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Thermomagnetic ferroconvection in an anisotropic permeable layer exposed to a modulated magnetic field

Abstract. The impact of a sinusoidal mode of magnetic field involving time-dependent on the threshold of magnetic smart liquid advection in a saturated Darcy-permeable framework is investigated using a regular perturbation technique. Aniso tropic permeability and thermal anisotropy are used to describe the flow through permeable medium. The regular perturbation technique is based on minimum amplitude of time-fluctuated magnetic field, the threshold condition is computed with regard to correction in a critical Rayleigh number and wavenumber. Correction in Rayleigh number is identified by modulating the magnetic field, modulation frequency, magnetic parameter, mechanical anisotropy, thermal anisotropy and Vadasz number. At intermediate frequency values, the impact of various physical factors is perceived to be noteworthy. It is found that by fine tuning the frequency of magnetic field modulation, we can either accelerate or postpone the onset of ferroconvection. The most sophisticated scientific application packages, Wolfram Mathematica 11.3 is used to extract the numerical values as well as plotting graphs. The problem sheds some light on convective heat transfer mechanisms in ferromagnetic fluid with time-varying magnetic field.
Keywords: Magnetic liquid, Anisotropy, Stability, Porous medium.

Introduction

A ferrofluid (magnetic nanofluid) is a liquid carrier that includes a solution of nanoscopic magnetic particles immersed in a surfactant coating. In comparison with conventional fluids, magnetic nanofluids are responsive to external magnetic fields even in the absence of gravitational force. Numerous studies on these fluids have been undertaken as an outcome of their diverse applications in computer disk drives, bio-medical, magnetic resonance, robotic systems and dynamic sound system, to mention a few [1, 2]. In order to improve the thermal conductivity of fluids, magnetic nanoparticles are suspended in them. Depending on the nanoparticle, fluids can often have a hundred times greater thermal conductivity than the carrier fluids. In this background, this article attempts a comprehensive review on magnetic fluid owing to its prospective value as a heat transfer phenomenon. An initial description of thermomagnetic convection was given by Finlayson [3], purely by showing how a horizontal surface of magnetic fluid with a variable magnetic susceptibility leads to a non-consistent force of the magnetic field. Future, many authors attracted towards the work of Finlayson and investigated the commencement of magnetic fluid convection under a variety of handy constraints [4-6]. According to recent work performed with the higher order Galerkin technique, it is clarified that the MFD viscosity plays a role in delaying the advent of ferroconvection in a sparsely packed permeable medium exposed to varying gravity fields [7].

In various sectors, such as charges in electrode materials and the resonance of a ferromagnetic field, modulation (oscillation) of a suitable parameter can affect the motion and can result in improved stability. Many theoretical and experimental investigation dealing with fluctuations in the magnetic field on the advection of a magnetic liquid and collision between harmonic and subharmonic conditions have been carried out by numerous authors [8-11] using the Floquet theory. In the articles [12, 13], it is reported that the nonzero flow field of the base state is caused by a double vortex reflecting an external magnetic field modulated symmetrically by two iron bars below and above a ferrofluid layer. The temperature distribution through an electrically charged liquid with internal heat source and couple stresses exposed to magnetic field fluctuation is discussed in detail [14]. Recently, a work is carried out on the advent of magnetic nanofluid under the influence of fluctuated magnetic field ferroconvection in a sparsely arranged permeable structure, it is revealed that convection can be delayed or advanced by controlling the parameters of the study [15].

Temperature profile through fluid-saturated nanopores has piqued the interest of many researchers due to its natural phenomenon and diverse applications in science and technology. This includes the use of geothermal energy resources, the eradication of nuclear excess, aquifers leftover removal, drying processes, and so on. Harton, Rogers, and Lapwood [16, 17] pioneered work on fluid-saturated permeable structures located between two identically flat surfaces and heated directly beneath, and the overall problem has been termed as "Horton-Rogers-Lapwood or Darcy-Benard". Moreover, numerous authors have addressed the topic in depth and the growing number of research in this area is extensively documented [18, 19]. The majority of scientific and experimental research on the advection of flow in porous environments has focused on isotropic materials. More than that, in many real scenarios, the mechanical and thermal assets of porous materials are anisotropic, which can be seen in several industrial and environmental situations as a result of irregular pattern of permeable matrix. Anisotropy can also be noticed in synthetic porous materials like nanoparticles used in chemical manufacturing techniques and coating materials.

The effect of Vadaz number on convection in a Darcy-permeable framework with rotating fluid surface is well explained in the articles [20, 21], it is noted that, unlike the problem in pure liquids, over stable advection in permeable medium at marginal stability is not limited to a specific range of Prandtl number values. By adopting the assumptions that the layer is anisotropic, homogenous, and has an infinite horizontal extent, [22] a theoretical examination of the thermal gradients in the permeable structure is handled. A permeability with anisotropy in thermal diffusivity produces two distinct convection cells when a symmetry axis is assumed and a $(90° - \theta)$ angle is made against perpendicular motion is discussed in detail [23]. In addition, anisotropic permeable matrix subjected to inclined layer, time–periodic temperature/gravity, rotation and double diffusivity has been reported in the literatures [25-28] respectively. The impact of thermal modulation on the advent of the ferroconvection in Darcian permeable materials confirms that subcritical point exists for balanced temperature fluctuation for minimum frequency. Moreover, for unbalanced and bottom wall fluctuation only supercritical state presents [29]. A weakly nonlinear unsteadiness in a rotary permeable anisotropic smart ferrofluid medium using Runge–Kutta–Gill numerical technique has been carried out in recent years [30].

Convection control is a phenomenon that is vital and intriguing in a wide range of magnetic fluid technologies, as well as conceptually challenging. The unamplified Rayleigh-Bénard advection in the ferromagnetic liquid has derived a plenty of attention. Notwithstanding, substantial attention turned out to be devoted to the combined impact of the modulated magnetic field and permeable anisotropy layer on the advent of ferroconvection. In this paper, the presented analysis is with reference to the presumption that the modulation dimension is very minimal and the convective currents are weak, allowing nonlinear effects to be ignored. Thus, depending on the frequency of magnetic field modulation, the advent of ferroconvection can be advanced or delayed in the presence of Darcian-anisotropic permeable medium. Present work aims to provide an introduction to vertical harmonic vibrations, magnetic factors, and anisotropy as they relate to natural convection.

Mathematical model

Permeable medium is considered, which is bound between two plates kept separate by a distance d (see Fig. 1). In mechanical and thermal aspects, the permeable medium is presumed to be closely packed and have vertical anisotropy. A vertical downward gravity force, as well as a uniform temperature difference ΔT between the two surfaces, act on the fluid. The reference rectangular coordinate frame's origin is at the bottom, with the z–axis pointing up vertically.

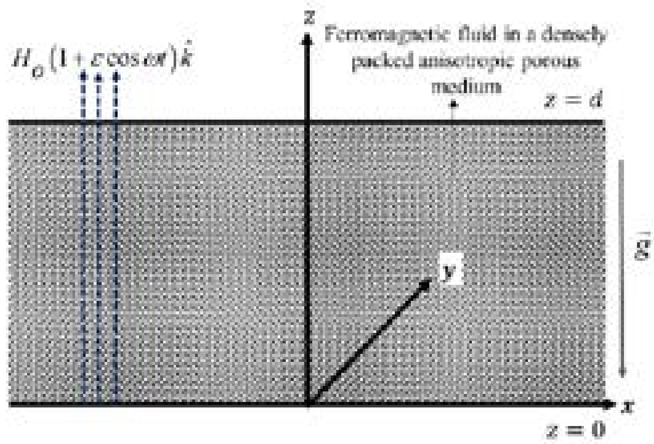

Figure 1 – Physical configuration

The magnetic field imposed externally is time-dependent and is used as

$$\vec{H}_o^{Ext}(t) = H_o^{Ext}(t) = H_o(1+\varepsilon\cos\omega t)\hat{k} \quad (1)$$

where H_o is a uniform magnetic field, ε and ω are modulation amplitude and frequency respectively.

The equation of continuity is

$$\nabla \bullet \vec{v} = 0, \quad (2)$$

The conservation of linear momentum with anisotropic inverse permeability $\vec{K} = K_x^{-1}(\hat{\imath}\hat{\imath} + \hat{\jmath}\hat{\jmath}) + K_z^{-1}(\hat{k}\hat{k})$, for modified Darcy model is taken in the form [3, 25, 28]

$$\rho_o\left[\frac{1}{\varepsilon_p}\frac{\partial\vec{v}}{\partial t} + \frac{1}{\varepsilon_p^2}(\vec{v}\bullet\nabla)\vec{v}\right] = = -\nabla p + \rho\vec{g} + \nabla\bullet(\vec{H}\vec{B}) - \mu_f\vec{K}\bullet\vec{v}, \quad (3)$$

where $\vec{v}$ is the actual velocity component, ρ is the density, ρ_o is the reference density, ε_p is the porosity, p is the pressure, $\vec{g} = -g\hat{k}$ is the acceleration due to gravity, μ_f is the viscosity, $\vec{H}$ is the overall magnetic field, $\vec{B}$ is the magnetic induction.

We adopted the Oberbeck–Boussinesq approximation in the study. For the derivation of appropriate equations, giving a rigorous basis for the Oberbeck-Boussinesq approximation, one can refer [31]. According to the above-mentioned assumption, and for small departures from reference temperature T_o the density ρ, as a function of temperature T, the density equation of state involving constant coefficient of volume expansion β is given by

$$\rho = \rho_o\left(1 - \beta(T - T_o)\right), \quad (4)$$

In energy transport equation the thermal conductivity $\vec{K}_T = K_{T_x}(\hat{\imath}\hat{\imath} + \hat{\jmath}\hat{\jmath}) + K_{T_z}(\hat{k}\hat{k})$ is assumed to be anisotropic and is of the form

$$\varepsilon_p C_1\frac{DT}{Dt} + \left(1-\varepsilon_p\right)\left(\rho_0 C\right)_s\frac{\partial T}{\partial t} + \mu_o T\left(\frac{\partial\vec{M}}{\partial T}\right)_{V,H}\cdot\frac{D\vec{H}}{Dt} = \nabla\cdot\left(\vec{K}_T\nabla T\right), \quad (5)$$

where $C_1 = \rho_o C_{V,H} - \mu_o\vec{H}\cdot\left(\partial\vec{M}/\partial T\right)_{V,H}$, $C_{V,H}$ is the specific heat at constant volume and magnetic field.

Maxwell's equations, simplified for a non-conducting fluid with no displacement current, become

$$\nabla\bullet\vec{B} = 0, \nabla\times\vec{H} = 0 \quad (6)$$

and

$$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right) \quad (7)$$

We adopt that the magnetization $\vec{M}$ is aligned with magnetic field, but allows a dependence on the magnitude of the magnetic field as well as temperature,

$$\vec{M} = \frac{\vec{H}}{H}M(H, T) \quad (8)$$

The magnetic equation of state is linearized about the magnetic field H_o and an average temperature T_o to give

$$M = M_o + \chi_m(H - H_o) - K_m(T - T_o) \quad (9)$$

where χ_m and K_m are the differential magnetic susceptibility and the pyromagnetic coefficient respectively. The temperatures of bottom and top surfaces respectively are

$$T(0) = T_o + \left(\frac{1}{2}\right)\Delta T, T(d) = T_o - \left(\frac{1}{2}\right)\Delta T \quad (10)$$

We now look at the necessary conditions for heat flow to continue in the above-noted permeable layer saturated in nanoscopic magnetic liquid. An undisturbed medium will be quiescent and be provided by

$$\vec{v} = \vec{v}_b = \vec{0}, p = p_b(z), \rho = \rho_b(z), T = \\ = T_b(z), \vec{H} = \vec{H}_b = H_o(z,t) = H_o^{Ext}(t), \\ \vec{M} = \vec{M}_b = M_o(z,t), \vec{B} = \vec{B}_b = B_o(z,t) \quad (11)$$

The temperature $T = T_b(z)$ is a solution of

$$K_{T_x}\left(\frac{\partial^2 T_b}{\partial x^2} + \frac{\partial^2 T_b}{\partial y^2}\right) + K_{T_z}\frac{\partial^2 T_b}{\partial z^2} = 0 \quad (12)$$

The solution of (12) subjected to the boundary conditions (10) is

$$T_b = T_o + \Delta T\left(\frac{1}{2} - \frac{z}{d}\right) \quad (13)$$

The magnetic field, magnetization and the related magnetic induction equations followed by (13) and the stationary basic state quantities are

$$H_b = \left[1 + \frac{\chi_o H_o \Delta T \delta}{(1+\chi_o)T_o}\left(\frac{1}{2} - \frac{z}{d}\right)\right] \quad (14)$$

$$M_b = \left[M_o + \frac{H_o \chi_o \Delta T \delta}{(1+\chi_o)T_o}\left(\frac{1}{2} - \frac{z}{d}\right)\right] \quad (15)$$

$$B_b = \mu_o(M_o + H_o) \quad (16)$$

where $\delta = \frac{(1+\varepsilon\, Re\{e^{-i\omega t}\})}{(1+\chi_o)}$, Re stands for the real part. We do not record the expressions of p_b and ρ_b as these are not explicitly required in the remaining part of the paper.

Linear Stability Analysis

The stability of the system is studied by superimposing infinitesimal disturbances on the basic state and we now have

$$\vec{v} = \vec{v}_b + \vec{v}', p = p_b + p', \rho = \rho_b + \rho', T = \\ = T_b + T', \\ \vec{H} = \vec{H}_b + \vec{H}', \vec{M} = \vec{M}_b + \vec{M}', \vec{B} = \vec{B}_b + \vec{B}', \quad (17)$$

where the prime indicates that the quantities are infinitesimal perturbations.

Substituting (17) into (2) – (9), and using the basic state solutions, we get the linearized equations governing the perturbations in the form

$$\nabla \bullet \vec{v}' = 0, \quad (18)$$

$$\frac{\rho_o}{\varepsilon_p}\left[\frac{\partial \vec{v}'}{\partial t}\right] = -\nabla p' + \beta\rho_o g T'\hat{k} - \mu_f \vec{K} \bullet \vec{v}' + \\ +\mu_o(M_o + H_o)\frac{\partial \vec{H}'}{\partial t} - \\ -\left(\frac{\mu_o \chi_o H_o \delta \Delta T}{T_o d}\right)\frac{\partial \phi'}{\partial z}\hat{k} + \left(\frac{\mu_o \chi_o^2 H_o^2 \delta^2 \Delta T}{T_o^2(1+\chi_o)d}\right)T'\hat{k}, \quad (19)$$

$$C_3\frac{\partial T'}{\partial t} - \varepsilon_p C_2\left(\frac{\Delta T}{d}\right)w' - \frac{\mu_o \chi_o H_o \delta}{T_o(1+\chi_o)}T'\frac{\partial}{\partial t}H_o\delta + \frac{\varepsilon_p \mu_o \chi_o H_o^2 \delta^2}{T_o}\left(\frac{\partial T'}{\partial t} - w'\frac{\Delta T}{d}\right) - \\ -\delta\mu_o\chi_o\left(\frac{\partial \phi'}{\partial z}\right)\frac{\partial}{\partial t}H_o - \mu_o\chi_o H_o\delta\frac{\partial}{\partial t}\left(\frac{\partial \phi'}{\partial z}\right) + \frac{\mu_o \chi_o^2 H_o^2 \Delta T}{T_o(1+\chi_o)d}\delta^2 w' = K_{T_z}\left[\eta\nabla_1^2 T' + \frac{\partial^2 T'}{\partial z^2}\right], \quad (20)$$

$$(1+\chi_o)\nabla^2\phi' - \left(\frac{\chi_o H_o \delta}{T_o}\right)\frac{\partial T'}{\partial z} = 0 \quad (21)$$

Here ϕ is the magnetic potential and $\vec{H} = \nabla\phi'$, $C_3 = \varepsilon_p C_2 + (1-\varepsilon_p)(\rho_o C)_s$, $C_2 = \rho_o C_{V,H}$, $\vec{v}' = (U', V', W')$. For the ferromagnetic fluid layer and Darcy-anisotropic permeable medium, the boundaries are assumed to be stress-free, isothermal the boundary conditions at $z = 0$ and $z = d$ are

$$W' = \frac{\partial^2 W'}{\partial z^2} = T' = \frac{\partial \phi'}{\partial z} = 0, \quad (22)$$

By operating curl twice on (19), we omit p' from it, and then we render the resulting equation and (19) – (21) dimensionless by setting

$$(x^*, y^*, z^*)d = (x', y', z'), T^* = \left(\frac{T'}{\Delta T}\right),$$
$$W^* = \left(\frac{C_2 d W'}{K_{T_z}}\right), t^* = \left(\frac{K_{T_z} t}{C_2 d^2}\right),$$
$$\phi^* = \left(\frac{(1+\chi_o)\phi'}{K_m \Delta T d}\right), \omega^* = \left(\frac{C_2 d^2 \omega'}{K_{T_z}}\right), \quad (23)$$

to obtain non-dimesnional equations as (on dropping '*' for simplicity),

$$\left(\frac{1}{Va}\frac{\partial}{\partial t}\nabla^2 + \nabla_1^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)W =$$
$$= [R + RM_1\psi^2]\nabla_1^2 T - RM_1\psi^2\frac{\partial}{\partial z}(\nabla_1^2\phi), \quad (24)$$

$$\lambda_p\frac{\partial T}{\partial t} - W + M_2\left(\frac{T_o\psi^2}{\chi_o(1+\chi_o)}\right)\left(\frac{\partial T}{\partial t} - W\right) +$$
$$+ \frac{M_2}{\varepsilon_p}\delta^2 W - M_2\frac{1}{H_o}\frac{\partial}{\partial t}H_o\delta\left(\frac{\partial\phi}{\partial z}\right) -$$
$$-M_2\left(\frac{\psi^2}{(1+\chi_o)}\right)\frac{\partial}{\partial t}\left(\frac{\partial\phi}{\partial z}\right) - M_2\frac{1}{H_o}T\frac{\partial}{\partial t}H_o\psi =$$
$$= \eta\nabla_1^2 T + \frac{\partial^2 T}{\partial z^2}, \quad (25)$$

$$\nabla^2\phi = \frac{\partial T}{\partial z}, \quad (26)$$

where, $\psi = (1 + \varepsilon\, Re\{e^{-i\omega t}\})$, ω is the frequency of modulation, $\nabla^2 = \nabla_1^2 + \frac{\partial^2}{\partial z^2}$ and $\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$. The dimensionless parameters are $Va = \frac{\varepsilon_p \gamma d^2}{K_z \kappa}$, the Vadasz number, $R = \frac{\beta g \Delta T d^3 K_z}{\gamma\kappa}$, the Darcy-Rayleigh number, $M_1 = \frac{\mu_o \chi_o \Delta T H_o^2}{T_o(1+\chi_o)^3 \beta \rho_o g d^3}$, the buoyancy-magnetization parameter, $RM_1 = \frac{\mu_o \chi_o^2 (\Delta T)^2 H_o^2 K_z}{\mu_f \kappa (1+\chi_o)^3}$, the magnetic Rayleigh number and $M_2 = \frac{\mu_o \chi_o^2 H_o^2}{C_2(1+\chi_o)T_o}$, the magnetization parameter, $\xi = \frac{K_x}{K_z}$ is the mechanical anisotropy parameter, $\eta = \frac{K_{T_x}}{K_{T_z}}$ is the thermal anisotropy parameter, $\kappa = \frac{K_1}{C_2}$, $\gamma = \frac{\mu_f}{\rho_o}$, $\lambda_p = \frac{C_3}{\varepsilon_p C_2}$.

The parameter M_2 is equivalent to the order of 10^{-6} [3]. Hence M_2 is omitted in further calculations. For simplicity λ_p and ε_p is assumed to be one. At $z = 0$ and $z = d$ the boundary condition (22) in the non-dimesnional form is given by

$$W = \frac{\partial^2 W}{\partial z^2} = T = \frac{\partial\phi}{\partial z} = 0 \quad (27)$$

After eliminating the coupling between (24) – (26) we obtain a single differential equation for the vertical component of velocity Was

$$L\nabla^2 W = R\nabla^2\nabla_1^2 W + RM_1\psi^2\nabla_1^4 W \quad (28)$$

where

$$L = \left(\frac{1}{Va}\frac{\partial}{\partial t}\nabla^2 + \nabla_1^2 + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\left(\frac{\partial}{\partial t} - \eta\nabla_1^2 - \frac{\partial^2}{\partial z^2}\right)$$

The boundary condition (27) in terms of the vertical component of velocity at $z = 0$ and $z = d$ become [32]

$$W = \frac{\partial^2 W}{\partial z^2} = \frac{\partial^4 W}{\partial z^4} = \frac{\partial^6 W}{\partial z^6} = 0 \quad (29)$$

Solution procedure

In view of small amplitude ($\varepsilon < 1$) assumption, we now seek the eigenfunctions W and eigenvalues R of (28) for a modulated magnetic field that is different from the constant magnetic field. The eigenfunction W and eigenvalue R should be a function of ε and they should be obtained for a given Vadasz number Va, buoyancy-magnetization parameter M_1, mechanical anisotropy parameter ξ, thermal anisotropy parameter η and frequency ω. Hence, we figured out (28) followed by the assumption of Venazian [33] of the form

$$\begin{pmatrix} W \\ R \end{pmatrix} = \begin{pmatrix} W_0 \\ R_0 \end{pmatrix} + \varepsilon \begin{pmatrix} W_1 \\ R_1 \end{pmatrix} + \varepsilon^2 \begin{pmatrix} W_2 \\ R_2 \end{pmatrix} + +\varepsilon^3 \begin{pmatrix} W_3 \\ R_3 \end{pmatrix} + \cdots\cdots\cdots \quad (30)$$

On substituting (30) over (28) and comparing the correlative terms up to order of ε^2, yields

$$GW_0 = 0, \quad (31)$$

$$GW_1 = R_1 \nabla^2 \nabla_1^2 W_0 + R_1 M_1 \nabla_1^4 W_0 + +2\, Re\{e^{-i\omega t}\} R_0 M_1 \nabla_1^4 W_0, \quad (32)$$

$$GW_2 = R_1 \nabla^2 \nabla_1^2 W_1 + R_2 \nabla^2 \nabla_1^2 W_0 + +R_1 M_1 \nabla_1^4 W_1 + R_2 M_1 \nabla_1^4 W_0$$

$$+2\, Re\{e^{-i\omega t}\} R_0 M_1 \nabla_1^4 W_1 + +2\, Re\{e^{-i\omega t}\} R_1 M_1 \nabla_1^4 W_0, \quad (33)$$

where

$$G = L\nabla^2 - R_0 \left[\frac{\partial^2}{\partial z^2} + (1 + M_1)\nabla_1^2 \right] \nabla_1^2$$

The function W_0 is the solution of unmodulated Rayeligh-Benard problem in ferromagnetic fluids [3]. The marginally stable solution for that problem is

$$W_0 = \left[e^{i(\alpha_x x + \alpha_y y)} \right] \sin \pi z, \quad (34)$$

corresponding to the lowest mode of convection with the Rayleigh number R_0 is given by

$$R_0 = \frac{\left(\frac{\pi^2}{\xi} + \alpha^2\right)(\pi^2 + \eta\alpha^2)(\pi^2 + \alpha^2)}{\alpha^2[\pi^2 + (1 + M_1)\alpha^2]}, \quad (35)$$

Following the analysis of [29, 33], one obtains the first non-zero correction to R_0

$$R_{2c} = \frac{R_0^2 M_1^2 \alpha^6}{[\pi^2 + (1 + M_1)\alpha^2]} \sum_{n=1}^{\infty} \frac{Q_n}{L_n}, \quad (36)$$

where

$$Q_n = -2 \begin{pmatrix} (\eta\alpha^2 + n^2\pi^2)(n^2\pi^2 + \alpha^2)\left(\frac{n^2\pi^2}{\xi} + \alpha^2\right) - R_0 \alpha^2 [n^2\pi^2 + (1 + M_1)\alpha^2] \\ -\omega^2 \frac{1}{Va}(n^2\pi^2 + \alpha^2)^2 \end{pmatrix}$$

$$L_n = \begin{pmatrix} (\eta\alpha^2 + n^2\pi^2)(n^2\pi^2 + \alpha^2)\left(\frac{n^2\pi^2}{\xi} + \alpha^2\right) - R_0 \alpha^2 [n^2\pi^2 + (1 + M_1)\alpha^2] \\ -\omega^2 \frac{1}{Va}(n^2\pi^2 + \alpha^2)^2 \end{pmatrix}^2 + \omega^2 \left(-\frac{1}{Va}(\eta\alpha^2 + n^2\pi^2)(n^2\pi^2 + \alpha^2)^2 - (n^2\pi^2 + \alpha^2)\left(\frac{n^2\pi^2}{\xi} + \alpha^2\right) \right)^2$$

Results and discussion

The outcome of time-periodic magnetic field fluctuation on the onset of ferroconvection in a horizontal anisotropic densely arranged permeable layer is investigated using the linear stability analysis, the analytical solution was accomplished by means of the standard normal mode approach proposed by Venezian [33]. The shift in the correction to the critical Rayleigh number R_{2c} equation is computed by means of the regular perturbation technique as a function of the modulated magnetic field frequency ω, magnetic parameter M_1, mechanical anisotropy parameter ξ, thermal anisotropy parameter η and Vadasz number Va and the results are depicted with the help of Figures 2 through 5. The stabilizing or destabilizing impact of magnetic field fluctuation is determined by the sign of R_{2c}. A positive R_{2c} means supercritical instability occurs while a negative R_{2c} means subcritical instability occurs, in contrast to system without time-varying magnetic field.

Figure 2(a) and 2(b) shows the variability of R_{2c} on ω and M_1 at $\xi = 0.7$, $\eta = 0.5$ and $Va = 5$. Among these figures it is obvious that an increament in M_1 augments the magnitude of the R_{2c}, provided ω is minimum (see fig. 2(a)), while moderate and large ω (see fig. 2(b)) decrements the magnitude of

R_{2c}. It is proved in Fig. 2(a) that at weak modulation frequency, $R_{2c} < 0$ signifying that the magnetic field modulation destabilizes the physical framework while from Fig. 2(b), it is clear that $R_{2c} > 0$ for moderate and strong frequency. It implies that R_{2c} stabilizes the framework of the problem.

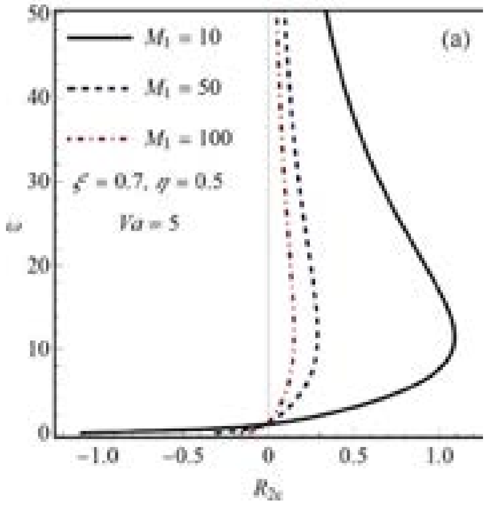

Figure 2(a) – Plot of small and moderate ω verses R_{2c} with variation in M_1.

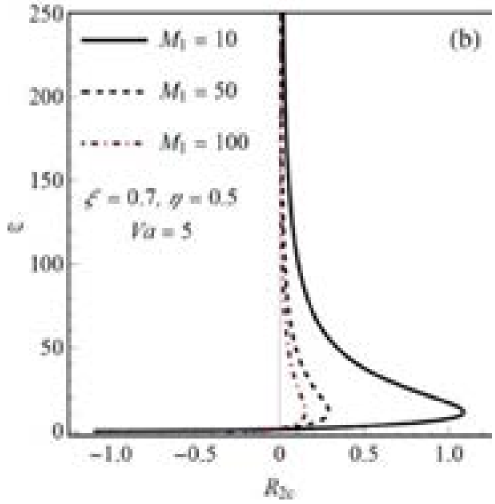

Figure 2(b) – Plot of large ω verses R_{2c} with variation in M_1.

The diversification of R_{2c} upon ω and Va for a specific term $\xi = 0.7$, $\eta = 0.5$ and $M_1 = 50$ is shown in Figures 3(a) and 3(b). We observe from this figures that as raising Va advances the range of R_{2c}. At $\omega = 10$, the peak point of R_{2c} spreads by enhancing Va. The force of Vadasz number Va on the steadiness of the mechanism is exactly opposite to M_1. The most notable outcome of the problem can be elucidated by exploring the outcomes of Figs. 2-3. Comparing the Vadasz number discrepancy with magnetic parameter, we reveal that the least value of R_{2c} is lower. This explicitly reveals that over M_1, the Vadasz number plays a vital role in augmenting ferroconvection and magnetic number is a crucial in postponing ferroconvection.

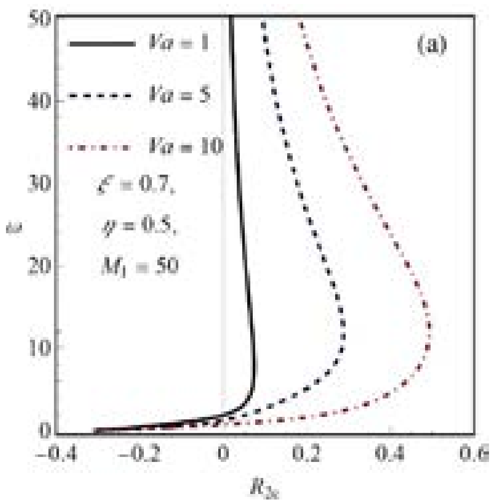

Figure 3(a) – Plot of small and moderate ω verses R_{2c} with variation in Va.

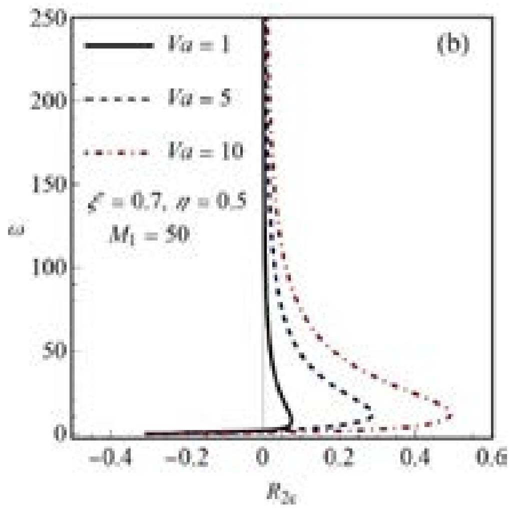

Figure 3(b) – Plot of large ω verses R_{2c} with variation in Va.

The force of mechanical anisotropy ξ on R_{2c} at $M_1 = 50$, $\eta = 0.5$ and $Va = 5$ is shown in Figures 4(a) and 4(b) for weak and moderately large ωrespectively. We note that a rise in the range of ξ results in a fall in the range of R_{2c}. This signifies that, the impact of growth in ξ, minimizes the outgrowth of time-varying magnetic field. It is meaningful to emphasize that at moderate and significant value of frequency, $\xi = 0.1,0.5,0.7$ experience a strong destabilizing influence. Conversely, at small value of ω, mechanical anisotropy $\xi = 0.1,0.5,0.7$minimizes the fluctuation impact of magnetic force.

The result of thermal anisotropy η is shown in 5 (a) and 5 (b) to elucidate the system's stableness for a fixed value of $M_1 = 50$, $\xi = 0.7$ and $Va = 5$. According to our observation, the large value of η delays the onset of convection as expected when η increases as a function of R_{2c}.

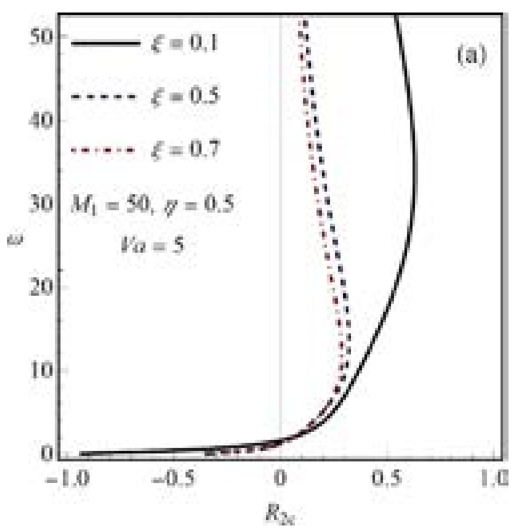

Figure 4(a) – Plot of small and moderate ω verses R_{2c} with variation in ξ.

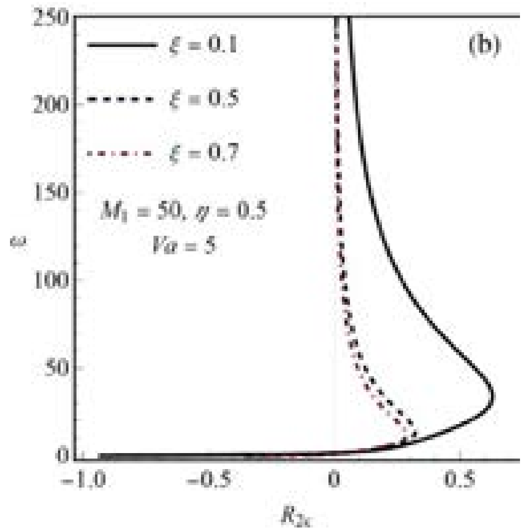

Figure 4(b) – Plot of large ω verses R_{2c} with variation in ξ.

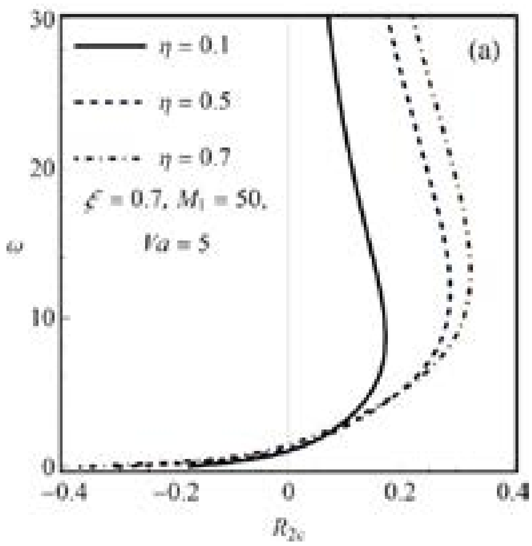

Figure 5(a) – Plot of small and moderate ω verses R_{2c} with variation in η.

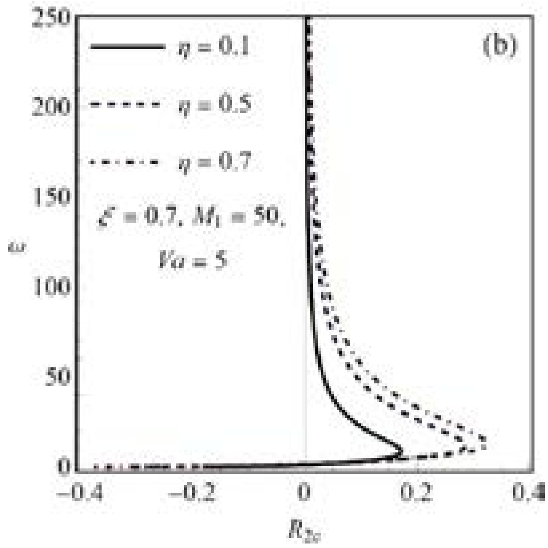

Figure 5(b) – Plot of large ω verses R_{2c} with variation in η.

Conclusions

The impact of magnetic field fluctuation on the advent of nanoscopic magnetic liquid advection in a thickly condensed anisotropic saturated permeable configuration is carefully elucidated adopting stability test and the succeeding conclusions are outlined:

- The weak frequency ω of magnetic field fluctuation is destabilizing while strong frequency of modulated magnetic field is continuously stabilizing.
- The effect of magnetic mechanism M_1 on magnetic field modulation is to stabilize at minute frequency and destabilize at balanced and strong frequency.
- The outcome of Vadasz number Va makes system stable expect for minute values of ω in the modulated magnetic field.
- At moderate and large ω, an increase in mechanical anisotropy ξ strengthens the impact of magnetic field fluctuation, whereas an increase in thermal anisotropy η weakens the impact of fluctuated magnetic field. However, at low ω, enhancing the parameters ξ and η gives opposite result of modulated magnetic field.
- The outcome of magnetic field fluctuation vanishes at high ω in each case.

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