UDC 539.171

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Peculiarities of nuclear and photonuclear reactions in crystalline structure of neutron star crusts

Abstract. Nuclear reactions in three-body systems consisted of neutron and two heavy nuclei and the appearance of the new type of few-body resonances in crystalline structures in lower layers of outer crust and upper layers of inner crust in neutron star envelopes have been considered.
The peculiarities of nuclear reactions in this area owe to small distances between the nuclei that are much smaller than the Bohr radius. In this region there are possible new types of reactions, such as the resonances in few-body systems, which are impossible in conventional terrestrial conditions.
The simple model of three-body system consisted of one light particle and two heavy particles interacting with separable pair potentials has been investigated and gives the solutions in analytical forms. Within the model, new quasi-stationary states and unusual transitions have been found that are exotic for the two-body problems.
An important feature of new resonances is that they occur under certain distances between the heavy particles. In the problem of neutron scattering on the subsystem of two heavy nuclei, the resonant amplitude has the complicated dependence on the neutron-nucleus resonance parameters and the distance between the nuclei.
Keywords: Nuclear reactions, photonuclear reactions, effective interactions, neutron resonances, few-body systems, nuclear astrophysics, neutron star crusts, electron capture by nucleus, Born-Oppenheimer approximation.

Introduction

Our Universe contains many strange and enigmatic objects. One of them is a neutron star whose substance is composed mainly of neutrons. Neutron star is formed from a giant star when its central part ceases to radiate energy. Then gravity compresses the star forming a super dense matter in the central part, and reactions of nuclear fusion amplified many times lead to huge release of energy in the outer regions of the star. This results in a huge explosion called supernova explosion, which gives birth to neutron star – a super-dense remnant of the central part of a giant star, and nebula - matter of the outer regions of a giant star dissipated in space.

Neutron stars and processes taking place in them are of considerable interest to researchers because they as well as white dwarfs and black holes are classified as compact stellar objects and are considered to be the final stage of star life. The radiating neutron stars are called pulsars. They can emit in different wavebands, ranging from radio waves to X-rays and gamma radiation. The physical cause of the radiation is still far from clear understanding [1].

Neutron stars are balanced by equality between gravity forces – the compression factor, and the pressure of a degenerate gas in the stars interior – the expansion factor. Stable neutron star has 1÷3 of solar mass ($M_\odot$) and a radius of about 10 km. A neutron star consists of several areas: the atmosphere about few centimeters; the crystalline crusts of iron atoms of several kilometers; the intermediate region of neutrons, the central core of the heavy particles. There is also a magnetosphere that can accelerate particles. The size of neutron star must be hundred times smaller than the white dwarf [1-3].

In this paper, we investigate the reactions and processes that occur in crystalline crusts of the neutron star. These reactions and processes play an important and perhaps catalytic role in generation of the radiation from neutron star. In the considered density region different nuclear and

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photonuclear reactions may take place, with neutrino participation, electron capture, etc. Many of these reactions are well known from laboratory experiments, and others can be assessed by following the fundamental laws of quantum theory, the theory of nuclear and photonuclear reactions, and the electron Fermi liquid, the state of matter under extreme conditions and the quantum theory of few-body systems.

The following section provides a brief overview of neutron star properties and types of radiation, highlighting some open problems and current problems associated with neutron stars. Then, in Section 2 the two-body and few-body interactions in neutron star crusts are described in frame of theory of few-body quantum mechanics systems.

In Section 3 the solutions of three-body problem and simplified model are given, based on the Born-Oppenheimer approximation and separable form of neutron-nucleus interactions.

In Section 4, we present an analysis of structural neutron resonances. Section 5 gives the results of calculations of neutron structural resonances and resonant radiation in overdense crystal structures with the nuclei of iron group.

In conclusion, we discuss the main findings related to the resonance processes occurring in the neutron star crusts.

The radiation and the main properties of neutron stars

The surface temperature of the neutron star can reach several million degrees, but because of the small area of radiating surface neutron stars are very weak in the visible light. Pulsar is a neutron star, with a near-perfect frequency emitting pulses in the radio. Radiation is in the form of two relatively narrow beams from the magnetic poles, and we can fix this radiation only if we are on the path of one of these beams. Because of partial mismatch of magnetic and rotational poles the neutron star light beam may be observed as periodically appearing pulses. To radiate, the neutron star must have a powerful magnetic field and rotate rapidly. Radio pulsars are different as normal and millisecond pulsars [1-4].

Ordinary pulsars are characterized by relatively large period of rotation, and the most famous of these is the Crab Nebula pulsar. It originated in the supernova explosion in 1054, and currently has a rotation period of 33 ms. At this explosion the fibrous nebula occurred with the expansion rate agrees well with the time of the supernova explosion.

Pulsation has a large constant, although the energy waste by radiation uniformly slows down the pulsar rotation. For example, the oscillation period of the Crab pulsar annually grows by 13.5 ms. For every 100 years the orbital period increases by 1.3 ms and initially it was 20 ms. But sometimes a pulsar rotation accelerates dramatically: from gradual deceleration its flatness decreases, and this leads to star quakes, after what the star slightly compresses and begins to rotate faster [2-4].

Binary radio pulsars are the same radio pulsars, but are part of a binary system. By the beginning of the 21st century only two such objects were known. They are interesting for they provide an opportunity to study them in detail: measure the mass of neutron stars, observe their precession, check subtle effects predicted by general relativity (for example, the emission of gravitational waves). The rotation speed of the pulsars is related to their age, which suggests the young and old pulsars.

Millisecond pulsars are objects with a particularly high rate of rotation. They are contrasted to ordinary pulsars. They always are members of close binary systems. The magnetic field of millisecond pulsars is much weaker than that of other pulsars, so they lose rotational energy slowly, and time of their possible life is comparable with the age of the observable universe [3, 4].

X-ray pulsars are members of close binary systems, in which the second companion is a normal star. Substance from this normal star constantly flows to the neutron star, which is accelerated in a strong gravitational field, and after hitting the surface of neutron star glows in X-rays. Substance falls in a spiral and, in addition, because of the strong magnetic field of the neutron star, settles on the surface only in the vicinity of the magnetic poles. This loss is not uniform and leads to the observed X-ray pulsations.

Radio quiet neutron stars are the opposite of radio pulsars. Pulsed radiation cannot appear because of initially slow rotation of the star, as in this case there is no strong magnetic field. For the same reason, there are no pulsations of the old pulsar that has "slow down" too much. Such pulsars are called extinct, and there should be about a thousand times more of them than of the "working" pulsars [3].

Since the gravitational energy is an essential part of the rest energy of star, neutron stars are relativistic objects; space-time is curved substantially within them and near the surface. Therefore, a neutron star can be seen as a unique astrophysical laboratory of overdense matter.

Neutron stars are the sources of electromagnetic radiation at all wavelengths from radio to hard gamma radiation. Neutron stars are also powerful high-energy particle accelerators. Their birth in supernova explosions is accompanied by a powerful neutrino pulse. Moreover, merging neutron stars are considered as the most promising targets for the detection of gravitational radiation.

Thus, neutron stars are the subject of intense theoretical and observational studies. Main mystery of neutron stars is the equation of state of dense matter in their cores. This equation cannot be derived from first principles because of the lack of precise theory of nuclear interactions and many-body theory to describe the collective effects in the overdense matter. In the absence of the theory, many models of equation of states have been built [1-3]. As a result, there is a wide variety of equations of state from mild to moderate to severe, leading to large differences in the expected structure of neutron stars. So many attempts are undertaken to obtain restrictions on the equation of state of neutron stars for comparisons between theory and observations.

Usually use measurements of mass in close binary systems containing a neutron star are used, or measurements of minimum rotation periods of millisecond pulsars. As well the determination of the orbital frequency is used, which corresponds to a neutron star near a stable orbit, the frequency measurements of kilohertz quasi-periodic oscillations, and other methods.

Another example of the uncertainty of the results obtained in the model of microscopic calculations is the nucleon super fluidity in the crusts and cores of neutron stars. The very existence of superfluidity in neutron stars is strongly dependent on the model of the nucleon-nucleon interactions and the model used to describe the many-body effects [4].

Despite the high density of the neutron stars, they are completely transparent to neutrinos in about 20 seconds after birth. Neutrinos are produced in many reactions, freely emitted from the neutron star, providing a powerful channel for cooling. To model the thermal evolution of neutron stars requires detailed knowledge of the processes of neutrino emission in the different reactions. Recently, however, there was a need to systematize the mechanisms of neutrino emission of neutron stars, the revision of a number of mechanisms studied with insufficient completeness and consideration of new mechanisms [3,4].

On the other hand, in the last decade, there is a rapid accumulation of observational data on the thermal radiation from the surface of isolated neutron stars. Beginning with the discovery of thermal radiation from a number of neutron stars by orbiting X-ray observatory EXOSAT and ROSAT, these studies are continuing and expanding rapidly at the moment. Modern X-ray space station ASCA, RXTE, XMM-Newton, Chandra are conducting systematic search and observations of isolated neutron stars. A large number of observational data obtained in the near infrared, optical and ultraviolet ranges, also comes from the largest terrestrial (BTA, ESO-NIT, Keck, VLT, SUBARU, etc.) and orbit (HST) optical telescopes. The comparison of the results of new multiwavelength observations of neutron stars with the current theoretical models of the thermal evolution gives the possibility to study the internal structure of neutron stars.

This work is carried out intensively by many theoretical groups worldwide. Therefore a significant progress in the study of the physical properties of the neutron stars can be expected in the coming years.

In this paper, we turn to the analysis of processes and nuclear reactions that occur in neutron star crusts. This is the area, where the substance is compressed so strongly that the atoms lose their electrons and become bare nuclei. The distances between the nuclei are much smaller than the Bohr radius, but remain still many times larger than the size of nuclei. Electrons form a degenerate electron Fermi gas, in which the bare nuclei are “frozen”. Together, they create a tight, nearly perfect crystalline structure, which retains overall electrical neutrality of matter and opposes gravitational contraction. At the crusts of neutron star the nuclei are identical to nuclei on Earth, so to describe them, we can use the available experimental data. But to describe the electronic components we have to use quantum mechanical model of strongly degenerate Fermi gases and the theory of the Fermi electrons, i.e. theory of elementary excitations in the electron Fermi liquid [1,2].

As for the nuclear reactions and nuclear transformations, new types of reactions, which are impossible in conventional terrestrial conditions, become possible in this region. In addition, here the reactions and phenomena such as neutron rich nuclei, the resonances in few-body systems become significant, while in normal circumstances their formation is not possible.

New types of reactions and phenomena can significantly affect the speed and power of the processes occurring in the neutron star crusts. This leads to an understanding of some interesting effects, such as the explanation of the oscillation and microstructure in pulsar impulses.

Two-body and few-body interactions in neutron star crusts

It is known that with increasing pressure, the substance is acquiring more universal properties as well as more ordered structure by increasing the internal energy. The pressure accessible in the laboratory experiments mainly supports this pattern. But properties of overdense matter are not yet sufficiently studied. This is one of the important problems of modern astrophysics [1-4].

Usually, the dynamic picture of changes in a matter state is considered from the pressure and temperature on the Earth to extreme conditions in the overdense stars, particularly, in neutron stars. Results of widespread experimental investigations and observation data of planets and stars are also taken into account. Then by theoretical considerations researchers are trying to propose a scenario of matter state evolution under extreme conditions.

In the pictures, as a rule, the analysis is based on the effective interactions between particles of matter, considered within the framework of two body quantum mechanics. These effective interactions result from the consideration of different particle interactions: electrons, protons and neutrons, ions of various sorts, etc. In this consideration the important results already are obtained, for example, from comprehensible phenomena of ionization and the destruction of the outer electron shells, up to the appearance of the neutron matter component, the formation of paste, voids and other structural varieties in neutron stars. At densities more than the nuclear density, the formation of strange and quark stars are predicted and then the formation of black holes [4].

We consider the region of densities where the distances between nuclear particles and ions are much smaller than the atom size, but still much larger than nuclear radius. The fact is that the evolution of the matter state in this region (from $10^{-12}\,m$ up to $10^{-14}\,m$) cannot proceed as smoothly as it is now considered.

The two-body interactions are undoubtedly important and essential at usual pressures and temperatures, and applicable even more broadly, but may not be sufficient for extremely high densities of matter. The reason is the resonance effects in few-body physics, which previously were not included in consideration in stellar dynamics. It was also believed that the few-body dynamics could only give a minor contribution into the process.

The three-body solutions were obtained in analytical form in the framework of one particular model [5]. It turned out that the solutions could have a resonance behavior [5-7], and in certain circumstances became governing contributions. The model has a sufficiently broad perspective of applications, particularly, in the problem of overdense stars, including neutron stars.

Let's adduce here some non-trivial effects which arise even in simple three-body systems and are well known in the few body quantum physics. First, it is the Thomas's effect [8] – the collapse into a center in the system of three identical particles whose pair interactions have a zero range (pointlike interactions). Furthermore, if the range of forces is finite, but the length of pair scattering is increasing, a lot of new levels appear in such system under the scattering threshold. And the growth of their number increases logarithmically as the ratio of the pair scattering length to the range of pair forces. This is the Efimov effect [9]. Lately, experiments were performed, which confirmed the existence of this effect [10]. Note also the investigation of threshold anomalies in cross sections and phenomena of a long-range character in three body systems [11,12].

These results indicate a complex picture of the phenomena and effects arising from quantum physics of few body systems. The impact of these phenomena on the state of overdense matter may prove decisive in some cases, but they cannot be considered in the framework of simple two-body interactions.

Let us turn to the model of three-body system where the mass of one of the particles is extremely small in relation to the masses of the other two particles and where the pair potentials of interaction between the particles involved are separable. In this model solutions of three-body system are determined in analytical forms, and their analysis is clear enough. Within the model, new quasi-stationary states and unusual transitions have been found that are exotic for the two-body scattering problems [5].

An important feature of new quasi-stationary states or resonances is that they occur under certain (almost discrete) values of the distances between the heavy particles. In the problem of neutron scattering on the subsystem of two heavy nuclei, the resonant amplification of amplitudes has the complicated dependence on the distance between the nuclei.

We call these resonances as "the structural neutron resonances" to mention explicitly their dependence on the target structure consisted of two and more nuclei, and to distinguish from ordinary neutron resonances.

It should be noted that the resonance enhancement of the scattering amplitude of neutrons on subsystem of few nuclei takes place not only in the elastic channel, but in the inelastic reactions too, particularly, in the radiation neutron capture [7].

The quantum three-body problem and simplified model

The basis of the three-body quantum theory is the system of Faddeev equations which give for the amplitudes corresponding to transitions between different channels of pair states [13,14]:

$$T_{ij} = t_i\,\delta_{ij} + \sum_k t_i\,G_0\,\bar{\delta}_{ik}\,T_{kj} \quad , \quad i,j,k = 1,2,3 \quad (1)$$

where t_i - matrices are given as known quantities, $t_i = V_i + V_i G_0 t_i$ are the pair t_i −matrices associated with pair interaction potentials V_i and determined in the space of three particles, $G_0(E)$ - is the Green's function for three free particles, $\bar{\delta}_{ij} = I - \delta_{ij}$, δ_{ij} is the Kronecker delta. The total T -matrix is the sum over the indices *i* and *j*, $T = \sum' T_{ij}$.

Formally, the index *i* on the element T_{ij} corresponds to the number of the last surviving pair in the left asymptotic region - that is, it corresponds to the number of the particle that leaves the interaction region first. Similarly, the index *j* corresponds to the number of the last interacting pair in the right asymptotic region. The Faddeev equations (1) guarantee uniqueness and existence of solutions [13].

Our model is based on two simple approximations:

- two body t-matrices are taken in the form characterizing the "compound" systems;
- in three-body problem the Born-Oppenheimer approximation is used.

The first approximation gives the exact solutions of the two-particle problem at once in an analytical form. This form is used in many problems of low-energy nuclear physics, for instance, for amplitude near the isolated Breit-Wigner resonance and even as solutions in the case of separable interactions like the Yamaguchi potential.

The Born-Oppenheimer approximation results in the simplification of the three-particle problem in the assumption that one particle is light, while the others are heavy. In particular, the Born-Oppenheimer approximation greatly simplifies the determination of solutions by dividing them into two sets of equations: "electronic" equations for the light particle and "nuclear" equations for heavy particles. The terms borrowed from quantum molecular physics, where light and heavy particles are electrons, and nuclei of atoms, respectively [15].

The first approximation immediately sets the exact analytical form for the pair amplitude, while the second one leads to a simplification of the three-body equations (1) and allows now finding the analytical form of solutions for the three-body problem.

For example, in the case of Yamaguchi potentials: $V_i = |\nu_i > \lambda_i < \nu_i|$, $\lambda_i -$ the coupling constant of the potential, solutions can be written in the simple form:

$$t_i = |\nu_i > \eta_i < \nu_i| \ , \qquad (2)$$

where the enhancement factor equals:

$$\eta_i^{-1} = \lambda_i^{-1} - A_i(E) \ , \ \text{with}$$
$$A_i = <\nu_i\,|\,G_0(E)\,|\,\nu_i> \ . \qquad (3)$$

Note that the sum of a finite number of separable terms can describe the short-range potentials of complicated forms. The solutions in these cases can be also written in an analytical form.

In the case of the Born-Oppenheimer approximation, when $m/M \to 0$, where m and M are masses of light particle and heavy particles, respectively (heavy particles are assumed here identical, for simplicity), the "electronic" equations become the main equations in problem. The total energy of the system becomes equal to the energy of the light particle, for instance, the initial energy of system will be $E = p_0^2/2m$, where $\vec{p}_0$ is initial impulse of the light particle (for simplicity, $c = \hbar = 1$ are employed). Moreover, the form factors for the pair potentials between the light particle and any of the heavy particles cease to depend on the heavy-particle momentum, that is $\nu_i(\vec{q}) \to \nu_i(\vec{p})$, where $\vec{p}$ is impulse of the light particle in an intermediate state. The enhancement factors in the *t*- matrices in eq. (2) for these pairs become functions only of the initial energy of the light particle, that is, they are functions of its initial momentum, namely $\eta_i(E) \to \eta_i(p_0),\, i = 2,3$. The heavy particles are labeled with indices 2 and 3, and then we have $m_2 = m_3 = M$.

Under these two approximations, exact solutions can be easily obtained in a simple analytical form. It means that the problem can be solved completely, as they say “the problem can be solved to the end”. It is even easier to find the exact solutions in the case of the scattering of the light particle on the system of two fixed centers.

By means of analytic solutions we can demonstrate how the new resonances appear in these three-body systems. It is remarkable that energies and widths of the new three-body resonant states depend on distance between two fixed centers.

The “electronic” equations come to the following form ($i, j \neq 1$)

$$M_{ij} = \Lambda_{ij} + \sum_{k=2,3}{}' \Lambda_{ik} M_{kj} ,$$

$$\Lambda_{ij} = < \nu_i \,|\, G_0(E) \,|\, \nu_j > \bar{\delta}_{ij} . \qquad (4)$$

The amplitudes M_{ij} correspond to the effective interactions between two fixed centers created by the neutron rescattering on them. In case of the subsystem of two heavy particles a direct interaction will supplement with the effective interaction [5]. For example, the direct interactions can be the ordinary Coulomb repulsive forces between heavy nuclei.

In order to obtain solutions of eq. (4) it is better to make the Fourier transform in coordinate space and obtain

$$M_{lk}(\vec{r}, \vec{r}\,') = J_{lk}(r\,; p_0)\delta(\vec{r} + \vec{r}\,') + \sum_{\rho} J_{l\rho}(r\,; p_0)\eta(p_0) M_{\rho k}(-\vec{r}, \vec{r}\,') \qquad (5)$$

where $M_{ij}(\vec{r}, \vec{r}\,')$ - the Fourier representative of the three-body amplitudes in eq. (4) with the initial pair i and final pair j. The element of matrix J_{ij} is the Fourier image of the Born interaction in three body system:

$$J_{ij}(\vec{r}; p_0) = 2m \int d\vec{p} \exp(i\vec{p}\vec{r}) \frac{\nu_i(\vec{p}) \cdot \nu_j(\vec{p})}{(p_0^2 - p^2 + i\gamma)}, \qquad i \neq j , \qquad (6)$$

and here $\vec{p}_1 + \vec{p}_2 + \vec{p}'_3 = 0$, where $\vec{p}_1 = \vec{p}$ is the impulse of the light particle. Then $M_{ij}(\vec{r}, \vec{r}\,')$ can be represented as

$$M_{ij}(\vec{r}, \vec{r}\,') = M_{ij}^{+}(\vec{r}\,)\delta(\vec{r} + \vec{r}\,') + M_{ij}^{-}(\vec{r}\,)\,\delta(-\vec{r} + \vec{r}\,') . \qquad (7)$$

Since delta-functions remove the integration on the right-hand side in eq. (5), the equations for $M_{ij}^{\pm}(\vec{r}\,)$ are reduced to an extremely simple form, and the solutions of the problem of the light-particle scattering on two fixed centers can be written in the analytical form

$$M_{ij}^{+}(\vec{r}) = \frac{1}{D_{ii}(\vec{r}; p_0)} J_{ij}(\vec{r}; p_0), \;\; i \neq j \qquad (8)$$

and

$$M_{ii}^{-}(\vec{r}) = \frac{1}{D_{ii}(\vec{r}; p_0)} J_{ik}(\vec{r}; p_0)\eta_k(p_0) J_{ki}(-\vec{r}; p_0), \quad i = j . \qquad (9)$$

Matrix D in the indices of particle pairs is diagonal. Its elements are $D_{ij}(\vec{r}; p_0) = 0$, if $j \neq i$, and $D_{ii} = 1 - J_{ik}(\vec{r})\eta_k J_{ki}(-\vec{r})\eta_i$, if $j = i$.

It is important that there are new options for the position of initial $\vec{r}$ and final $\vec{r}\,'$ of scatterer centers [7]. The appearance of additional parameters is expected, since the description of three-body system requires not only the initial energy of the light particle (we will assume that it is a neutron) but also the location of heavy particles (nuclei). In fact, it suffices to know the distance between heavy particles: $\vec{d} = \vec{r} - \vec{r}\,'$.

To write the final solution of the problem of neutron scattering on the scattering centers the amplitudes of the “electronic equation” can be written in the form:

$$V_{ij}^{ef} = |\, \nu_i > \eta_i M_{ij} \eta_j < \nu_j \,| , \qquad (10)$$

and sandwich the expression in eq. (10) between the wave functions for the initial and final states of the system. The wave functions are the product of the wave functions of the two fixed heavy nuclei and free the neutron. After sandwiching the $\delta-$ functions in eq. (7) are removed, and $\vec{r}$ and $\vec{r}\,'$ will determine the coordinates of the scattering centers. Furthermore, the expression of eq. (7) automatically picks out the origin of the system, where the origin must be in the center of symmetry between the heavy particles. Thus, the scattering amplitude takes the parametric dependence on $d-$ the distance between the scattering centers.

If the solutions of the "electronic equation" are known this means the effective potential for the "nuclear equation" is determined. Then the solutions of the "nuclear equation" are not difficult [5,15].

In our case the effective potential is given by eq. (10). This potential is the complicated function of the initial neutron energy, and also depends on the distance between the nuclei.

From eqs. (8) and (9), it is seen that the amplitudes have pole singularities at the same values of r and p_0 when the determinant of the matrix D vanishes. The dependence of D on the initial energy of the light particle is contained explicitly in the enhancement factors $\eta_i(p_0)$ and rescattering form-factors $J_{ij}(\vec{r}, p_0)$. In the case of the neutron scattering in the crystal the heavy nuclei are fixed in the lattice nodes at $r = d/2$, where d is the lattice constant.

We consider the problem of neutron scattering on two fixed centers subsystem if the two-body scattering amplitudes have the resonant form:

$$t_i = \ |\nu_i> \frac{E_i^R}{E - E_i^R + i\Gamma_i/2} <\nu_i|. \qquad (11)$$

The energy and width of resonance are determined with real and imaginary parts of resonance wave number: $E_i^R = (p_R^2 - p_I^2)/2m$, $\Gamma_i = -4p_R p_I/2m$. We can assume that the form-factor ν_i is almost a constant value in a sufficiently wide range around the resonance point $E \approx E_i^R$. For illustration we consider t_i-matrices in S-wave with the following form-factors and enhancement factors:

$$\nu_i^2(E) = \frac{\pi}{pm}\frac{\Gamma_i}{E_i^R}$$

and

$$\eta_i = \frac{E_i^R}{E - E_i^R + i\Gamma_i/2}, \qquad (12)$$

which correspond with the Breit-Wigner form.

It is important to note that the main contribution to the three body resonances is given by the enhancement factor - η_i. And rescattering factors - J_{ij} together with η_i create the group of structural neutron resonances in the energy region near the energy of the pair neutron resonance.

The structural neutron resonances.

Indeed, the function J_{ij}, following eq. (6), can be expressed in a very simple form:

$$J_{ij} = J_{ji} = J(r) = -\frac{\Gamma}{2}\cdot\frac{\exp(ip_0 r)}{p_R r}, \qquad (13)$$

where $\Gamma_i = \Gamma_j = \Gamma$ and $E_i^R = E_j^R = E_R$, because heavy nuclei are considered identical here.

With simplifications of eqs. (11) and (12), the zeros of the determinant of the matrix D are defined by the simple relationship $(J\eta)^2 = 1$ which results in the following equation:

$$\left(\mathrm{E} - \mathrm{E_R} + \mathrm{i}\frac{\Gamma}{2} - \mathrm{J(r)}\right)\cdot\left(\mathrm{E} - \mathrm{E_R} + \mathrm{i}\frac{\Gamma}{2} + \mathrm{J(r)}\right) = 0. \qquad (14)$$

Expressions of eqs. (13) and (14) reveal the emergence of the three-body resonance of two families of the levels with energies $E^{st}_{res,n}(r; p_0) = E^{st}_{R,n} - i\Gamma^{st}_n$, where

$$E^{st}_{R,n} = E_R \pm \frac{\Gamma}{2}\cdot\frac{\cos(p_0 r)}{p_R r},$$

$$\Gamma^{st}_n = \frac{\Gamma}{2}\{1 \mp \cdot\frac{\sin(p_0 r)}{p_R r}\}. \qquad (15)$$

Note that some of the structural neutron resonances may have the width equal to zero, i.e. $\Gamma^{st}_{n=k}(r_k; p_{0;k}) = 0$. Here, the index k points out the proper energy levels and also marks the corresponding energies and the distances between the heavy nuclei. We call these levels as "quasi-bound states". Such levels are not realized in an ordinary two-body problem. If the light particle (neutron) penetrates in the structure with an unusual small lattice constant, this structure can play a role of quantum traps for the particle.

The generalization of the resonance neutron scattering on a heavy nucleus in the presence of inelastic channels is well known. For example, if the reaction of the neutron radiation capture by nucleus is possible, i.e. inelastic channels are opened, then the pair scattering amplitude near the resonance energy can be written in the following simple form:

$$t_{i;nn}(E \approx E_i^R) \approx \frac{1}{p_0\, m_n} \cdot \frac{\sqrt{\Gamma_{i;n}} \cdot \sqrt{\Gamma_{i;n}}}{E - E_i^R + i\,\Gamma_i / 2},$$

$$t_{i;n\gamma}(E \approx E_i^R) \approx \frac{1}{p_0\, m_n} \cdot \frac{\sqrt{\Gamma_{i;n}} \cdot \sqrt{\Gamma_{i;\gamma}}}{E - E_i^R + i\,\Gamma_i / 2}, \tag{16}$$

where $\Gamma_{i;n}$ is the width of the resonance in the elastic channel, $\Gamma_{i;\gamma}$ is the width of the gamma emission process and Γ_i is their sum.

Description of resonance scattering in the presence of inelastic channels in our three-body model is as well simple. Solutions have the form similar to eq. (16), but with a different term. The enhancement factor will be common to elastic and inelastic channels and equal to $M_{ij}(\vec{r}, \vec{r}\,')$ that contains the structural features of the nuclear system. Indeed, paired inelastic transitions will not take part in creation of the matrix D.

This is due to the fact that the radiation capture $n + A \to A'^{*} + \gamma$ aborts the neutron rescattering process in the heavy nuclei subsystem. We assume here that the reverse process is unlikely, because the number of excited nuclei with a different mass will be small. But it is also important that the reverse three-body reaction, i.e. $\gamma + A'^{*} + A \to n + A + A$, will have a resonant behavior for other values d, i.e. other values of the lattice constant.

Therefore, the process of neutron rescattering forms the whole matrix D. And the properties of the compound system are independent from the initial or final partial channels. But then, the elastic and inelastic resonance amplitudes can be written in the form similar to eq. (16):

$$T_{ij;nn} = \frac{1}{p_0 m_n} \sqrt{\Gamma_{i,n}}\, M_{ij} \sqrt{\Gamma_{j,n}},$$

$$T_{ij;n\gamma} = \frac{1}{p_0 m_n} \sqrt{\Gamma_{i,n}}\, M_{ij} \sqrt{\Gamma_{j,\gamma}}. \tag{17}$$

The matrix M_{ij} is determined as shown in eqs. (6) - (9).

Note that the structural neutron resonance scattering on the subsystems of three or more heavy nuclei, can also be defined in an analytical form [7].

Structural neutron resonances in the subsystems of the nuclei of the iron group

In this section we consider the results of calculations for the structural neutron resonances. The target nuclei are assumed fixed at the nodes of the crystal lattice. Note that the effective mass of nucleus in crystal equals to the mass of crystalline structure, i.e. the effective mass will be many times larger than the mass of individual nucleus, like in Mossbauer Effect.

Our calculations are based on the well-known data of neutron-nucleus resonances (see, for example, [16, 17-19]). The lowest neutron-nucleus resonances, which are well isolated from each other, have been taken into account.

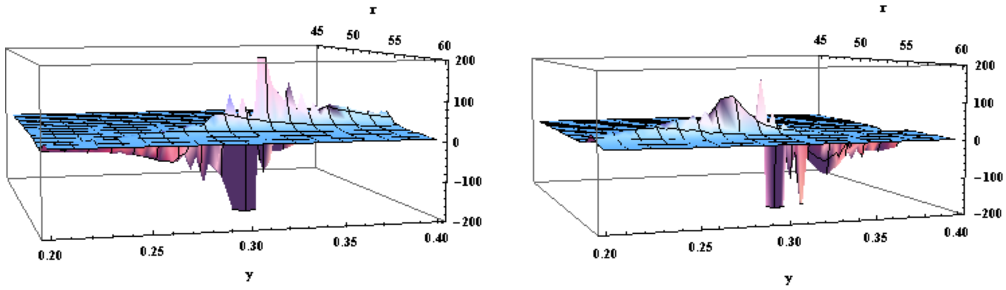
The important quantity of the three-body problem $K = (\det D)^{-1}$ is calculated, because this is the universal factor expressed in eqs. (8) and (9). Moreover this factor determines the ratio of three-body and two-body amplitudes under the same conditions. The condition $\det D(r; p_0) = 0$ gives the poles of the three-body amplitude. It allows finding the value of the structure factors $r = r_n^{st}$ and wave numbers $p_0 = p_n^{st}$ corresponding to this pole.

Note that every neutron-nucleus resonance creates own structural resonance levels in our three body problem. It is interesting that previous results of calculations are hardly changed by taking into account the two-body neutron-nucleus resonances of higher levels (with the energy higher than energies of lowest levels). But they introduce the new groups of structural neutron resonances in our problem. This means that the number of structural neutron resonances in the three-body problem is many times as large as the number of neutron resonances appears in the usual two-body problem.

It is also important to note the positions of structural neutron resonances on the scale of distances between heavy nuclei. Normally these resonances are located at very small distances, much smaller than the Bohr radius, but larger than the radius of the nuclear forces. However, if the energy of the two-body lowest level is very small (it may be the bound, virtual or resonant state with a very low energy), the structural neutron resonances can appear at larger distances up to a few hundred fm.

These resonances are of particular interest in practical applications. And they provide us with the first step to lead to an unusual structural transformation of matter in the process of compression during the formation of neutron stars. Let's consider the examples of crystal structures of the isotopes of the iron element group. This choice is motivated by the fact that these elements are considered as essential in the composition of the surface crust of neutron stars and structural formations in the interior of stars.

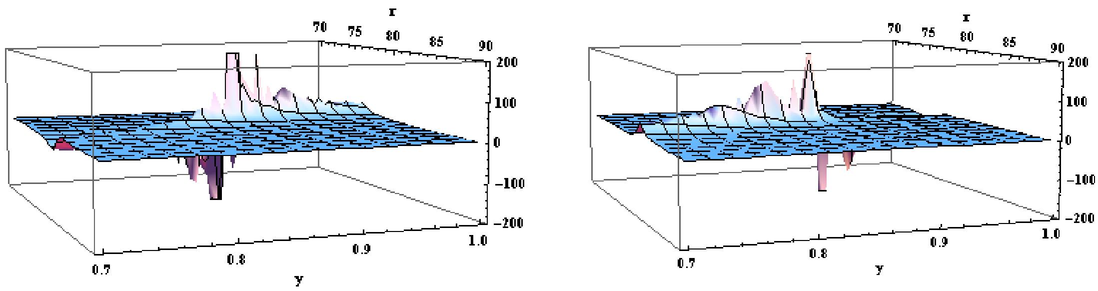
Figs 1 - 4 show the universal enhancement factor K as the function of variables r and y in the cases of various isotopes. The variable y is the shift of neutron wave number $p_0 = p_R(1+y)$ relative to p_R being the neutron wave number of the lowest resonance in neutron-nucleus pair. The distance between nuclei is $d = 2r$, where r in *fm*. Note that the property of structural neutron resonances, described by r_n^{st} and p_n^{st}, strongly depend on the individual characteristics of the isotopes. The value of p_R determines the region where the group of structural neutron resonances is placed.



a) b)

Figure 1 – The enhancement factor: Re K (a) and Im K (b), in system «n+ Mn55+Mn55». In two-body system «n+Mn55»: J = 3 and l = 0. The range of neutron energy: 2.67 - 3.64 keV.

The system of n+ Mn55+Mn55, where the two-body (n+Mn55) resonances with J = 3 and l = 0 are taken into account. Fig. 1 shows the enhancement factor K. The structural neutron resonances are obtained in the region of the energy range 2.67 - 3.64 *keV* and the lattice constant d in the range 106 - 114 *fm*.



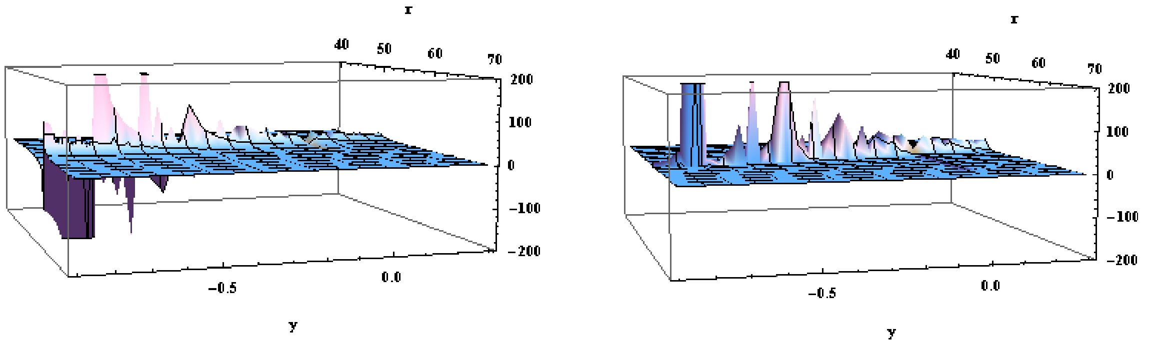
a) b)

Figure 2 – The enhancement factor: Re K (a) and Im K (b), in system of «n+ Mn55+Mn55». In two-body system «n+Mn55»: J = 2 and l = 0. The range of neutron energy: 1.08 - 1.21 keV.

Fig. 2 shows the enhancement factor K when the two-body (n+Mn55) resonances with J = 2 and l = 0 are taken into account. The structural neutron resonances are calculated in the region of the energy range 1.08 - 1.21 *keV* and d = 152 - 158 *fm*.

Figs. 3 shows the results for iron isotopes Fe56. In two-body system «n+Fe56»: the bound state Eb=- 6.52 *keV*, the resonances: E_{R1} = 1,1474 *keV* and E_{R2} = 12,45 *keV*. The structural neutron resonances are placed at larger values of d which are in the range of hundreds *fm*.

Note that the structural neutron resonances at greater distances are generated by two-body resonances with lower energies. So, the structural neutron resonances placed in the region of d at hundreds or thousands fm are generated by the two-body neutron-nuclear resonance of the lowest energy (*eV* or less). On the other hand, the structural neutron resonances placed at several tens of *fm* are generated by the two-body resonances of *keV* energies. Moreover, the two-body resonances with narrower widths produce the larger number of structural neutron resonances, some of them with more powerful and very narrow peaks.



a) b)

Figure 3 – The enhancement factor: Re K (a) and Im K (b), in system of «n+ Fe56+Fe56».
In two-body system «n+Fe56»: the bound state Eb=- 6.52 keV, the resonances ER = 1,1474 keV and ER = 12,45 keV.

Figs. 4 and 5 show the results for iron isotopes Fe57 and Fe58. The three-body system with the isotopes of Ni61 is worthy of note Fig. 6. The two body n+Ni61 system with J=2 and l=0 has a bound state with a very low energy - 9.5 *eV* and the resonance with energy 7.5 *keV* and width 225 *eV*. They generate the structural neutron resonances appear at very large distances of 2.8 – 2.95 *pm*, i.e. 0.028 – 0.0295 Å . The lowest structural neutron resonance has a very low energy ~ 0.1 *eV*.

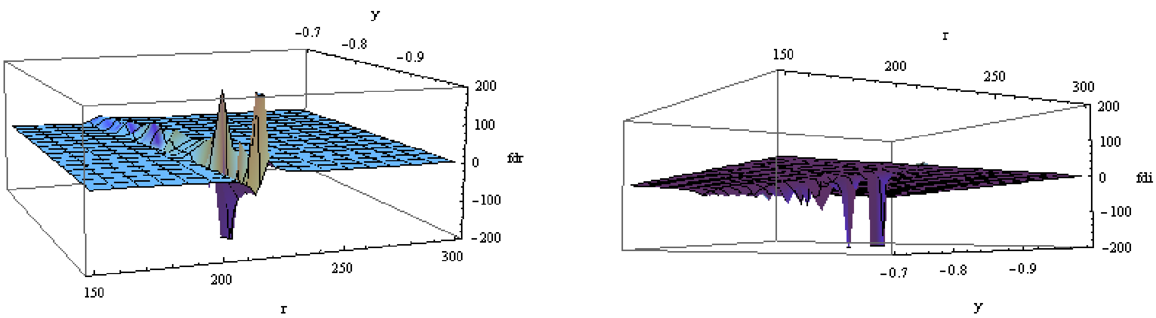


Figure 4 – The three-body system: n+Fe57+Fe57. The three lowest resonances are taken into account in two-body subsystem: n+Fe57 : E_b= - 0.44 keV in S-wave, J=1, l=0; the first resonances: E_{R1} = 1,63 keV and Γ = 0,11 eV Γ_n = 0.11 eV ; J=2, l =1; E_{R2} = 3,95 keV Γ = 108 eV ; Γ_n = 107 eV; J=0,l=0

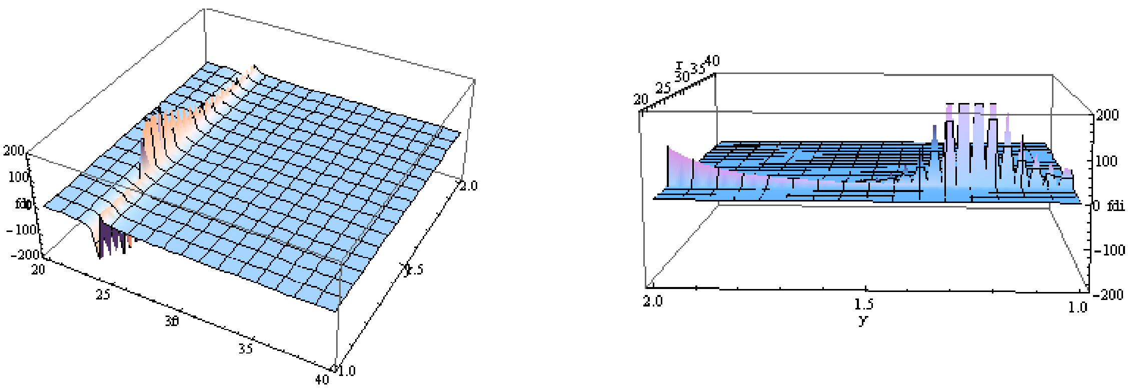


Figure 5 – The system: n+Fe58+Fe58. The lowest resonances in n+Fe57: E_b=- 35 keV in S-wave, E_{R1} = 0,2298 keV and Γ = 0,002 eV Γ_n = 0.002 eV; E_{R2} = 0,3586 keV and Γ = 0,021 eV Γ_n = 0,021 eV.

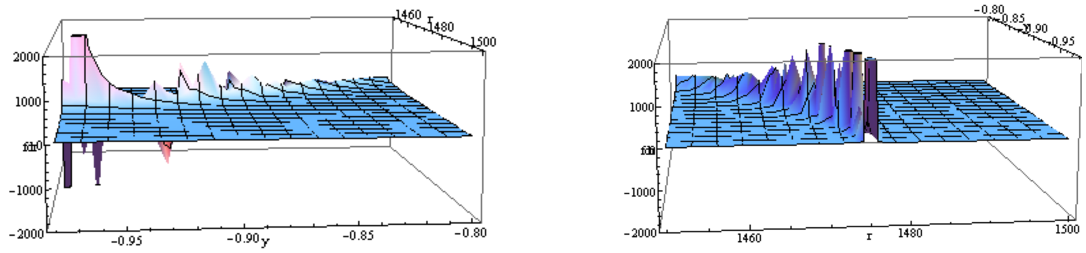


Figure 6 – The system: n+Ni61+Ni61. In two-body subsystem: n+Ni61 (J = 2, l = 0): E_b= - 0.0095 keV, E_R = 7,545 keV and Γ = 227,3 eV, Γ_n = 225 eV.

The isotope Ni62 gives a different picture (Fig.7).

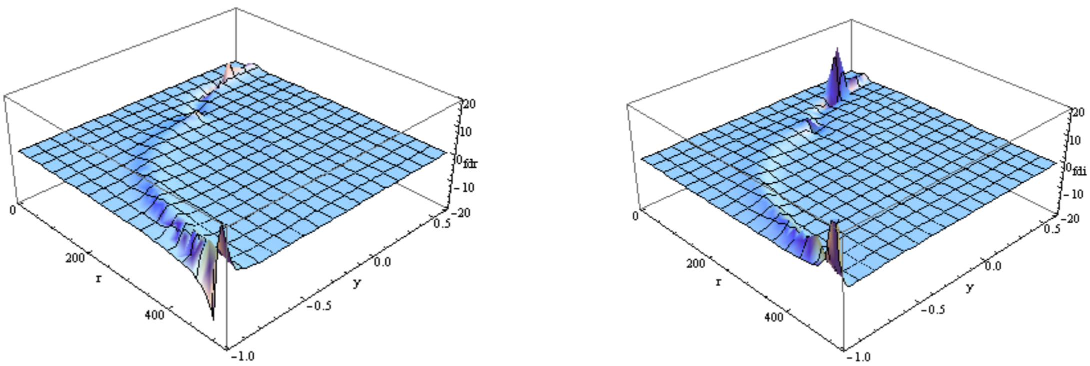


Figure 7 – The system n+Ni62+Ni62. In two-body subsystem n+Ni62 (J = 1/2, l = 0) : E_b=- 0.077 keV, E_R = 4,54 keV and Γ = (1880+0.76) eV; Γ_n = 1880 eV.

So, in two-body problems the different elements have quite different spectra of neutron resonances. Their isotopes also have fundamental differences in neutron spectra. In three-body problems they generate very rich and complicated spectra of the structural neutron resonances.

We also estimated the nuclear jitter effect in the crystal lattice. Fluctuations were considered to be small so far, but they may be considered by selecting an appropriate wave function describing the heavy nucleus state in the lattice node [5].

A bigger peak of the function K is evaluated on changing of the magnitude of the nuclear jitter. It turned out that small fluctuations do not alter essentially the character of resonant peaks. If the nuclear jitter gives $\Delta d / d \approx 0.05$ the structural resonance peak is slightly reduced in the maximum from the value of 10^9 to value of 10^7 and becomes more widespread, and when $\Delta d / d \approx 0.1$ this peak is reduced to 10^4 and the width becomes $\sim \Delta d$.

Conclusion

Let's consider the possible manifestations and consequences of the emergence of structural neutron resonances.

As it is well-known the outer crust of neutron stars consists of iron-group elements and has a crystalline structure. The crust squeezes the liquid mantle, which is propping up the bottom of the inner crust. The existence of internal and external solid crust is assumed. Matter of the star is squeezed with a powerful gravitational field, so neutron matter in star is formed. Neutron-rich nuclei are constructed to different structures having the form of pastes, plates, etc. Deeping to the center, matter becomes denser and reaches the nuclear densities, and even denser.

In this picture there is an attractive opportunity to use the appearance of the structural neutron resonances and their impact on the environment in almost all stages from the interaction of neutrons with the structure of the neutron star crusts.

Here, we pay attention to the effective potential between the nuclei, described by eq. (9), which is arising as a result of multiple rescattering of the neutron with heavy nuclei. This is a solution at least of three-body problems. The effective potential has a resonance behavior, which defines the structure and properties of the nuclei forming the structure. Forces created by the effective interaction potential will seek to bring together heavy nuclei, when the distance between them is a little more than the resonance value $d_n^{st} = 2 \cdot r_n^{st}$, and vice versa, push them apart, if this distance is slightly less than the resonance value.

The resonant nature of the effective interaction may lead to the following interesting phenomena:

1. If the distance between the nuclei is close to one causing the resonance structure and there are enough neutrons with the energy near E_n^{st} in the environment, the resonance will be evolving rapidly. The structure will have plenty of structural resonance neutrons. Their multiple rescattering on nuclei of structure will be accompanied by the radiation capture of neutrons. Released energy will heat the close layers of neutron star crust. Radiation is scattered by electrons will produce neutrino-antineutrino pairs.

The resonance cycles will stimulate the resonance process of the neutron enrichment of nuclei within the structure. With the change of isotope composition of the structure the resonance processes will go out or be replaced with other structural resonances.

From this it follows that the "nuclear life", i.e. the effect of nuclear interactions, is active in dense structures, even when the distance between heavy nuclei are still many times as large as the sizes of these nuclei themselves. The presence of neutrons and the existence of nuclei structure give the nuclear resonances opportunity to display their interactions of the "long-range order".

2. With increasing density when the distances between nuclei in the solid structures become near hundred or even tens of fm, the number of new structural neutron resonances increases. Forces created by effective potentials of eq. (10) will tend to "pull apart" or "stretch" the nuclei into connected fragments or clusters according as the resonance shapes of these potentials. They can form ordered chain of clusters of nuclear matter in the inner crust. There are a lot of cluster configurations of nuclei, which can vary greatly their shapes in the reactions with capture of neutrons [18-21]. The cluster chains of neighboring nuclei can overlap in space, forming new more heavy nuclear configurations.

3. The "extended" nuclear units can exist a long time and enough deep inside the star. In the case of the "star eruptions" or explosions, they may become free outside the star and then shrink to almost normal forms, transform to well-known nuclei, like super heavy nuclei with large mass numbers [22].Thus, the neutron star and three-body effects present us an additional opportunity to nucleosynthesis of super heavy nuclei. We can say that neutron stars can be "a factory" that creates a whole range of nuclei and nuclear entities well-known in nature and experiments.

4. The structural neutron resonances can also appear in the outer crust of neutron star. Let's assume that the deformation of the solid crust occurs periodically, for example, due to the gravitational influence of a satellite or surface waves. Then the lattice constants will be also changed, and if the distance between the nuclei crosses one of the values d_n^{st} and there are enough neutrons with energy close to p_n^{st}, the structural neutron resonances will appear, too. The generation of energy and radiation occurs in this local area. Gamma rays which accompany neutron structural resonances will behave in unusual ways. They will interact nonlinearly between each other's and with the environment and create excited states of the Fermi electrons. They will generate neutrino-antineutrino pairs and heat the local layers of crusts. These effects will be investigated and the results will be presented in subsequent papers.

This project IPS: 15/2012 is supported by Ministry of Education and Sciences of Republic of Kazakhstan, and carried out in the framework of the collaboration between Al-Farabi Kazakh National University and Hokkaido University.

Acknowledgment

The authors thank the participants of the center "Asia Database of Nuclear Reactions" for useful discussion; also thank professors M.W. Snow, N. Kawamoto, A. D'Adda and H. Quevedo for valuable advices. N.Takibayev, A. Sarsembaeva and M. Takibayeva thanks the Hokkaido University for their hospitality and support.

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