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Scattering phase shifts of lithium isotopes

Abstract. Investigation of few body cluster systems is very important in nuclear physics. Problems appearing in few body systems can in principle be divided into two classes: bound state problems and scattering problems. The bound state problems are usually related to the spectroscopy of such systems while scattering problems describe their reactions. The main focus in the work is the scattering problem for systems consisting of two cluster systems. The single channel two body scattering problem is considered in the framework of different spin parity states for lithium isotopes.

Scattering phase shifts on negative and positive parity states of ^{5}Li, ^{6}Li and ^{7}Li nuclei are calculated applying two-body $\alpha + p$, $\alpha + d$ and $\alpha + t$ systems and the complex scaling method. ^{6}Li and ^{7}Li are stable nuclei and their ground and low-lying excited states are considered in this work.

In this study, we calculated scattering phase shifts of the negative parity $J^\pi = 3/2^-$ and $J^\pi = 1/2^-$ states for p −wave of ^{5}Li, $J^\pi = 7/2^-$, $5/2^-$, $3/2^-$ and $1/2^-$ states for $p-$ and f −waves of ^{7}Li and the positive parity $J^\pi = 1^+$, 2^+, 3^+ states for $s-$ and $d-$ waves of ^{6}Li.

Key words: phase shifts, structure of light nuclei, two-body system, low-lying excited states, ground state.

Introduction

\During last several decades, scientists have tried and failed to provide a complete solution to scattering in complex nuclear system, one of the most fundamental phenomena in nuclear physics. Nuclear physics is one of the most rapidly developing fields of natural science in terms of theoretical and experimental research, many important and interesting issues remain still unclear in this area. The nuclei are complex objects consisting of several interacting nucleons where neutrons and protons have been arranged with different combinations. Light nuclei have exotic properties owing to peculiarities of the nuclear forces and quantum states of nucleon systems. To understand the characteristic properties of every nucleus, we use appropriate nuclear models and effective nuclear and Coulomb interactions [1-2].

The nuclear models can contain quasistationary or virtual states of nuclei, as well as their excited states located on the complex energy plane close to the real physical region of existence of the nuclei [3-7]. Nuclear models not only focused on the description of nuclear structures and reactions, but also considered nuclear fission and nuclear decay. At the beginning of development for nuclear models, it was known that the nucleons tend to group into clusters were extremely important in determining the structure of light nuclei. Consequently, the cluster structure of nucleus ground and excited (resonance or virtual) states became the focus of theoretical and experimental studies. Light nuclei are loosely bound and change their configurations when they interact with nucleons or other nuclei at relatively small distances. It was informed that a nucleus cluster structure is displayed in reactions with neutrons at low energies and with protons at energies higher than

Coulomb potential barrier. In the reactions of neutron scattering for various nucleus in the low-energy region is quite well measured by experimentally but the measured data for proton scattering on light nuclei at low energies is rare.

In this work, we apply the complex scaling method (CSM) [8-9] to the $\alpha + p$, $\alpha + d$ and $\alpha + t$ two-body models for obtaining ground and low-lying excited states of ^{5}Li, ^{6}Li and ^{7}Li nuclei. Applying the CSM and $\alpha + p$ two-body model for the $J^\pi = 3/2^-$ and $1/2^-$ states of ^{5}Li, $\alpha + d$ two-body model for the $J^\pi = 1^+$, 2^+, 3^+ states of ^{6}Li and $\alpha + t$ two-body model $J^\pi = 7/2^-, 5/2^-, 3/2^-$ and $1/2^-$ states of ^{7}Li.

For $\alpha + p$ and $\alpha + t$ systems the negative parity states of $p-$ and $f-$ waves are considered for the calculation of scattering phase shifts. The phase shifts for positive parity states in $s-$ and $d-$ waves of $\alpha + d$ system is calculated.

Complex Scaling Method and Two Body Model

The *Schrödinger* equation, $\hat{H}\Psi = E\Psi$, is transformed as

$$H^\theta \Psi^\theta = E^\theta \Psi^\theta, \qquad (1)$$

where the complex scaled wave function is defined as

$$\Psi^\theta = U(\theta)\Psi = e^{\frac{3}{2}i\theta}\Psi(re^{i\theta}). \qquad (2)$$

The factor $e^{\frac{3}{2}i\theta}$ comes from the Jacobian of the coordinate transformation for r. The Hamiltonian in Eq. (1) is

$$H^\theta = U(\theta)HU^{-1}(\theta). \qquad (3)$$

To solve Eq. (1), we expand the wave functions $\Psi^\theta(k,r)$ to a finite number of L^2 basis functions, the Gaussian basis functions $u_i(r)$ for $i = 1, 2, \cdots, N$,

$$\Psi^\theta(k,r) = \sum_i^N c_i(k,\theta)\, u_i(r,b_i)\,. \qquad (4)$$

The coefficients $c_i(k,\theta)$ and the discrete spectra are obtained by solving the eigenvalue problem

$$\sum_i^N H_{ij}^\theta\, c_j(k,\theta) = E^\theta c_i(k,\theta)\,, \qquad (5)$$

$$H_{ij}^\theta = \langle \tilde{u}_i | H^\theta | u_j \rangle, \qquad (6)$$

where H_{ij}^θ are the matrix elements of the complex scaled Hamiltonian given in Eq. (3).

Applying the CSM to two-body $\alpha + p$ model the Hamiltonian is expressed as

$$\hat{H} = \sum_{i=1}^2 \hat{T}_i - \hat{T}_{c.m.} + V_{ap}^{Nucl}(r) + V_{ap}^{Coul}(r), \quad (7)$$

where $\hat{T}_i$ and $\hat{T}_{c.m.}$ are the kinetic energy operators of the $i-$th cluster and the center-of-mass of the total system, respectively. V_{ap}^{Nucl} is alpha-proton potential, $V_{ap}^{Coul}(r)$ is Coulomb potential. For the $\alpha + d$ and $\alpha + t$ two-body models the same Hamiltonian given in Eq. (7) is applied.

For each partial wave, we use Gaussian functions with different size parameters as basis functions

$$u_i^\ell(r,b_i) = N_\ell(b_i)\, r^\ell \exp\left(-\frac{1}{2b_i^2}r^2\right), \qquad (8)$$

$$N_\ell(b_i) = \frac{1}{b_i^{\ell+3/2}}\left\{\frac{2^{\ell+2}}{(2\ell+1)!!\,\sqrt{\pi}}\right\}^{1/2}, \qquad (9)$$

where the parameters $(b_i: i = 1, 2, \cdots, N)$ are give by a geometrical progression of the form

$$b_i = b_0 \gamma^{i-1}\,, \qquad (10)$$

where b_0 and γ are the first term and the common ratio, respectively.

Results and Discussion

$\alpha + p$ two body system

Phase shifts of the elastic scattering of proton from an alpha particle are shown in Figure 1. Calculated phase shifts for the $J^\pi = 3/2^-$ and $J^\pi = 1/2^-$ states generated by the total orbital momentum $L = 1$, which has a resonant state for each partial state. We obtained resonance state energy 0.74 MeV with decay width 0.59 MeV for $J^\pi = 3/2^-$ and resonance state energy 2.10 MeV and its decay width 5.82 MeV for $J^\pi = 1/2^-$ states. As can be seen from Figure 1 a), a narrow decay width state is calculated for the $J^\pi = 3/2^-$ state and the calculated scattering phase shifts of the $J^\pi = 3/2^-$ state increases rapidly from 1 MeV due to the small decay width. A resonance energy with large decay width for the $J^\pi = 1/2^-$ state is obtained and the calculated phase shifts approaches $\pi/2$ which shows a clear resonance behavior in Figure 1 b).

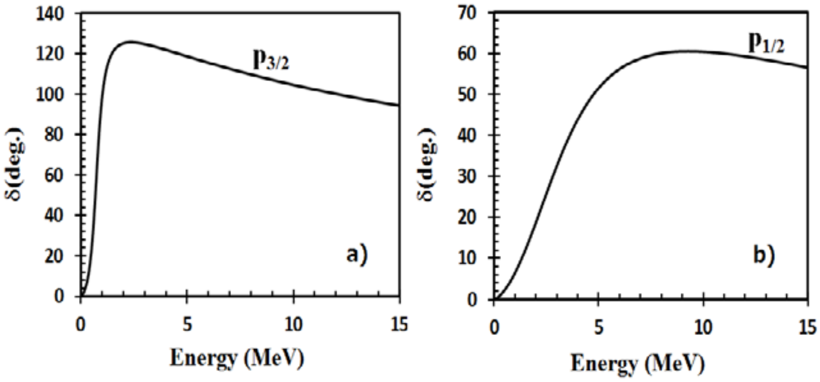

Figure 1 –The scattering phase shifts of $\alpha + p$ system / for $J^{\pi} = 3/2^{-}$ (a) and $J^{\pi} = 1/2^{-}$ (b).

$\alpha + d$ two body system

^{6}Li is a stable nucleus and excited energy levels are observed by experimentally. Calculated phase shifts for the elastic $\alpha + d$ scattering of the orbital momentum $L = 0$ and 2 , the total angular momentum J are presented in Figure 2. In this calculation we consider only even parity states of ^{6}Li and ignored odd parity states because phase shifts for negative parity states are very small as comparing with even parity states. The $J^{\pi} = 1^{+}$ for the orbital momentum $L = 0$ is a ground state of ^{6}Li. The calculated phase shifts of the $J^{\pi} = 1^{+}$ for $L = 0$ indicate an attractive interaction nature and it is displayed by dotted line in Figure 2.

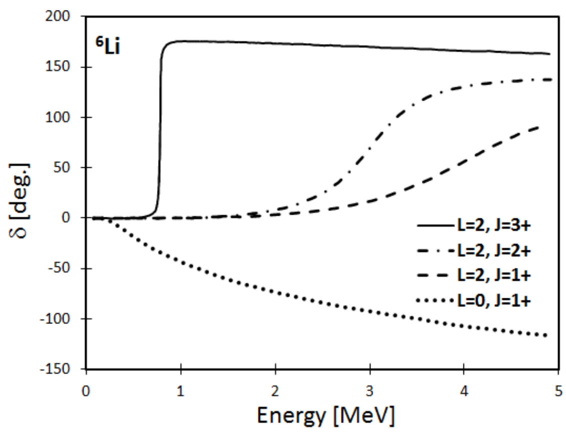

Figure 2 – The scattering phase shifts of $\alpha + d$ system of the $J^{\pi} = 1^{+}$ state for $L = 0$, and the $J^{\pi} = 3^{+}, 2^{+}, 1^{+}$ states for $L = 2$. The dotted, dashed, dotted-dashed and solid lines denote the calculated phase shifts of the $J^{\pi} = 1^{+}$ state for $L = 0$, and the $J^{\pi} = 1^{+}, 2^{+}, 3^{+}$ states for $L = 2$, respectively.

In the case of the $J^{\pi} = 3^{+}$ state for $L = 2$, the resonance energy is obtained and the calculated phase shifts for this state shows a sharp resonance behavior because of the very small resonance width. The calculated phase shifts of the $J^{\pi} = 3^{+}$ state increases sharply ~1 MeV and it approaches π which is displayed by solid line in Figure 2, and it implies a resonance state with small decay width. In Figure 2, the dotted-dashed and dashed lines represent the calculated phase shifts of the $J^{\pi} = 2^{+}$ and $J^{\pi} = 1^{+}$ states for $L = 2$, respectively. It can be seen from Figure 2, the calculated phase shifts for the $J^{\pi} = 2^{+}$ and $J^{\pi} = 1^{+}$ states express resonance behavior. Phase shifts of the $J^{\pi} = 2^{+}$ state presents by the dotted-dashed line, and it increases gradually from 3 MeV and approaches $5\pi/6$. The dashed line expresses the calculated phase shifts of the $J^{\pi} = 1^{+}$ state and it increases smoothly from 4.5 MeV and approaches $\pi/2$.

$\alpha + t$ two body system

We display phase shifts of the elastic $\alpha + t$ scattering in Figure 3. In this time, we study only negative parity states of ^{7}Li for $L = 1$, and 3 waves. The positive parity states are negligibly small as comparing with odd parity states.

Due to the Coulomb interaction, phase shifts are very small at the energy range $0 < E < 0.5$ MeV. Phase shifts for the negative parity state and for the total angular momentum $J^{\pi} = 7/2^{-}$ and $J^{\pi} = 5/2^{-}$ for $L = 3$ exhibit a resonance behavior which approach π and $5\pi/6$. As can be seen from Figure 3, the solid line expresses the calculated phase shifts of the $J^{\pi} = 7/2^{-}$ state and it implies a sharp resonance state obtained. The dotted-dashed line displays the results of the $J^{\pi} = 5/2^{-}$ state and it shows resonance behavior too.

The phase shifts of the $J^{\pi} = 3/2^{-}$ and $J^{\pi} = 1/2^{-}$ state for $L = 1$ are drawn by dotted and dashed lines in Figure 3. The phase shifts behaviors for $L = 1$ state show attractive nature.

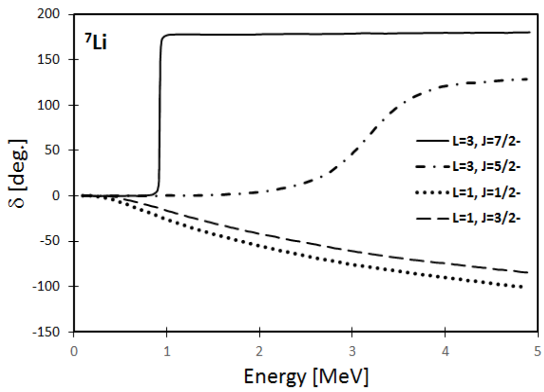

Figure 3 – The scattering phase shifts of $\alpha + t$ system of the $J^{\pi} = 3/2^{-}$ and $J^{\pi} = 1/2^{-}$ state for $L = 1$ and the $J^{\pi} = 7/2^{-}, 5/2^{-}$ states for $L = 3$. The dotted, dashed, dotted-dashed and solid lines denote the calculated phase shifts of the $J^{\pi} = 3/2^{-}$ and $J^{\pi} = 1/2^{-}$ state for $L = 1$ and the $J^{\pi} = 7/2^{-}, 5/2^{-}$ states for $L = 3$.

Conclusions

In this work we discussed the calculated scattering phase shifts for the different spin parity states of ^{5}Li, ^{6}Li and ^{7}Li nuclei applying two body model. The negative parity states of $L = 1$ and $L = 3$ waves are considered for $\alpha + p$ and $\alpha + t$ systems. The scattering phase shifts are calculated in positive parity states of $L = 0$ and $L = 2$ waves for $\alpha + d$ system.

The $J^{\pi} = 3/2^{-}$ and $J^{\pi} = 1/2^{-}$ states of $\alpha + p$ system have a resonant state for each partial state and the calculated phase shifts show resonance behavior.

We calculate scattering phase shifts of $\alpha + d$ system for $L = 0$ and 2 waves where only even parity states are considered. The phase shifts for negative parity states are very small as comparing with even parity states in $\alpha + d$ system. ^{6}Li has the $^{3}_{2}He + t$ cluster configuration and it reported in Ref. [10], the ($^{3}_{2}He + t$) configuration of ^{6}Li is only slightly less probable than the ($\alpha + d$) configuration.

^{7}Li nuclei is modelled as $\alpha + t$ two clusters and scattering phase shifts of $L = 1$, and 3 waves are calculated. The positive parity states are negligibly small as comparing with odd parity states in $\alpha + t$ system.

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