IRSTI 41.17.41 https://doi.org/10.26577/ijmph.2022.v13.i2.01

Namrata I. Jain

Maharshi Dayanand College of Arts, Science and Commerce, Mumbai, India
e-mail: namrata.jain.jain2@gmail.com
(Received 18 May 2022; received in revised form 2 December 2022; accepted 15 December 2022)

The early universe scenario in FRW and K-K models: A study

Abstract. The early Universe has been a subject of research for many cosmologists and it is reviewed and analyzed rigorously in cosmology recently. Thermal history and theory of phase transition coordinate us to elucidate the evolution of the Universe at its early stages and is interesting to get a gist of early universe behavior. At phase transition, effective potential plays a significant role for the kinds of transition that occurred during the evolution. The temperature, at which phase transition occurs, can be determined at minimum effective potential. It is also known fact that temperature of the Universe has changed considerably with its evolution. The present work investigates the time-temperature relation in four- dimensional and five-dimensional cosmological models. A comparison of time- temperature relation in FRW model with that of Kaluza-Klein (K-K) model demonstrates that temperature decreases faster in Kaluza-Klein model. The investigation demonstrates the important role played by extra dimension in the study of time-temperature relation at the early Universe scenario.
Keywords: The Early Universe, Phase transition, effective potential, FRW model, Kaluza-Klein cosmology.

Introduction

Man is always curious about secrets of the Universe. Birth of the Universe, Origin, its behavior at early stages are long standing issues which are still being explored by researchers. In this regard, major revolution is caused by Big-Bang theory which is the most successful yet incomplete, as it is unable to explain certain features of the Universe, such as, presence of Dark Matter, Dark Energy, Large Scale Structure, Accelerated Expansion etc. Emergence of the universe from nothing has been an amazing situation. In this regard, Lamaitre has proposed 'hypothesis of primeval atom' in 1927 [1]. The theory has also predicted that just after 10^{-37} second when the temperature and pressure had been very high enough to cause cosmic inflation [2]. The Universe was in the soup of matter and radiation which were in thermal equilibrium with each other. Thermal history of the Universe revealed that it had undergone several phase transitions during its evolutions. In this regard, a detail information with deep theoretical analysis have been provided by the published literature [3 – 5]. It is interesting to note that theoretical analysis of thermal history have been done by mechanism of Higg's boson [3] which has been discovered recently [4]. Weinberg [5] has ingeniously elucidated various stages of phase transitions at the early era of the universe.

First order phase transition is similar to process of thermal equilibrium at bubble walls [6]. If pressure difference across the bubble wall is different than bubble wall can be broken and energy is released which can be in other forms. Consequences of First order transition give rise to formation of domain walls, generation of gravitational waves and other certain topological defects. Cosmic strings are also topological defects which came into existence before the Electro – Weak Transition.

Second Phase transition [7] occurred which resulted in Quark-Gluon -Plasma State at 10^{-6} sec. After Big-Bang, although temperature was not so high, pair production and annihilation was happening that resulted into production of quarks and leptons. Today quarks and leptons are basic building blocks for elementary particles. Baryogenesis [8], i.e. generation of Baryons continued with the evolution of the Universe. It has also seen that Baryogenesis even violated the conservation of baryon number in the process of matter creation so as to have some structure for the Universe. Although matter creation at early stages of the Universe sounds to be appealing but it is found that the natur has preferred matter generation over antimatter creation. In the Universe, matter-antimatter ratio is unequal. This Problem is yet to be solved.

With the time evolution of the Universe, due to pair annihilation it was filled with photons, neutrinos, electrons, protons. The Second Order phase transitions are also called crossover transition that took place at time 10^6 sec where the temperature is about 1 GeV. As Universe expanded, it cooled down so the quarks- Hadron interaction resulted into its appearance in bound form in baryons and mesons. Before this transition, quarks were free to move in space which is called as Quark -Gluon plasma state [9]. The impact of the transition on quarks is to interact in such a way that led to the formations of baryons and mesons. In this way, they became building blocks for Hadrons or baryons. In other words, Second Order Transition led to confinement of Quarks, so also the formation of Hadrons. The inflation caused the Universe to expand continuously. As a result, the temperature of the Universe has fallen to several Kelvin. It had been predicted by several workers [10, 11, 12] that the Universe at very early stages was initially anisotropic, later due to its expansion it becomes isotropic.

There are several effects of the phase transitions in the normal matter. The major effect of the Phase transition is the symmetry breaking. In normal matter, it is observed that water is more symmetric than ice, steam is more symmetric than water. From here, we also observe that symmetry of the matter is related to the temperature. There is more symmetry for high temperature. Thus, for the Universe, a fraction of second after its birth, it was highly symmetric as its temperature was very high. Phase transition is well understood from thermal history of the Universe. Evolution of the Universe with its thermal history has been discussed by Prokopec [13]. Besides phase transition, attempt has been made to unite all forces as it is believed that just before the Big-Bang all forces except gravity are together as per Grand Unified Theory (GUT) which had been first forth by Guth [14-15]. Weinberg [16] has indigenously illustrated these forces which had been later put together as standard model in particle physics. Later on Kaluza in 1921[17] and Klein in 1926 [18] have made indigenous effort for it and propounded Kaluza-Klein theory which proved a milestone for the further development in the early Universe cosmology.

Thermal history successfully depicts evolution of the Universe [19] but the cause of phase transition cannot be understood by it. Theoretical aspects of phase transition have been explained by Toy model [20, 21] which illuminates phase transition by inclusion of effective potential. The theory of phase transition [19-21] provides the relation between effective potential and critical temperature and it has been given by

$$V_{eff}(\varphi,T)=\frac{\lambda}{4}\varphi^4+\frac{g^3T}{4\pi}\varphi^3+ \\ +\frac{1}{2}\left[\left(\frac{\lambda}{3}+\frac{g^2}{4}\right)T^2-\lambda v^2\right]\varphi^2 \qquad (1)$$

where λ, v, are constants. g is the charge, φ is gauge potential and T is the temperature. It is also found that effective potential is minimized at critical temperature. There are three minima of potentials observed at different temperatures which are termed as critical temperatures. The order of phase transition depends upon these temperatures. As discussed previously, structure, geometry and present universe scenario are the consequences of phase transitions. The effective potential and critical temperature can be obtained from Friedmann-Robertson-Walker (FRW) model also. The FRW model is also called as steady state model under certain conditions.

Big Bang model had been challenged by 'Steady state model' (FRW) [22]. The Steady state model of the Universe explained the Universe and its behavior with the help of 'Cosmological Principle' which is termed as perfect Cosmological principle. The expansion of the Universe, its isotropic nature, matter creation etc. are well explained by the steady state model. However certain observations created a major setback to steady state model. In this regard, Big-Bang model is proved to be more successful than Steady state model. The observations by Deep space radio telescope indicated that the Universe at its early stages was quite different than the present Universe [23, 24]. Cosmic microwave background for the Universe as per COBE satellite [25, 26] was inferred by which the Universe appeared to be permeated uniformly by Cosmic radiation in background, thus, it appears to be isotropic in present time. Apart from Big-Bang model, higher dimension theory as a consequence of string theory in the field of Cosmology brought a revolution which can be mainly applied in the study of the early Universe [27,28]. Many researchers worked on Kaluza-Klein theory [29-31] and set up the model so as to study the early universe.

Motivated with the above discussion, time-temperature relations have been obtained in Friedmann Robertson Walker (FRW) and Kaluza-Klein (KK) cosmological models in this paper. We have also studied effective potential for four dimensional and five dimensional models. It is observed that with evolution, temperature is decreased. The present study also emphasizes the implication of extra dimension on time temperature.

The organization of the paper is in five sections. We began with Introduction in first section followed by discussion on determination of time-temperature relation in FRW metric in section 2 and in K-K model in section 3. Section 4 comprises of discussion followed by conclusion in the 5th section.

Time-Temperature relation in FRW metric

Friedmann derived field equations in 1922 with the help of Einstein's general theory of relativity in the context of expansion of the universe, assuming homogeneous and isotropic. Universe. These equations are called as Friedmann equations. In 1926 Lemaitre independently carried out the similar work explaining expansion of the universe. Later in 1930s, Robertson and Walker independently obtained metric equation in order to explain complete geometrical properties of the universe. Using Robertson-Walker metric, the Einstein Field Equations (EFE) have been derived which resembled the equations obtained by Friedmann, The cosmological model representing Friedmann equations thus named as Friedmann-Lemaitre-Robertson-Walker cosmological model (FLRW), or FRW model in more general form. The FRW model is also called standard model as it can explain most of the features of the universe as discussed in previous section. Here, let us have a glance at FRW model for the study of time-temperature relation, which will enable us to set up a new model in higher dimension, to be discussed in next section.

Consider FRW metric [32,33] as given below,

$$ds^2 = -dt^2 + \\ +R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2\right] \quad (2)$$

Let $x^0 = -t$, $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$ are space –time coordinates. In a curved space time, the line element is given as, $ds^2 = g_{ij}dx^i dx^j$ where g_{ij} is a 4×4 metric tensor. k is a curvature constant, $k = 0$ for flat universe, $k = 1$ for closed universe and $k = -1$ for open universe. The Einstein field equations can be obtained from the following Eq. (3)

$$R_j^i - \frac{1}{2}Rg_j^i = -8\pi G T_j^i + \Lambda g_j^i \quad (3)$$

R^i_j – Ricci tensor, R – Ricci scalar, T^i_j – nergy-momentum tensor, Λ – Cosmological constant and G – gravitational constant. Field Equations (Eq. 3) can also be written as,

$$G_j^i = -\frac{8\pi G}{c^2}T_j^i + \Lambda g_j^i \quad (4)$$

where $G_j^i = R_j^i - \frac{1}{2}Rg_j^i$.

Energy momentum tensor T^i_j is represented as given below.

$$T_j^i = (p+\rho)u^i u_j + g_j^i p \quad (5)$$

$$u_i = \frac{dx_i}{dt}$$

is the 4 – vector velocity component such that $\mathbf{u}^i\mathbf{u}_j = -1$, when $i = 0,1,2,3$ (space-time coordinates); p and ρ are pressure and density of matter distribution of the universe, respectively. Hence, from above equation, the energy momentum tensor is given by $T^i_j = (-\rho, p, p, p)$. We have assumed $\hbar = c = 1$ for deriving field equations in accordance with the cosmological principle. Field equations for FRW cosmological model are obtained by solving Eqs (2-5) as follows,

$$3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} = 8\pi G\rho + \Lambda \quad (6)$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} = -8\pi G p + \Lambda \quad (7)$$

Let $H = \frac{\dot{R}}{R}$ then Eq. (6) and (7) are modified as:

$$3H^2 + 3\frac{k}{R^2} = 8\pi G\rho + \Lambda \quad (8)$$

$$(1-2q)H^2 + 3\frac{k}{R^2} = -8\pi G p + \Lambda \quad (9)$$

In Eq. (9), q is deceleration parameter defined as,

$$q = -\left|\frac{R\ddot{R}}{\dot{R}^2}\right|$$

Here Λ the Cosmological constant plays a significant role in the study of early Universe as it represents Vacuum Energy. At the early stage of the Universe k can be taken zero (k = 0) i.e. the universe at its early stage can be assumed to be flat. In order to find T_C, Λ is assumed to be constant factor at the early universe although it is not really constant which was inferred through the observations recently [34, 35]. To determine T_C, we consider energy conservation $T^i_{j;j} = 0$ which gives the following equation.

$$\dot{\rho} + (\rho + p)3\mathrm{H} = 0 \qquad (10)$$

Substituting k=0 and constant Λ in equation (4) and solving it with Eq. (10) we get,

$$\frac{d}{dR}(\rho R^3) + 3pR^2 = 0 \qquad (11)$$

For radiation dominated Universe , p = 1/3ρ, substituting p in above equation and solving it, we get $\rho \propto R^{-4}$. let us consider radiation density ρ = U for past epoch in the early era. The radiation density for past epoch R is given by $U = U_0 \frac{R_0^4}{R^4}$ (where U_0, R_0 are the initial radiation energy density and initial epoch at t=0 respectively.

If we consider the early universe as perfect black body then Energy density in the perfect black body is given by

U= σT^4 ,where σ is radiation constant and T is the Temperature of it . In this situation we obtain $= \frac{K}{R}$, where K is constant depending upon σ and constant of proportionality. At very early epoch we neglect Λ as compared to very high temperature. Substituting ρ = U =σT^4 , Eq.(6) is rewritten as,

$$\frac{\dot{R}^2}{R^2} = \frac{8\pi G\sigma T^4}{3} \qquad (12)$$

On substitution of *T = A/R* and assuming at t=0 R=0,

$$R = A\left(\frac{3}{32\pi G\sigma}\right)^{\frac{-1}{4}}\left(t^{\frac{1}{2}}\right) \qquad (13)$$

So,

$$T = A\left(\frac{3}{32\pi G\sigma}\right)^{\frac{1}{4}}\left(t^{-\frac{1}{2}}\right) \qquad (14)$$

Above equation gives direct relation between T and t. It is known to us that at early stage of the Universe, the particles are relativistic and it is assumed to behave as relativistic gas at high temperature. If particle interaction is slower than expansion rate H [20,21], density is modified as

$$\rho = \frac{\pi^2}{30} g^* T^4 \qquad (15)$$

From Eq. (8) considering k=0 and neglecting Λ , we write Eq. (8) as

$$3H^2 = 8\pi G\rho(T) \qquad (16)$$

Assuming Plank's mass $M_{pl} = \frac{1}{\sqrt{8\pi G}}$

$$H = \frac{\dot{R}}{R} = \left(\frac{\pi^2}{90}\right)^{\frac{1}{2}} (g^*)^{\frac{1}{2}} \frac{T^2}{M_{pl}^2} \qquad (17)$$

As found earlier T∝ 1/R and we can get an expression,

$$R\dot{R} = \left(\frac{\pi^2}{90}\right)^{\frac{1}{2}} (g^*)^{\frac{1}{2}} \frac{1}{8\pi G} \qquad (18)$$

Assuming 8πG =1, integrating and rearranging the terms in above equation, we get,

Solving above equation we obtain the following relation between time and temperature

$$t = \sqrt{\frac{90}{\pi^2}\frac{1}{\sqrt{g^*}}}\left(\frac{1}{T^2}\right) \qquad (19)$$

In above equation, g^* is number of degrees of freedom of the particles.

Direct information of the critical temperature could not be revealed from Eq.(14) and (19), but Calculations of T_C can be made simple by knowing g* at the phase transitions. Consider the GUT transition, $g^* = g_b + g_f$ where g_b and g_f are internal degrees of freedom for bosons and fermions respectively. From the Particle Data group [36,37] g^* is calculated as 106.75 at the time of GUT transition while g^* = 17.25 at QCD transition. For different g^* at the phase transitions, temperature T_C's are calculated which are different at GUT, electroweak and QCD transitions. The relation (14) is not obeyed strictly at the transitions as it represents continuous t-T variations. In the next section, we obtain the relation between time and temperature for Kaluza-Klein cosmological model.

Time – Temperature relation in Kaluza- Klein Cosmological model

The Kaluza-Klein metric is the 5*D* FRW metric i.e. four dimensional FRW metric with an extra dimension [38,39] which is as given below:

$$ds^2 = -dt^2 + +R^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\theta^2 + r^2 sin^2\theta d\varphi^2\right] + +A^2(t)d\psi^2 \quad (20)$$

where, k is a curvature constant equal to 0, 1 , -1 for flat , closed and open universe, respectively. $R(t)$ and $A(t)$ are fourth and fifth dimensional scale factors. The five-dimensional coordinates in above equation are given by $x^0 = t$, and $x^1, x^2, x^3, x^4 = r, \theta, \varphi, \psi$, respectively. Following the Eqs. (3-5) for Kaluza-Klein metric, the field equations are obtained as given below.

$$3\frac{\dot{R}^2}{R^2} + 3\frac{\dot{R}\dot{A}}{RA} + 3\frac{k}{R^2} = 8\pi G\rho + \Lambda \quad (21)$$

$$2\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{k}{R^2} + 2\frac{\dot{R}\dot{A}}{RA} + \frac{\ddot{A}}{A} = -8\pi Gp + \Lambda \quad (22)$$

$$3\frac{\ddot{R}}{R} + 3\frac{\dot{R}^2}{R^2} + 3\frac{k}{R^2} = -8\pi Gp + \Lambda \quad (23)$$

In above field equations A is the extra dimension. For early Universe k can be taken as zero as The Universe is almost flat at very early Universe and assuming Λ is constant, Field equations can easily be solved to get the relation between R and A. In order to get an expression for density, we consider five dimensional energy momentum tensor as $T^i_j = (-\rho, p, p, p, p)$. Energy conservation relation hence obtained as given below.

$$\dot{\rho} + (\rho + p)\left(3\frac{\dot{R}}{R} + \frac{\dot{A}}{A}\right) = 0 \quad (24)$$

As per Cosmological Principal the Universe is filled with perfect fluid so Equation of State for it is given by p= (γ-1) ρ so we get an expression as follows

$$\rho = \rho_0 R^{-3\gamma} A^{-\gamma} \quad (25)$$

Here, we assume ansatz $A = R^n$ ('n' is constant) [The universe is anisotropic at its early stages, so $\sigma^2 \propto \theta$, where σ is shear scalar and θ is expansion scalar. Due to this the metric potentials are related by power relations [40]]. Substituting A in Eq. (25), we get,

$$\rho = \rho_0 R^{-\gamma(n+3)} \quad (26)$$

In this case Temperature dependence on scale will be given by

$$T = T_0 R^{-\frac{\gamma(n+3)}{4}} \quad (27)$$

[[$T_0 = \left(\frac{\rho_0}{\sigma}\right)^{1/4}$], σ is the Stefan's constant.]

Assuming 8πG =1 and neglecting Λ as compared to density and pressure of the universe, Solving field equations (21) – (23) we get,

$$R = d\left[\frac{\gamma(n+3}{2}t - C\right]^{\frac{2}{\gamma(n+3)}} \quad (28)$$

If we assume initial conditions i.e. at t = t_0, R= R_0, H=H_0 we obtain

$$C = \frac{\gamma(n+3)}{2}t_0 - \frac{1}{H_0},$$

$$d = (H_0)^{\frac{2}{\gamma(n+3)}} R_0$$

So time-temperature relation can be obtained as,

$$T = T_0\left[\frac{\gamma(n+3)}{2}H_0(t - t_0) + 1\right]^{-\frac{2}{\gamma(n+3)}} \quad (29)$$

From above equation, temperatures at Radiation dominated phase, matter dominated phase can be determined.

Discussion

Comparing Eq. (14) and Eq. (29) it is observed that in FRW model $T \propto t^{-1/2}$ while in K-K cosmological model, temperature of the universe depends upon *n* which is an index factor for extra dimension. Since early Universe is supposed to be in radiation dominated phase, so, for radiation dominated phase time-temperature expression depicts dependency of temperature on time. To determine it, consider γ = 4/3 as universe is radiation dominated at its early stage, therefore equation (23) is modified in the following form.

$$T = T_0\left[\frac{3(n+3)}{2}H_0(t-t_0)+1\right]^{-\frac{3}{2(n+3)}} \quad (30)$$

Irrespective of constants in equations (14) and (29), it is observed that temperature in five dimensional model is lower than that four dimensional model. The temperature in FRW model is proportional to $t^{-1/2}$ while it is proportional to $t^{-3/2(n+3)}$. This can be due to the presence of extra dimension in the early Universe. We also observe that T depends upon γ. Hence at different phases, temperatures of the universe can be calculated. Although above expression explains Time-temperature relation in higher dimension but it does not provide any clue to find critical temperature at the phase transition. To determine critical temperature in five-dimensional Universe, it is necessary to account number of degrees of freedom in five-dimension for both Fermions and Bosons which were supposed to be major Constituents at early stage of the Universe. The work by Dienes et.al, Emel'Yanov [37, 41] had explained the implications of extra dimension for t-T relation in higher dimension by calculating effective potential for four as well as in five-dimensional physics. They have shown that effective potential in four-dimensional has been quite different than that of five-dimensional models. Consequently, critical temperature in five dimensional model has been modified and can be compared with that of in five-dimensional model. Critical temperatures in four-dimensional model and in five-dimensional model have been obtained [41]as:

$$(T_C)_{D=4} = 2\frac{\mu}{\sqrt{\lambda}} \quad (31)$$

and,

$$(T_C)_{D=5} = \left(\frac{2\pi}{3\zeta(3)}\frac{\mu^2}{\lambda r}\right)^{\frac{1}{3}} \quad (32)$$

In above equations μ, and λ are the bare mass term for fermions and bosons and coefficient of quadratic quantum field associated with effective potential respectively. Above equations clearly demonstrates the difference in critical temperatures in four and five dimensional physics.

Conclusions

The Universe at early stages had gone through several phases. Phase transition and Critical temperature at different phases in four as well as five dimensional model are obtained and compared in this paper. It is observed that temperature decreases faster with time in Kaluza-Klein model. The extra dimension plays a very important role during phase transition. It is also observed that that constant lambda Λ (cosmological constant) does not affect time-temperature relation as such at the early stage of the universe. It is well known that the present Universe is expanding and accelerating, so, determination of time – temperature relation for Kaluza-Klein cosmological model with variable cosmological constant can reveal the present universe scenario.

Acknowledgements

My sincere gratitude to anonymous referee for providing me their valuable suggestions for improvement of this manuscript.

References

1. Lemaître, G. "Un univers homogène de masse constante et de rayon croissant, rendant compte de la vitesse radiale des nébuleuses extra-galactiques". *Ann. Soc. Sci. Bruxelles, série A*, (1927): 47–49.
(New English translation and Editorial Note in J.-P. Luminet, *Gen. Rel. Grav*, (2013); 45, 1619.
2. Lamaitre, G. "A Homogeneous Universe of Constant Mass and Increasing Radius accounting for the Radial Velocity of Extra-galactic Nebulæ", *M.N.R.A.S.* (1931): 91, 483
3. Kapusta, J, I, and Gale, C. *Finite-Temperature Field Theory*, Cambridge University Press (2006).
4. "ATLAS; CMS Collaborations, Measurements of the Higgs boson production and decay rates and constraints on its couplings from a combined ATLAS and CMS analysis of the LHC pp collision data at √s = 7 and 8 TeV", *Journal of High Energy Physics* (2016) : 2016 (8): 45. [arXiv:1606.02266 [hep-ex]].
5. Weinberg, S. First Three minutes, Basic books Inc., New York, (1977)
6. Steinhardt, P, J. "Relativistic Detonation Waves and Bubble Growth in False Vacuum Decay", *Phys. Rev. D* (1982): 25, 2074.

7. Ignatius, J. Cosmic phase transitions, [arXiv:hep-ph/9312293v1] (introductory part of Ph.D. dissertation, (1993): pg. 53)

8. Barrow J, D. Turner, M, S. "Baryosynthesis and the origin of galaxies", *Nature Physics*, (1981); 291 (5815), 469–472

9. Van Hove, Léon Charles, Prudent. "Theoretical prediction of a new state of matter, the quark-gluon plasma" 17th International Symposium on Multi-particle Dynamics, Seewinkel, Austria, 16 – 20 Jun 1986, (1987); pp.801–818

10. Smoot G, F. et al. "Structure in the COBE Differential Microwave Radiometer First-Year Maps", *Astrophys. J.* (1992); 396, L1

11. Bennett, C, L. et al. "Cosmic temperature fluctuations from two years of COBE differential microwave radiometers observations", *Astrophys. J.* (1994); 436–423.

12. Ajaiab, M. "Anisotropic to Isotropic Phase Transitions in the Early Universe", arXiv:1202.3668v2 (astro-ph.CO.) (2012)

13. Prokopec, T. "The standard cosmological model", *Lecture notes for cosmology* (http://www.phys.uu.nl/prokopec/2scm.pdf) (2008).

14. Ross, G. *Grand Unified Theories*. Westview Press. (1984); ISBN 978-0-8053-6968-7.

15. Guth, A., Inflationary universe: A possible solution to the horizon and flatness problems, *Phys. Rev. D*, (1981); 23, 347

16. Weinberg, S. "Recent progress in gauge theories of the weak, electromagnetic and strong Interactions", *Rev. Mod. Phys*., (1974); 46, 255

17. Kaluza, T. "Zum Unitätsproblem in der Physik", *Sitzungsber. Preuss. Akad. Wiss. Berlin. (Math. Phys.)*, (1921): 966–972

18. Klein, O. "Quantentheorie und fünfdimensionale Relativitätstheorie", *Zeitschrift für Physik A*. (1926); 37 (12), 895–906.

19. Sticlet, D., "Master Program of Theoretical Physics 'Student Seminar in Cosmology", Drd. Maarten VAN DE MEENT (www.physics.umd.edu/courses/)

20. Gleiser, M. "Phase Transitions in the Universe", (1998): Preprint, arXiv : hep-ph/9803291/v1

21. Rajantie, A. "Proceedings Cosmological Phase Transitions and Topological Defects", Porto, Portugal, [arXiv: hep-ph/03112622003] (2003)

22. Bondi, H. "Gold T. The Steady-State Theory of the Expanding Universe", *M.N.R.A.S.* (1948): 108, 252

23. Hubble E, P. "A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae", *Proc.Nat.Acad. Sci*, (1929): 15, 168

24. Hubble E, P. "Distribution of luminosity in elliptical nebulae". *Ap. J.*, (1930): 71, 231

25. Penzias, A, A, and Wilson R, W. "A Measurement of Excess Antenna Temperature at 4080 Mc/s", *Ap.J.*,; (1965); 142, 419

26. Dicke R, H, and Peebles P, J, E. "Roll P. G., Wilkinson D. T., A Measurement of Excess Antenna Temperature at 4080 Mc/s" , *Ap. J.*, (1965): 142, 414

27. Gubser S, S, and Lykken J, D. "Strings, branes and extra dimensions", World scientific Publishing Co Pte. Ltd. , (2004): ISBN 981-238-788-9

28. Ellis G, F, R, and Maartens, R. and MacCallum M, A, H. *Relativistic Cosmology* ,Cambridge University press, (2012): ISBN 978-0-521-38114-4

29. Bailin, D, and Love A, "Kaluza-Klein theories", *Rep. Prog. Phys.*, (1987): 50, 1087

30. Wesson, Paul, S. *Space–Time–Matter: Modern Kaluza–Klein Theory*, Singapore: World Scientific. (1999): ISBN 978-981-02-3588-8.

31. Appelquist, T, and Chodos, A, and Freund P, G, O. *Modern Kaluza–Klein Theories*, Menlo Park, Cal.: Addison–Wesley. (1987): ISBN 978-0-201-09829-7

32. Mohajan H, K. "Friedmann Robertson-Walker (FRW) model in cosmology", *Journal of environmental treatment techniques*, (2013): 1(3), 158-164

33. Jain, N. and Bhoga S, S. "Friedman-Robertson-Walker model: A study", *Int. J. Math. Phys. Sci. Res*., (2014): 2(2), 70–86

34. Weinberg S. The Cosmological Constant Problem, *Reviews of Modern Physics* (1989): 61 (1), 1–23.

35. Carroll S. M., Press W. H., Turner E. L., The Cosmological Constant, *Annual Review of Astronomy and Astrophysics*. (1992); 30 (1), 499–542.

36. Patrignani C.et al. (Particle Data Group), Big-Bang nucleosynthesis, *Chin. Phys. C*. (2016); 40, 100001.

37. Dienes K. R., Dudas E., Gherghetta T., Riotto A., Cosmological Phase Transitions and Radius Stabilization in Higher Dimensions, *Nucl. Phys. B*, (1999); 543, 387–422, [arXiv: hep-ph/9809406 (1998)]

38. Jain N. I., Bhoga S. S., Khadekar G. S., Implications of time varying cosmological constant on Kaluza- Klein cosmological model, *Int. J. Th. Physics*, (2013); 53, 4416

39. Jain N.I., Bhoga S. S., Kaluza-Klein. Bulk Viscous Cosmological model with time dependent Gravitational constant and cosmological constant, *Int. J. Theo. Phys*., (2015); 54, 2991

40. Collins C.B., Global structure of the Kantowski–Sachs cosmological models, *J. Math. Phys*.(1977); 18, 2116

41. Emel'Yanov V. M. Physics in Multidimensional Spaces and the Beginning of Metagalaxy, *Physics Reports*, (1986); 143, 01

Funded by Al-Farabi KazNU.