\right) = k_z w\cos\vartheta+\nu\omega_c-\Omega_2\left(k_z\right).$$

Let the equation $G\left(k_z\right)=0$ have ℓ simple zeros [14] $\hat{k}_z = p_\ell\left(\nu,k_\perp\right)$ so that the derivative with respect to k_z, $G'\left(p_\ell\left(\nu,k_\perp\right)\right)$ does not vanish, $G'\left(p_\ell\left(\nu,k_\perp\right)\right)\neq 0$, for all these zeros. Then

$$W_\nu(w,\mathbf{k},\mathbf{B}) = = \sum_\ell \frac{\Omega_2^2\left(p_\ell\left(\nu,k_\perp\right),k_\perp\right) m_2\left(p_\ell\left(\nu,k_\perp\right),k_\perp\right)}{\left(k_\perp^2+p_\ell^2\left(\nu,k_\perp\right)\right)\left|G'\left(p_\ell\left(\nu,k_\perp\right)\right)\right|},$$

depends only on the orthogonal component of the wavevector. Thus, we reduce the calculation of the stopping power to the following relatively simple expression

$$-\frac{dE}{dx} = \frac{2(Ze)^2}{\pi}\sum_{\nu=-\infty}^{\infty}\sum_\ell \int_0^\infty k_\perp J_\nu^2\left(\frac{k_\perp w\sin\vartheta}{\omega_c}\right) N_{\nu,\ell}\left(k_\perp\right) dk_\perp, \tag{26}$$

$$N_{\nu,\ell}\left(k_\perp\right) = \frac{\Omega_2^2\left(p_\ell\left(\nu,k_\perp\right),k_\perp\right) m_2\left(p_\ell\left(\nu,k_\perp\right),k_\perp\right)}{\left[k_\perp^2+p_\ell^2\left(\nu,k_\perp\right)\right]\left|G'\left(p_\ell\left(\nu,k_\perp\right)\right)\right|}, \tag{27}$$

which contains only one integral of the square of the (first kind) Bessel function of integer order ν and only two summations, one of which is a finite sum.

Conclusion

It is obvious that the present work is only a first step to the solution of the complicated problem of evaluation of the stopping power of collisional plasmas with significant correlational effects, both related to the Coulomb and exchange interactions. We have at least deduced the difficulty of the calculation of the stopping power to the level characteristic for the classical plasmas [2, 5]. The next step will consist in the employment of the non-canonical solutions of the loss-function Hamburger moment problem [15-19], which is now straightforward since the lower-order moments are now calculated. This problem along with the determination of the slow- and fast-projectile [20-23] and other asymptotic forms of the stopping power are beyond the scope of the present short publication.

Acknowledgments

The work was carried out with the support of the grant of the Ministry of Education and Science of the Republic of Kazakhstan AP09260349.

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