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Energy loss of relativistic projectiles in non-ideal electron liquids

Abstract. The energy loss of relativistic projectiles in collisional one-component plasmas is analyzed within the method of moments. Both the canonical and non-canonical solutions of the Hamburger moment problem corresponding to five convergent power frequency moments of the electron plasma loss function are employed with the static, purely imaginary, Nevanlinna parameter with the imaginary part iqual to the collision frequency calculated within the Green-Kubo formalism in terms of static structure factors evaluated in the HNC approximation using the Deutsch effective potential.Thus we take into account the dissipation processes in the plasma. It is pointed out that the correlations only slightly influence the deviation of the stopping power with the relativistic corrections taken into account from the classical Bethe-Bohr-Larkin asymptotic form.
Keywords: stopping power, relativistic velocity, sum rules, method of moments.

Introduction

Stopping power is a characteristic of primary interest for different areas of physics such as nuclear physics, condensed matter physics and plasma physics, as it arises when studying the interaction of charged particles with matter. In 1930 Bethe derived his seminal formula for the fast projectile energy losses assuming that the atoms of the medium behave as quantum-mechanical oscillators [1]. Later, Larkin [2] showed that when fast ions permeate an electron gas, an analogous formula is applicable, but with the mean excitation frequency replaced by the plasma frequency ω_p:

$$-\frac{dE}{dx} \underset{v \gg v_F}{\simeq} \left(\frac{Z_p e \omega_p}{v} \right)^2 \ln \frac{2mv^2}{\hbar \omega_p}, \qquad (1)$$

where Ze and v stand for the charge and velocity of the projectile, and $\omega_p = \left(4\pi n e^2 / m\right)^{1/2}$, m being the electron mass. This formula is usually employed to determine experimentally the number density of electrons, n, in a charged particle system. Particularly, its applicability seems to be more promising in the field of plasma physics [3,4,5] for two reasons: first, in an ionized medium the energy loss is mainly caused by the free electrons, leading to an enhancement of the stopping power compared to the cold target [3,4,5]; secondly, this technique appears as the only suitable candidate for the diagnosis of hot and dense ($n \gtrsim 10^{19}\ cm^{-3}$) plasmas, because most of the other methods fail under these conditions [5].

Leaving the ionization losses aside, to calculate the stopping power of a fast projectile passing through a Coulomb fluid we will adopt the polarizational picture, which becomes more accurate as the kinetic energy of the projectile increases. In 1954 Lindhard obtained an expression relating the polarizational stopping power with the medium (longitudinal) dielectric function [6]. This expression can be generalized further by applying the Fermi golden rule to obtain [7,8,9]:

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$$-\frac{dE}{dx}=\frac{2\left(Z_pe\right)^2}{\pi v^2}\int_0^\infty\frac{dk}{k}\int_{\alpha_-(k)}^{\alpha_+(k)}\omega n_B\left(\omega\right)\left(-\mathrm{Im}\varepsilon^{-1}\left(k,\omega\right)\right)d\omega, \qquad (2)$$

$\alpha_\pm\left(k\right)=\pm kv+\hbar k^2/2M$, where M is the mass of the projectile (here we will work with heavy projectiles, $M\gg m$), and $n_B\left(\omega\right)=\left(1-\exp\left(-\beta\hbar\omega\right)\right)^{-1}$ [1], β^{-1} being the system temperature in energy units. In addition, unmagnetized Coulomb fluids are considered and, hence, the dielectric function effectively depends only on the wavevector modulus. Expression (2) is valid only if the interaction between the projectile and the plasma is so weak that it can be treated as a linear effect and no plasma relativistic effects need to be taken into account, i.e., when the energy lost by a projectile is much less than its kinetic energy, which, in turn, is assumed to be much smaller than its rest energy[2].

The literature on the polarizational stopping power is very extensive. The problem has been analyzed within the random-phase approximation (RPA) [7] and beyond, introducing an analytic formula for the local field correction (LFC) factor [10]. In addition there are also nonlinear polarization effects [11], which are beyond the scope of this work. Whereas we assume that the coupling between the projectile and the target one-component plasma can be treated perturbatively, we do not impose any restriction on the value of the coupling parameter, $\Gamma=\beta e^2/a$ ($a=(4\pi n/3)^{-1/3}$ being the Wigner-Seitz radius), with the proviso that the latter remains in the liquid phase[3]. The modeling of its dielectric properties constitutes a difficult task, because its characteristic lengths, i.e., Wigner-Seitz radius and Debye radius, $\lambda_D=(4\pi ne^2\beta)^{-1/2}$, are of the same order of magnitude (in a non-ideal plasma $\Gamma\gtrsim1$, which makes mean field theories, such as the RPA, and perturbative treatments no longer valid) and, at the same time, the electronic system is degenerate.

[1] Notice that $n_B(\omega)+n_B(-\omega)=1$.

[2] For instance, in the experiments reported in Refs. [3, 4, 5] the plasma temperature was of the order of a few eV (in Ref. [4] it is said to be below 500 eV), whereas the projectiles where protons and deuterons at around 1 MeV.

[3] Non-ideal plasmas are known to crystallize at large values of coupling forming an anisotropic phase.

Lately, the problem of energy losses of relativistic protons has arised [12] and our aim here is to determine the relativistic corrections to the asymptotic form of energy losses of fast projectiles in non-ideal electron liquid.

The framework

Our dielectric formalism is based on the method of moments [13,14,15], which allows to determine the dielectric function $\varepsilon(k,\omega)$ from the first known frequency moments or sum rules. The sum rules we employ are actually the power frequency moments of the loss function (LF)

$$\mathcal{L}\left(k,\omega\right)=-\omega^{-1}\mathrm{Im}\varepsilon^{-1}\left(k,\omega\right)$$

defined as

$$C_\nu(k)=\pi^{-1}\int_{-\infty}^{\infty}\omega^\nu\mathcal{L}\left(k,\omega\right)d\omega,\quad \nu=0,1,\ldots.$$

Due to the parity of the LF, all odd-order frequency moments vanish. The even-order frequency moments are determined by the static characteristics of the system. After a straightforward calculation one obtains [16,13,15]:

$$C_0(k)=(1-\varepsilon^{-1}(k,0)),\quad C_2(k)=\omega_p^2,$$

$$C_4(k)=\omega_p^4(1+K(k)+U(k)+H),$$

with

$$K(k)=\left(\left\langle v_e^2\right\rangle k^2+\hbar^2k^4/\left(2m\right)^2\right)/\omega_p^2,$$

$\left\langle v_e^2\right\rangle$ being the average squared characteristic velocity of the plasma electrons. The last term in the fourth moment stems from the interaction contribution to the system Hamiltonian and can be expressed in terms of the structure factor $S\left(k\right)$:

$$U\left(k\right)=\left(2\pi^2n\right)^{-1}\int_0^\infty p^2\left(S\left(p\right)-1\right)f\left(p,k\right)dp,$$

where we have introduced

$$f\left(p,k\right)=5/12-p^2/\left(4k^2\right)+$$
$$+\left(k^2-p^2\right)^2\ln\left|\left(p+k\right)/\left(p-k\right)\right|/\left(8pk^3\right).$$

The Nevanlinna formula of the theory of moments expresses the dielectric function which satisfies the known sum rules $\{C_{2\nu}\}_{\nu=0}^{2}$ [13]:

$$\varepsilon^{-1}(k,z)=1+\frac{\omega_p^2(z+q)}{z(z^2-\omega_2^2)+q(z^2-\omega_1^2)}, \qquad (3)$$

where

$$\omega_1^2=\omega_1^2\left(k\right)=C_2/C_0,\quad \omega_2^2=\omega_2^2\left(k\right)=C_4/C_2,$$

in terms of a function $q=q(k,z)$, which is analytic in the upper complex half-plane Imz>0 and having there a positive imaginary part. It must also satisfy the limiting condition: $(q(k,z)/z)\to 0$ as $z\to\infty$ for $\mathrm{Im}z>0$. In an electron liquid this Nevanlinna parameter function plays the role of the dynamic LFC $G\left(k,\omega\right)$. In particular, the Ichimaru visco-elastic model expression for $G\left(k,\omega\right)$ is equivalent to the Nevanlinna function approximated as i/τ_m, τ_m being the effective relaxation time of the Ichimaru model [17]. Thus we choose the Nevanlinna parameter function to be equal to *iν*with the collision frequency calculated within the Green-Kubo formalism in terms of the plasma species static structure factors as it was suggested in [18]. The static structure factors were computed within the HNC approximation using the Deutsch effective potential [19].

The corrected Bethe-Larkin formula

Let us choose a model function q satisfying the conditions mentioned after the Nevanlinna formula (3) that would permit to treat the stopping power calculation analytically. If we put simply $q(k,\omega)=i0^+$, then we get the following particular solution of the moment problem:

$$\frac{\mathcal{L}\left(k,\omega\right)}{\pi C_0\left(k\right)}=\frac{\omega_2^2-\omega_1^2}{\omega_2^2}\delta\left(\omega\right)+$$
$$+\frac{\omega_1^2}{2\omega_2^2}\left[\delta\left(\omega-\omega_2\right)+\delta\left(\omega+\omega_2\right)\right]. \qquad (4)$$

Physically, Eq. (4) describes an undamped collective excitation mode (Feynman approximation) at $\omega_2\left(k\right)$ with an additional central peak accounting for hydrodynamic diffusional processes. The applicability of this expression is justified provided that the damping of the collective excitation is small enough making this mode to act as the main energy transfer channel. Thus we can disregard the details of the rest of the excitation spectrum. If we introduce expression (4) into the Lindhard formula (2), it immediately reduces to:

$$-\frac{dE}{dx}\underset{v\gg v_F}{\simeq}\frac{\left(Ze\omega_p\right)^2}{v^2}\ln\frac{k_2}{k_1}, \qquad (5)$$

where the "cut-off" wavenumbers k_1 and k_2 are such that the inequality $0<\omega_2\left(k\right)<kv$ is satisfied with $v/v_F\to\infty$ and $\omega_2\left(k\right)$ understood as the plasma Langmuir mode dispersion law $\omega_L(k)$. For a weakly coupled plasma the RPA dispersion law is valid which neglects the correlational contributions to $\omega_L(k)$:

$$\omega_L(k)=\left(\omega_p^2+\left\langle v_e^2\right\rangle k^2+\hbar^2k^4/\left(2m\right)^2\right)^{1/2}.$$

Then, if v is asymptotically large, we have $k_1=\omega_p/v$, $k_2=2mv/\hbar$, and we recover the Bethe-Larkin result [1,2]. Notice that in the above-mentioned inequality for ω_2, we have presumed that $kv\gg\hbar k^2/2M$, which is equivalent to disregard, at most, terms of the order of m/M.

To take into account all Coulomb and exchange interactions in the system analytically, we might use for the electron-electron contribution $U\left(k\right)$ its long- and short-range asymptotic forms,

$$U(k \to 0) \simeq -v_{ee}^2 k^2 / \omega_p^2,$$

$$U(k \to \infty) \simeq -h_{ee}(0)/3,$$

where $v_{ee}^2 = -4E_{ee}/(15nm)$ is defined by the plasma electron-electron interaction energy density E_{ee} of the plasma [9], $h_{ee}(0)$ being equal to the previous expression for $U(k)$, but with the function $f(p,k)$ replaced by unity. If we interpolate the plasma mode dispersion law as

$$\omega_L(k) = \left(\omega_p^2 + wk^2 + \hbar^2 k^4/(2m)^2\right)^{1/2},$$

with $w = 2\langle v_e^2 \rangle - v_{ee}^2$, then the "cut-off" wavenumber k_1 still equals ω_p / v, for $v/v_F \to \infty$, so that the fast projectile stopping power relmais equal to

$$-\frac{dE}{dx} \underset{v \gg v_F}{\simeq} \left(\frac{Ze\omega_p}{v}\right)^2 \ln \frac{2mv^2}{\hbar\omega_p}. \qquad (6)$$

Energy loss of relativistic projectiles

Relativistic corrections to the Lindhard formula were studied in [20]:

$$-\frac{dE}{dx} = -\frac{(Z_p e)^2}{\pi v^2} \int_0^\infty \frac{dk}{k} \int_{-kv}^{kv} \omega Im \left(\varepsilon^{-1}(k,\omega) \frac{\varepsilon^{-1}(k,\omega) - \frac{v^2}{c^2}}{\varepsilon^{-1}(k,\omega) - \frac{\omega^2}{k^2c^2}} \right) d\omega, \qquad (7)$$

It can be easily seen that when the speed of light $c \to \infty$ and $M \gg m$, (7) turns into (6). Now, observe that

$$\mathrm{Im}\left(\varepsilon^{-1}(k,\omega) \frac{\varepsilon^{-1}(k,\omega) - \frac{v^2}{c^2}}{\varepsilon^{-1}(k,\omega) - \frac{\omega^2}{k^2c^2}} \right) = \left(\mathrm{Im}\varepsilon^{-1}(k,\omega)\right) \times \left(1 + \frac{\left(\frac{\omega}{kc}\right)^2 \left(\left(\frac{v}{c}\right)^2 - \left(\frac{\omega}{kc}\right)^2\right)}{\left(\mathrm{Re}\varepsilon^{-1}(k,\omega) - \left(\frac{\omega}{kc}\right)^2\right)^2 + \left(\mathrm{Im}\varepsilon^{-1}(k,\omega)\right)^2} \right)$$

and that, due to (4),

$$\mathrm{Im}\varepsilon^{-1}(k,\omega) = \frac{\pi\omega_p^2}{2\omega_2^2} \omega_2(k) \times \left[\delta(\omega + \omega_2(k)) - \delta(\omega - \omega_2(k))\right],$$

so that (7) simplifies into:

$$\left.\frac{-dE}{dx}\right|_{v \to c} \simeq \left(\frac{Ze\omega_p}{v}\right)^2 \ln \frac{2mv^2}{\hbar\omega_p} + \left(\frac{Ze\omega_p}{c^2}\right)^2 \int_{\omega_p/v}^{2mv/\hbar} \frac{dk}{k^3} \frac{\omega_2^2(k)\left(1 - \frac{\omega_2^2(k)}{k^2v^2}\right)(\omega_2^2(k) - \omega_1^2(k))^2}{\Omega^4(k) + \left(\frac{\omega_p^2 \omega_2(k) \mathrm{Im} q(k, \omega_2(k))}{|q(k,\omega_2(k))|^2}\right)^2}, \qquad (8)$$

Where

$$\Omega^2(k) = \omega_p^2 + \left(\omega_2^2(k) - \omega_1^2(k)\right)\left(1 - \frac{\omega_2^2(k)}{k^2c^2}\right) + \frac{\omega_p^2 \omega_2(k) \mathrm{Re} q(k, \omega_2(k))}{|q(k,\omega_2(k))|^2}.$$

We have analyzed numerically the relative importance of (8) as compared to (6), the results are presented in figures 1–4. The one-component plasma static characteristics were estimated in the HNC approximation [19]. The continuous lines correspond to the expression (8), while the discontinuous lines do not include the realtivistic corrections, formula (6). The velocity is measured in terms of the Fermi velocity

$$v_F = \sqrt[3]{\frac{9\pi}{4}}\frac{e^2}{r_s\hbar},$$

where

$$r_s = a / a_B, a = \sqrt[3]{3/4\pi n}$$

is the Wigher-Seitz radius, and $a_B = \hbar^2 / me^2$ is the Bohr radius. The plasma coupling parameter

$$\Gamma = \frac{e^2}{ak_BT},$$

T being the plasma temperature.

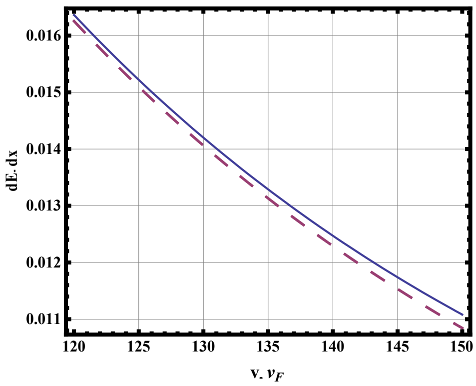

Figure 1 – The stopping power absolute value at $\Gamma = 10.776$, $r_s = 2.5256$ for very fast projectiles.

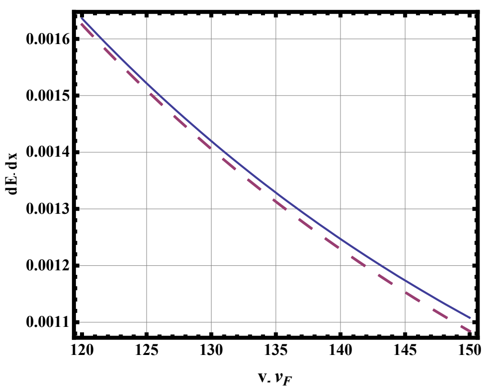

Figure 2 – The stopping power absolute value at $\Gamma = 1.077$, $r_s = 2.5256$ for very fast projectiles.

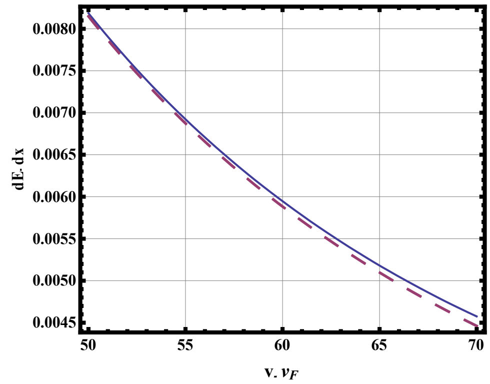

Figure 3 – The stopping power absolute value at $\Gamma = 2.321$, $r_s = 1.172$ for very fast projectiles.

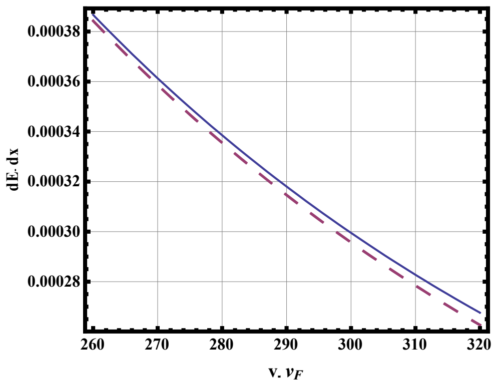

Figure 4 – The stopping power absolute value at $\Gamma = 0.5$, $r_s = 5.441$ for very fast projectiles.

Conclusions

In this paper we have studied how the Bethe-Larkin asymptotic expression (2) for the electron plasma stopping power is modified when the projectiles possess a relativistic velocity, up to about 85% of the light speed c. We observe that in non-ideal plasmas, i.e., at higher densities, the relativistic efects become slightly more pronounced. The results can be used to diagnose plasmas within the method of proton radiography.

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