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Spin-up and spin-down evolution in general relativistic rotating white dwarfs

Abstract. In this work we study spin-up and spin-down evolution of super-and sub-Chandrasekhar mass white dwarfs by angular momentum loss. We show that super-Chandrasekhar white dwarfs can only spin-up, whereas sub-Chandrasekhar white dwarfs can only spin down by angular momentum loss. Moreover we show that the white dwarfs of mass close to the Chandrasekhar mass limit can both spin-up and spin-down depending on the initial rotation period.
Keywords: general relativistic rotating white dwarfs, stability, spin-up and spin-down evolution.

Introduction

White dwarfs are stellar remnants composed mostly of electron-degenerate matter. They are very dense; a white dwarf's mass is comparable to that of the Sun, and its volume is comparable to that of the Earth. White dwarfs (WDs) are intermediate objects between main sequence stars and compact objects (neutron stars, strange stars etc.) in the sense that the relativistic effects are not noticeable in low mass white dwarfs whereas in the massive ones the effects are crucial. For this reason they are usually considered both within Newton's gravity and general relativity (GR) [1]. Recently, equilibrium configurations of non-rotating (static) ^{4}He, ^{12}C, ^{16}O and ^{56}Fe WDs within GR have been constructed by Rotondo et. al. 2011 [2]. The white dwarf matter has been there described by the relativistic generalization of the Feynman-Metropolis-Teller (RFMT) equation of state (EOS) obtained by Rotondo et al. 2011 [3]. A new mass-radius relation that generalizes both the works of Chandrasekhar 1931 [4] and Hamada & Salpeter 1961 [5] has been there obtained, leading to a smaller maximum mass and a larger minimum radius with respect to the previous calculations. In addition, it has been shown how both GR and inverse β-decay are relevant for the determination of the maximum stable mass of non-rotating WDs.

Further generalization of the results of Rotondo et. al. 2011 [2] to the case of rotation was investigated by Boshkayev et. al. 2011 [6] in the simplified case when microscopic Coulomb screening is neglected in the EOS, following the Chandrasekhar [4] approximation by describing the matter as a locally uniform fluid of electrons and nuclei. The average molecular weight in the Chandrasekhar EOS is $\mu = A/Z = 2$, where A is the mass number and Z is the number of protons in a nucleus. Moreover we calculated in [7] the maximum mass of rotating ^{4}He, ^{12}C, ^{16}O and ^{56}Fe WDs using the Salpeter [8] and the RFMT EOS [2]. As a result we obtained there different maximum mass for different chemical composition of WD matter. Then we investigated the stability of general relativistic uniformly rotating ^{4}He, ^{12}C, ^{16}O and ^{56}Fe white dwarfs against secular and dynamical instabilities. We determine the maximum mass and minimum rotation period of stable white dwarfs depending on chemical composition, taking into account the Coulomb interactions as well as the nuclear interactions and the electroweak equilibrium at high densities, within the Salpeter and RFMT EOS [9].

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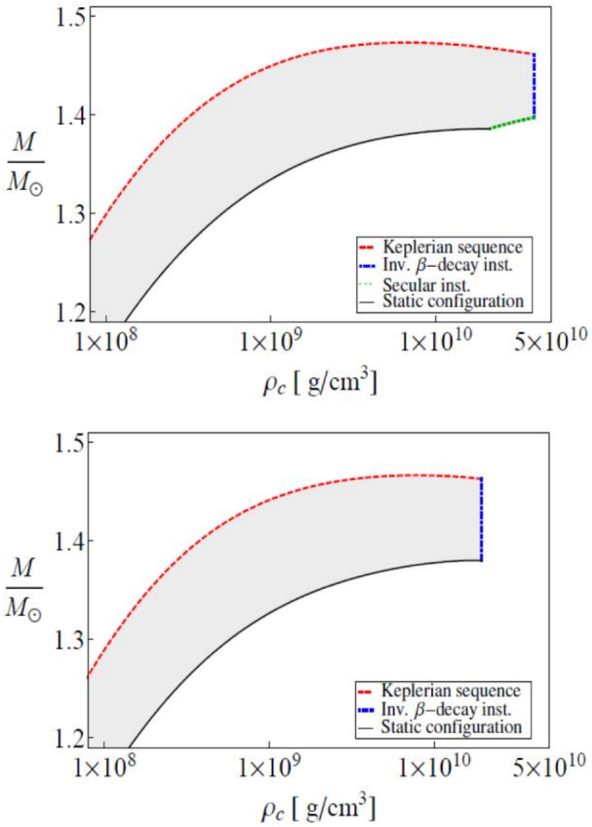

Figure 1 – Mass in solar masses versus the central density for ^{12}C (left panel) and for ^{16}O (right panel) WDs. The solid curve corresponds to the mass of non-rotating WDs, the Keplerian sequence is the red thick dashed curve, the blue thick dotted-dashed curve is the inverse β instability boundary, and the green thick solid curve is the axisymmetric instability boundary. All rotating stable WDs are in the shaded region.

We summarized all the above results in [10], where we analyzed in detail the stability of rotating white dwarfs (RWDs) both from the microscopic and macroscopic point of view. Besides the inverse β-decay instability, we also study the limits to the matter density imposed by zero-temperature pycnonuclear fusion reactions using up-to-date theoretical models of Gasques et. al. 2005 and Yakovlev et. al. 2006 [11, 12]. The importance of GR to study the stability of RWDs was considered in [13]. Moreover one can restrict to the Newtonian case when estimating the minimum period of uniformly RWDs. In case of the maximum rotating mass and in the analyses of stability, general relativity indeed becomes relevant. We make use of Hartle's formalism [14, 15] for the construction of uniformly rotating stars in GR. As a WD matter we consider the RMFT EOS [2, 3]. The main parameters characterizing RWDs such as the total mass M, angular momentum J, quadrupole moment Q are obtained from the matching procedure between the internal and external solutions of the field equations and the equations of hydrostatic equilibrium [14, 15]. The main objective of this work is to investigate spin-up and spin-down evolution of sub- and super-Chandrasekhar mass white dwarfs in general relativity following the procedure used in [10]. For this, we construct mass-central density, mass-radius relations. Moreover we compute constant rest mass sequence satisfying all the stability criteria for uniformly rotating white dwarfs in the following sections.

Stability region of RWDs

In figure 1 we construct mass-central density relation for uniformly RWDs making use of Hartle's formalism [16] for RFMT EOS. The shaded region is called the stability region which shows that all stable, static and rotating WDs can exist only in this region. Out of the stability region all WDs are subjected to one of the following instabilities:

1) above the Keplerian sequence WDs will be unstable due to the mass-shedding. In this case, the centrifugal force will prevail over the gravitational force, and the matter will start shedding away from the equatorial plane (surface) of the star;

2) on the right hand side of the inverse β-decay line WDs will collapse into a neutron star. Since at higher densities electrons become ultra-relativistic and start interacting with protons inside nuclei, forming thus neutrons. This process is also called neutronization process;

3) below the axisymmetric instability boundary (line) WDs probably will explode either as type Ia supernova or collapse into a neutron. This issue is very delicate. Further investigations are required.

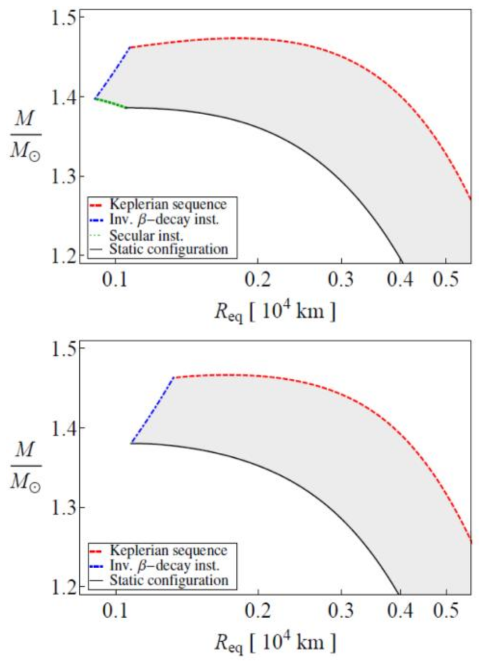

Figure 2 – Mass in solar masses versus the equatorial radius in units of 10^4 km for ^{12}C (left panel) and for ^{16}O (right panel) WDs. The left and right panels show the configurations for the same range of central densities of the corresponding panels of Figure 1

In figure 2 we show mass-radius relation in the density range corresponding to figure 1. The more massive white dwarf the more compact its size, as expected. On both figures 1-2 we considered ^{12}C (left panel) and ^{16}O (right panel) WDs, since they are the most observed.

Spin-up and spin-down

The spin-down of a fast rotating star is qualitatively different from that of a slowly rotating one [16]. A slowly rotating star is essentially spherical and loss of angular momentum J results in a decrease in the angular velocity Ω of a star, but no substantial change in the moment of inertia I. A fast rotating star, however, may be highly oblate, and the loss of angular momentum can result in a significant decrease in I. Since $\Omega=J/I$, the sign of the angular velocity Ω clearly depends on the relative rates of decrease of J and I. The energy and angular momentum of a rotating star can slowly decrease by such mechanisms as the emission of electromagnetic or gravitational radiation. For example, the emission of magnetic dipole radiation is generally accepted as the dominant spin-down mechanism for radio pulsars and magnetic white dwarfs [17].

It is known that at constant rest mass M_0, entropy S, and chemical composition (Z, A), the spin evolution of an RWD is given by (see [17], for details)

$$\dot{\Omega} = \frac{\dot{E}}{\Omega}\left(\frac{\partial \Omega}{\partial J}\right)_{M_0,S,Z,A}, \tag{1}$$

where $\dot{\Omega} = d\Omega/dt$ and $\dot{E} = dE/dt$, and E is the energy of the star.

Thus, if RWD is losing energy by some mechanism (for instance, via magneto-dipole radiation) during its evolution, that is $\dot{E} < 0$, then the change of the angular velocity Ω in time depends on the sign of $\partial\Omega/\partial J$; RWDs that evolve along a track with $\partial\Omega/\partial J > 0$ will spin down ($\dot{\Omega} < 0$), and the ones following tracks with $\partial\Omega/\partial J < 0$ will spin up ($\dot{\Omega} > 0$).

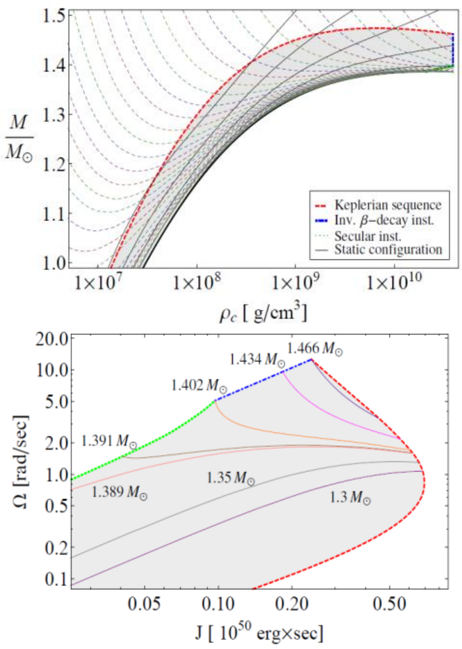

Figure 3 – Left panel: mass versus the central density for ^{12}C RWDs. The solid black curves correspond to J = constant sequences, where the static case $J = 0$ is the thickest one. The color thin-dashed curves correspond to Ω = constant sequences. The Keplerian sequence is the red thick dashed curve, the blue thick dotted-dashed curve is the inverse β-decay instability boundary, and the green thick dotted curve is the axisymmetric secular instability boundary. Right panel: contours of constant rest mass in the $\Omega - J$ plane; RWDs that evolve along a track with $\partial\Omega/\partial J > 0$ spin down by losing angular momentum, while the ones with $\partial\Omega/\partial J < 0$ spin up.

In figure 3, we show (the left panel) the Ω =constant and J =constant sequences in the mass-central density diagram and, in the right panel, contours of constant rest mass in $\Omega - J$ plane.

The sign of $\partial\Omega/\partial J$ can be analyzed from the left panel plot of figure 3 by joining two consecutive J =constant sequences with a horizontal line taking into account the fact that J decreases from left to right and from top to bottom. Instead, the angular velocity Ω decreases from right to left and from top to bottom for super-Chandrasekhar WDs and, from left to right and from top to bottom for sub-Chandrasekhar WDs. We note that in the super-Chandrasekhar WDs region, Ω =constant sequences satisfy $\partial\Omega/\partial\rho_c < 0$ while, in sub-Chandrasekhar region, both $\partial\Omega/\partial\rho_c < 0$ and $\partial\Omega/\partial\rho_c > 0$ appear (see minima). Super-Chandrasekhar WDs can only either spin up by losing angular momentum or spin down by gaining angular momentum. In the latter case, the RWD becomes decompressed with time, increasing it radius and moment of inertia, and then super-Chandrasekhar WDs following this evolutionary track will end at the mass shedding limit (see figure 3). Some evolutionary tracks of sub-Chandrasekhar WDs and Super-Chandrasekhar WDs are shown in the right panel of figure 3. It is appropriate to recall here that Shapiro et al. (1990) [16] showed that spin up behavior by angular momentum loss occurs for rapidly rotating Newtonian polytropes if the polytropic index is very close to $n = 3$, namely, for an adiabatic index of $\Gamma \approx 4/3$. It was explicitly shown by Geroyannis & Papasotiriou (2000) [18] that these conditions are achieved only by super-Chandrasekhar polytropes. Besides the confirmation of the above known result for super-Chandrasekhar WDs in the general relativistic case, here we also report the presence of minima $\partial\Omega/\partial\rho_c = 0$ for some sub-Chandrasekhar masses (see, e.g., the evolution track of the RWD with $1.38M_\odot$ in the right panel of figure 3), which raises the possibility that sub-Chandrasekhar WDs (near the Chandrasekhar mass limit) can experience, by angular momentum loss, not only the intuitively spin-down evolution, but also spin-up epochs.

Conclusions

We analyzed the stability of RWDs within Hartle's formalism and computed constant mass sequences, using the RFMT EOS, particularly for WDs composed of ^{12}C and ^{16}O. We showed that, along these sequences by losing angular momentum, sub-Chandrasekhar (close to the Chandrasekhar mass limit) RWDs can experience both spin-up and spin-down epochs, while super-Chandrasekhar WDs can only spin up. These results are particularly important for the evolution of WDs whose masses approach, either from above or from below, the maximum non-rotating mass. The knowledge of the actual values of the mass, radii, and moment of inertia of massive RWDs are relevant for the computation of delay collapse times in the models of type Ia supernova explosions [19]. A careful analysis of all of the possible instability boundaries such as the ones presented in [10] have to be taken into account during the evolution of the WD at pre-SN stages [20]. All these results have very important astrophysical implications in investigating the progenitors of type Ia supernova explosions, millisecond pulsars and Soft-Gamma Repeaters along with Anomalous X-Ray Pulsars in view of the recent work by Malheiro et. al. 2012 [17].

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References

1 Boshkayev K., Rueda J.A., Ruffini R., Siutsou I. General Relativistic and Newtonian White Dwarfs // Proceedings of the Thirteenth Marcel Grossman Meeting on General Relativity, edited by Rosquist K., Jantzen R., Ruffini R. / World Scientific – Singapore, 2012. (in press)

2 Rotondo M., Rueda J.A., Ruffini R., Xue S.S. Relativistic Feynman-Metropolis-Teller theory for white dwarfs in general relativity // Physical Review D – 2011. – Vol. 84. – Issue 8. – id. 084007

3 Rotondo M., Rueda J.A., Ruffini R., Xue S.S. Relativistic Thomas-Fermi treatment of compressed atoms and compressed nuclear matter

cores of stellar dimensions // Physical Review C. – 2011. – Vol. 83. – Issue 4. – id. 045805.

4 Chandrasekhar S. The Maximum Mass of Ideal White Dwarfs // Astrophysical Journal. – 1931. – Vol. 74. – P. 81.

5 Hamada T., Salpeter E.E. Models for Zero-Temperature Stars // Astrophysical Journal. – 1961. – Vol. 134. – P. 683.

6 Boshkayev K., Rueda J., Ruffini R. On the Maximum Mass of General Relativistic Uniformly Rotating White Dwarfs // International Journal of Modern Physics E. – 2011. – Vol. 20. – P. 136–140.

7 Boshkayev K., Rueda J., Ruffini R. On the Maximum Mass of General Relativistic Uniformly Rotating ^{4}He, ^{12}C, ^{16}O and ^{56}Fe White Dwarfs // From Nuclei to White Dwarfs to Neutron Stars, edited by Mezzacappa A., Ruffini R. – 2011 (in press).

8 Salpeter E.E. Energy and Pressure of a Zero-Temperature Plasma // Astrophysical Journal. – 1961. – Vol. 134. – P. 669.

9 Boshkayev K., Rueda J., Ruffini R. On the Maximum Mass and Minimum Rotation Period of Relativistic Uniformly Rotating White Dwarfs // 3rd Galileo-Xu Guangqi Meeting – Proceedings, edited by Cao Zh., Chen X., Ruffini R., Xue Sh., Zhang C-M., Zhang Sh. – 2011 (in press).

10 Boshkayev K., Rueda J., Ruffini R., Siutsou I. On General Relativistic Uniformly Rotating White Dwarfs // Astrophysical Journal. – 2013. – Vol. 762. – Issue 2. – P. 117.

11 Yakovlev D.G., Gasques L.R., Afanasjev A.V., Beard M., Wiescher M. Fusion reactions in multicomponent dense matter // Physical Review C. – 2006. – Vol. 74. – Issue 3. – id. 035803.

12 Gasques L.R., Afanasjev A.V., Beard M., Chamon L.C., Ring P., Wiescher M. Pycnonuclear reaction rates between neutron-rich nuclei // Nuclear Physics A. – 2005. – Vol. 758. – P.134–137.

13 Boshkayev K. Rotating White Dwarf and Neutron Stars in General Relativity // PhD thesis. – 2012. http: // padis. uniroma1. It / bitstream / 10805 / 1934 / 1 / Thesis % 20 of % 20 Kuantay % 20 Boshkayev. pdf

14 Hartle J.B. Slowly Rotating Relativistic Stars. I. Equations of Structure // Astrophysical Journal. – 1967. – Vol. 150. – P. 1005.

15 Hartle J.B., Thorne K.S. Slowly Rotating Relativistic Stars. II. Models for Neutron Stars and Supermassive Stars // Astrophysical Journal. – 1968. – Vol. 153. – P. 807.

16 Shapiro S.L., Teukolsky S.A., Nakamura T. Spin-up of a rapidly rotating star by angular momentum loss // Astrophysical Journal Letters. – 1990. – Vol. 357. – P. L17–L20.

17 Malheiro M., Rueda Jorge A., Ruffini R. SGRs and AXPs as Rotation-Powered Massive White Dwarfs // Publications of the Astronomical Society of Japan. – 2012. – Vol. 64. – No.3. – P. 56.

18 Geroyannis V.S., Papasotiriou P.J. Spin-up and Spin-down of Rotating Magnetic White Dwarfs: A Straightforward Numerical Approach // Astrophysical Journal. – 2000. – Vol. 534. – Issue 1. – P. 359–366.

19 Ilkov M., Soker N. Type Ia supernovae from very long delayed explosion of core-white dwarf merger// Monthly Notices of the Royal Astronomical Society. – 2012. – Vol. 419. – Issue 2. – P. 1695–1700.

20 Ilkov M., Soker N. The number of progenitors in the core-degenerate scenario for Type Ia supernovae // Monthly Notices of the Royal Astronomical Society. – 2013. – Vol. 428. – Issue 1. – P. 579–586.