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Static structure factor of macroparticles in dusty plasmas

Abstract. Equilibrium distribution functions are studied based on the previously proposed pseudopotential model of dust particles interaction in the plasma, which takes into account both the finite-size and the screening effects. Consideration is made in the framework of the renormalization theory of plasma particles interaction leading to the so-called generalized Poisson-Boltzmann equation. The main idea is to re-use the renormalization theory to treat the dust component of the plasma. Initially, a generalized Poisson-Boltzmann equation is used to determine characteristics of the interaction between two isolated dust particles. The interaction potential obtained in that way does not contain the number density of dust particles and can be utilized for further theoretical considerations. In particular, this paper re-uses the Poisson-Boltzmann equation to derive equilibrium distribution functions of dust particles. Such an approach allows one to obtain analytical expressions for the static structure factor of the dust particles. Non monotonic behavior of the static structure factor of the dust particles is observed at different values of plasma parameters, which may indicate the short-range or even long-range order formation in the system.

Keywords: dusty plasma, pseudopotential model, generalized Poisson-Boltzmann equation, the renormalization theory of plasma particles interaction, static structure factor.

Introduction

At present investigations of dusty plasma properties are of great interest from the viewpoint of scientific and practical applications. Besides buffer plasmas particles such a plasma contains conductive or dielectric macro-sized particles, called dust grains. Under a variety of external conditions they are readily arranged in an ordered structure called a dusty plasma crystal [1]. Such a unique plasma state is characterized by long-lasting localization of grains at some quazi-lattice points [2-4].

It is believed that processes taking place in dusty plasmas may play a fundamental role in explanations of the current structure of the Universe including galaxies. Dust particles are formed in many plasma devices as a result of interaction of the buffer plasma particles with walls. They can also appear, for instance, in experiments intended for implementation of controlled thermonuclear fusion and have a significant effect on physical and chemical properties of walls material due to interaction with the near-surface layer plasma.

Being immersed in a buffer plasma, macroscopic dusty particles rapidly acquire very high negative charge caused by a high-mobility of electrons they absorb [5, 6]. Thus, the electric charge of dusty particles might reach hundreds or even thousands of the elementary charge which results in the occurrence of strongly coupling effects caused by interparticle interactions. This means that to correctly describe microscopic and macroscopic properties of dusty plasmas it is very important to establish an exact form of the interaction potential of dust grains with each other and with the buffer plasma particles as well [7-9].It should be noted that the Yukawa type potential is often used for this purpose [10-12].

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Dimensionless plasma parameters

In the following we consider the interaction between two dust grains placed into two-component hydrogen plasma consisting of free electrons with the electric charge $-e$ and the number density n_e and of free protons with the electric charge e and the number density $n_p = n_e = n$. It is assumed that macroparticles are simply solid spheres with the radius R and the electric charge $-Z_d e$, where Z_d stands for the charge number of dust particles.

To describe the state of the buffer plasma we introduce the effective coupling parameter defined as

$$\Gamma_R = \frac{e^2}{R k_B T}, \qquad (1)$$

where k_B is the Boltzmann constant, T denotes the plasma temperature.

Coupling parameter (1) is not conventional and represents the ratio of the Coulombic interaction energy of two electrons located at a distance R from each other to their average kinetic energy of chaotic thermal motion.

The dimensionless screening parameter is also introduced as

$$\kappa = \frac{R}{\lambda_D}, \qquad (2)$$

where $\lambda_D = \sqrt{k_B T / 8\pi n e^2}$ designates the Debye screening radius.

Knowledge of the charge number of dust particles Z_d and the dimensionless parameters (1) and (2) is perfectly enough to describe the interaction of two isolated hard spheres in a buffer plasma.

The coupling parameter for the dust particles Γ_D is related to the above defined effective coupling parameter Γ_R as follows

$$\Gamma_D = \frac{Z_d^2 e^2}{a_d k_B T} = \frac{Z_d^2 \Gamma_R}{D}, \qquad (3)$$

where the new dimensionless parameter $D = a_d / R$ represents the ratio of the average distance between the dust particles $a_d = (3/4\pi n_d)^{1/3}$ to their radius, and n_d is the number density of dusty particles.

Interaction model of two isolated macroparticles

The Coulomb potential is used as interaction micropotential of charged particles in the buffer plasma

$$\varphi_{ee}(r) = \varphi_{pp}(r) = -\varphi_{ep}(r) = \frac{e^2}{r}. \qquad (4)$$

In much the same way the micropotential of dusty component interaction is defined

$$\varphi_{ed}(r) = -\varphi_{pd}(r) = \frac{Z_d e^2}{r}, \quad \varphi_{dd}(r) = \frac{Z_d^2 e^2}{r}. \qquad (5)$$

It should be mentioned that micropotentials (4) and (5) are infinite at $r \to 0$, and sufficiently slowly decrease with distance which results in well-known difficulties in theoretical description of plasma properties.

To treat the finite size effects of dusty particles we make use of the transform

$$\varphi_{(p,e)d}(r) \to \varphi_{(p,e)d}(r+R),$$

$\varphi_{dd}(r) \to \varphi_{dd}(r+2R)$ in equation (5) to obtain:

$$\varphi_{ed}(r) = -\varphi_{pd}(r) = \frac{Z_d e^2}{r+R}, \quad \varphi_{dd}(r) = \frac{Z_d^2 e^2}{r+2R}. \qquad (6)$$

It is evident that such a transform simply eliminates reciprocal penetration of dust grains and penetration of the buffer plasma electrons and ions into the dusty particles either. The penetration of the buffer plasma particles into the dust grains leads to the changing their electric charge considered by introducing Z_d.

It is worth noticing that in contrast with the behavior of Coulomb potential expressions (6) and (7) are finite at $r \to 0$.

The Fourier transform of the Coulomb micropotential (4) is found as:

$$\tilde{\varphi}_{ee}(k) = \tilde{\varphi}_{pp}(k) = -\tilde{\varphi}_{ep}(k) = \frac{4\pi e^2}{k^2}. \qquad (7)$$

The Fourier transforms of the interaction micropotential of the charged plasma particles, i.e. protons and electrons, with the dusty particles and the latter with each other (6) are written as:

$$\tilde{\varphi}_{pd}(k) = -\tilde{\varphi}_{ed}(k) = -\frac{4\pi Z_d e^2}{k^2} + \frac{4\pi Z_d e^2 R}{k}\left[\mathrm{Ci}(kR)\sin(kR) + \frac{1}{2}\cos(kR)(\pi - 2\,\mathrm{Si}(kR))\right], \quad (8)$$

$$\tilde{\varphi}_{dd}(k) = \frac{4\pi Z_d^2 e^2}{k^2} - \frac{8\pi Z_d^2 e^2 R}{k}\left[\mathrm{Ci}(2kR)\sin(2kR) + \frac{1}{2}\cos(2kR)(\pi - 2\,\mathrm{Si}(2kR))\right]. \quad (9)$$

where, $\mathrm{Ci}(x) = -\int_x^\infty \frac{\cos t}{t}dt$ и $\mathrm{Si}(x) = \int_0^x \frac{\sin t}{t}dt$ are integral cosine and sine-functions of x, respectively.

In [13] there normalization theory of plasma particles interaction was put forward leading to the so-called generalized Poisson-Boltzmann equation of the form:

$$\Delta_i \Phi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b) = \Delta_i \varphi_{ab}(\mathbf{r}_i^a, \mathbf{r}_j^b) - \sum_{c=e,p} \frac{n_c}{k_B T} \int \Delta_i \varphi_{ac}(\mathbf{r}_i^a, \mathbf{r}_k^c)\Phi_{cb}(\mathbf{r}_j^b, \mathbf{r}_k^c) d\mathbf{r}_k^c, \quad (10)$$

where, n_c is the number density of particles of species c. Notice that in (10) the summation is implied over the buffer plasma electrons and ions only, i.e. $c = e, p$, the dusty particles number density is considered to be zero since we are interested in studying the interaction of the two isolated grains.

It is important to mention that the generalized Poisson-Boltzmann equation can strictly be obtained from the Bogolyubov hierarchy for equilibrium distribution functions in the pair correlation approximation [13]. It was successfully applied to various types of plasmas, such as quasiclassical [14-16], partially ionized [17, 18] and even dusty plasmas in the Debye approximation [19].

Equation (10) represents the relation that determines the pseudopotential Φ_{ab} in terms of the microscopic potential φ_{ab}. One can see the pseudopotential takes into account collective effects since it contains the number densities of various species.

In the Fourier space set of equations (10) turns into a set of linear algebraic equations whose solution is found as

$$\tilde{\Phi}_{dd}(\mathbf{k}) = \frac{\tilde{\varphi}_{dd}(k) - 2A\tilde{\varphi}_{ed}^2(k)(1 + A\tilde{\varphi}_{ee}(k)) + 2A\tilde{\varphi}_{ee}(k)(\tilde{\varphi}_{dd}(k) + A\tilde{\varphi}_{ed}^2(k))}{1 + 2A\tilde{\varphi}_{ee}(k)} \quad (11)$$

where, $A = n / k_B T$.

The interaction pseudopotential of macroparticles in configurational space is calculated from equation (11) by using the backward Fourier transform

$$\Phi_{dd}(\mathbf{r}) = \int \tilde{\Phi}_{dd}(\mathbf{k})\exp(-i\mathbf{kr})d\mathbf{k}. \quad (12)$$

Static structure factor

Pseudopotential (12) does not include the dusty particles number density because it represents the interaction of two isolated grains and the screening effect is realized by electrons and ions of the buffer plasma. This makes it possible to use that pseudopotential in well approved theoretical approaches and computer simulations of the dust component. In particular, it is reasonable to repeatedly use there-normalization theory which in this case yields the following generalized Poisson-Boltzmann equation:

$$\Delta_i \Psi_{dd}(\mathbf{r}_i,\mathbf{r}_j) = \Delta_i \Phi_{dd}(\mathbf{r}_i,\mathbf{r}_j) - \frac{n_d}{k_B T}\int \Delta_i \Phi_{dd}(\mathbf{r}_i,\mathbf{r}_k)\Psi_{dd}(\mathbf{r}_j,\mathbf{r}_k)d\mathbf{r}_k \,, \tag{13}$$

where, Ψ_{dd} stands for the interaction pseudopotential taking into consideration collective events in the dust particles interaction.

The solution of (13) in the Fourier space takes the next form:

$$\tilde{\Psi}_{dd}(\mathbf{k}) = \frac{\tilde{\Phi}_{dd}(\mathbf{k})}{1+\frac{n_d}{k_B T}\tilde{\Phi}_{dd}(\mathbf{k})} \tag{14}$$

It was shown in [13] that the static structure factor S_{dd} (k) of the dusty particles is expressed by (14) as follows

$$S_{dd}(\mathbf{k}) = 1 - \frac{n_d}{k_B T}\tilde{\Psi}_{dd}(\mathbf{k}) = \frac{1}{1+\frac{n_d}{k_B T}\tilde{\Phi}_{dd}(\mathbf{k})} \tag{15}$$

Thus, important analytical formula (15) has been obtained for static structure factor S_{dd} (k) of the dusty particles. Figures 1 and 2 show the corresponding dependence at various values of plasma parameters.

It is seen that an increase in the dimensionless screening parameter *k* leads to only slight quantitative change in the static structure factor of the dusty particles whereas a decrease of the parameter *D* provides the dramatic influence on the quantitative behavior of the static structure factor of dusty particles. All these can be attributed to the coupling parameter for the dust particles (3) which strongly depends on the parameter *D* and remains practically unaffected by the parameter *k*.

Figure 3 displays the three-dimensional dependence of the static structure factor both on the wave number and the effective coupling parameter whose increase leads to strengthening of correlations in the dusty particles subsystem which is again due to the change in the coupling parameter for the dust particles (3).

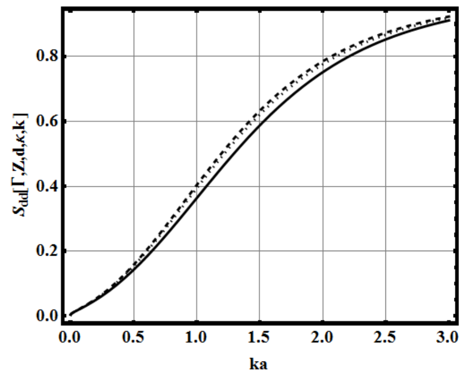

Figure 1 – Static structure factor against the wave number at Γ_R=0.1, Z_d=100, and *D*=5. Solid line: *k*=3; dotted line: *k*=5; dashed line: *k*=7.

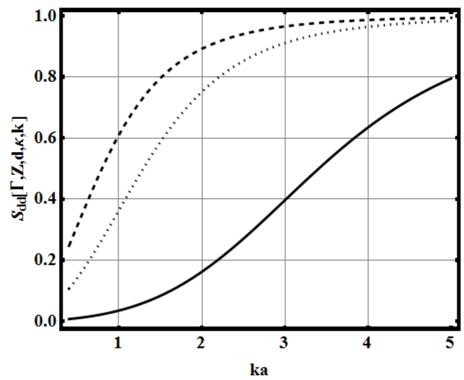

Figure 2 – Static structure factor against the wave number at Γ_R=0.1, Z_d=100, and *k*=3. Solid line: *D*=2; dotted line: *D*=5; dashed line: *D*=7.

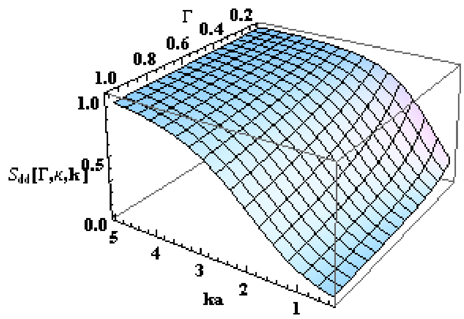

Figure 3 – Static structure factor against the wave number and the effective coupling parameter at *Zd*=100, *k*=1.5, and *D*=7.

Conclusions

In this paper an analytical expression has been presented for the static structure factor of the dusty particles and its behavior has been studied at various values of plasma parameters. An increase in both the screening parameter and the effective coupling parameter leads to strengthening of correlations in the dusty particles subsystem which can be interpreted as the short-range order formation.

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