IRSTI 29.05.45, 29.17.19, 41.29.00, 41.29.17, 41.29.21 https://doi.org/10.26577/ijmph.2022.v13.i1.011

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(Received 11 March 2022; received in revised form 18 May 2022; accepted 30 May 2022)

Starobinsky model with a viscous fluid

Abstract. The article considers some cosmological solutions of the Starobinsky model for a flat inhomogeneous viscous Universe. The first section contains a brief of the $F(R)$ theory of gravity. One of the most common examples of $F(R)$ gravity with a high degree of curvature is the Starobinsky model. For the Starobinsky $F(R) = \alpha R + \beta R^2$ model, the cosmological model of a flat and homogeneous Universe is considered. For the Friedmann-Robertson-Walker metric, the Lagrange function is defined, and the corresponding equations are determined by the Euler-Lagrange equations and the Hamilton energy condition. Using the equation of state for inhomogeneous viscous fluid, we considered two cases of the viscosity parameter when the state parameter is constant. Next, using the results obtained, we determined the dynamics of the Hubble parameter H. At constant viscosity, $\xi = \xi_0$ has a negative value of the Hubble parameter and decreases with time along a hyperbola, while $\xi = 3H$ has a positive value decreases along a hyperbola.

If we compare it with the well-known de Sitter solution describing the accelerated expansion of the Universe and take into account that time in physics should only be positive, then the change in the Hubble parameter for the viscosity $\xi = 3H$ occurs later. An analysis of this solution shows that at a certain point in time the acceleration of the Universe turns into a process of instantaneous compression. However, at the end, the result is similar to the de Sitter solution tends to zero, i.e. the Universe stops accelerating. Based on the results obtained, a graph was constructed with respect to the de Sitter solution. The analysis was carried out according to the graph. These results are useful for describing the accelerated expansion of the modern Universe and do not contradict modern astronomical observations.

Key words: viscous fluid, cosmology, Starobinsky model, FRW metric, $F(R)$ gravity.

Introduction

The general theory of relativity is the basic theory describing gravitational phenomena in nature. The correctness of this theory is confirmed by various experiments and observations. However, the general theory of relativity does not fully describe some aspects of the evolution of the Universe, for example, the current accelerated expansion of the Universe [1, 2]. The best theory to explain this expansion of the Universe is dark energy [3-5], but the nature of dark energy is still unknown.

The latest cosmological data limit the state parameter ω of dark energy to ω = -0.972 + 0.061 -0.060 so that various forms of dark fluid (phantom, quintessence, inhomogeneous fluids, etc.) can satisfy the corresponding equation of state. The study of non-ideal fluids in the Friedman-Robertson-Walker (FRW) universe can be justified by various arguments. First of all, even though many macroscopic physical systems, such as the large-scale structure of baryonic matter and radiation in the Universe, can be approximated as ideal fluids (with the equation of state p = ωρ, ω is constant), the description of dark energy does not exclude a more complex equation of state, since its nature is still unknown.

Moreover, interest in modified theories of gravity has increased in recent years. Such theories suggest changing not only the nature of dark energy, but also a different approach to Einstein's spacetime or gravity by replacing the curvature of spacetime in the classical Hilbert-Einstein formula with more generalized variants (Riemann tensor, Weyl tensor, Ricci tensor, etc.). One of the most common examples of modified gravity is the model Starobinsky. Various applications of Starobinsky models in cosmology are presented in the literature [17].

In this paper, we study the dynamics of a viscous fluid [18-20] in the Starobin gravitational field. The corresponding equations of motion are determined and the evolution of the Hubble parameter for two types of viscous fluids is obtained. Moreover, the results obtained are compared with de Sitter's solution and allow us to describe the late evolution of the universe.

It should be noted that we fully adopt the natural system of units by taking 8πG=c=h=1. Indices i,j,l run from 1 to 4 throughout this paper.

Action and equations of motion

In this section, we consider the Starobinsky model for FRW metric. In the general case, the action $F(R)$ gravity can be written as follows

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2k}F(R) + L_m\right), \quad (1)$$

where $k = \frac{8\pi G}{c^4}$, g is the determinant of the metric tensor $g_{\mu\nu}$, $F(R)$ is some function of the Ricci scalar R, L_m is the Lagrangian matter. The dependence of the function $F(R)$ on the Ricci scalar is given in this paper similarly to the Starobinsky model $F(R) = \alpha R + \beta R^2$, where $\alpha, \beta = const$.

Then consider the FRW metric with action (1)

$$ds^2 = -dt^2 + a^2(t)\left(dx^2 + dy^2 + dz^2\right), \quad (2)$$

where $a(t)$ is a scale factor that depends only on time t. For this metric, we obtain the following equations

$$\sqrt{-g} = a^3, \ R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right),$$

where the dot denotes the differentiation in time t.

Therefore, for metric (2) action (1) can be rewritten as follows

$$S = \int d^4xa^3\left[\alpha R + \beta R^2 - \lambda\left(R - 6\frac{\ddot{a}}{a} - 6\frac{\dot{a}^2}{a^2}\right)\right]. \quad (3)$$

If we take a variation of this action with respect to R, we can determine the Lagrange multiplier λ

$$\lambda = \frac{\mathrm{d}F(R)}{\mathrm{d}R} = \alpha + 2\beta R.$$

Thus, we can write the point-like Lagrangian as follows

$$L = \beta R^2a^3 + 12a^2\dot{a}\beta\dot{R} + 6(\alpha + 2\beta R)a\dot{a}^2. \quad (4)$$

Using the Euler-Lagrange equation, we find the pressure for the considered model as follows

$$p = -2\dot{H} - 3H^2 = \frac{-\frac{\beta R^2}{2} + 2\beta\ddot{R} + 4\beta\dot{R}H}{\alpha + 2\beta R}. \quad (5)$$

Using the energy condition, we define our energy density as follows

$$\rho = 3H^2 = \frac{R^2\beta}{2(\alpha + 2\beta R)} - \frac{6H\beta\dot{R}}{\alpha + 2\beta R}, \quad (6)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble parameter.

If we equate equations (5) and (6), we obtain the following equation

$$\dot{R} - \frac{1}{12H}R^2 + HR + \frac{\alpha}{2\beta}H = 0. \quad (7)$$

Model of inhomogeneous viscous fluid

The pressure p introduced into the Friedmann equations and the energy density ρ must satisfy the following conservation law

$$\dot{\rho}+3H(\rho+p)=0. \tag{8}$$

For our model, we study the general form of the equation of state for an inhomogeneous viscous fluid [17-20]

$$p=\omega(\rho)\rho-B(a(t),H,\dot{H},...), \tag{9}$$

where $\omega(\rho)$ parameter of the equation of state can depend on the energy density, and the mass viscosity $B(a(t),H,\dot{H},...)$ is a function of its arguments. Consider $B(a(t),H,\dot{H},...)=\xi(H)$ for a viscous fluid

$$p=\omega(\rho)-3H\xi(H), \tag{10}$$

thus, $\xi(H)$ – is the bulk viscosity.

Thus, if we substitute the pressure in equation (5) to the law of conservation of energy (6), we will obtain an additional equation of motion describing a viscous fluid. As a result, we obtain a system of equations of motion

$$\dot{\rho}+3H\rho(1+\omega(\rho))=3H\xi(H), \tag{11}$$

$$\rho=3H^2=\frac{\frac{\beta R^2}{2}-6H\beta\dot{R}}{\alpha+2\beta R}, \tag{12}$$

$$R=6\dot{H}+12H^2, \tag{13}$$

$$p=-2\dot{H}+3H^2=\frac{\left(-\frac{\beta R^2}{2}+2\beta\ddot{R}+4\beta\dot{R}\right)}{(\alpha+2\beta R)}. \tag{14}$$

Now consider two cases related to these given equations of motion. Let's transform these received formulas.

Cosmological solution

For thermodynamic reasons $\xi(H)$ is usually chosen to be positive. Therefore, various forms of viscosity can be used to numerically or accurately solve the Hubble parameter. Next, consider two types of viscosity parameter $\xi(H)$.

First case: $\omega=const$, $\xi(H)=const$

In this case, for $\xi(H)=\xi_0=const$, we obtain the following equation using equations (11)-(14)

$$\dot{H}=-\frac{3}{2}(\omega+1)H^2+3\xi_0 H, \tag{15}$$

as a result, we get the following solution

$$H=\frac{2\xi_0}{1+\omega-e^{-3\xi_0(t-t_0)}}. \tag{16}$$

If we consider $\omega=-1$ and the early Universe for vacuum, then the time $t_0\cong 0$, i.e.

$$H=-\frac{2\xi_0}{e^{-3\xi_0 t}}, \tag{17}$$

Having determined the dependence of the scalar curvature on time, substituting this solution (13) and solving equation (7) $\alpha=1$, $\beta=1$, $\xi_0=1$, we determine the Hubble parameter for the Starobinsky model

$$H=-10\frac{\left(-18e^{-3t}+\sqrt{6}\sqrt{e^{-3t}\left(55e^{-3t}-120\right)}\right)e^{3t}}{-120+e^{-3t}}. \tag{18}$$

Second case: $\omega=const$, $\xi=3H$

Consider the case where the equation of state parameter for vacuum is $\omega=-1$ and the bulk viscosity depends only on the Hubble parameter, then

$$H=\frac{2}{3(\omega-2)(t-t_0)}. \tag{19}$$

After some actions, analogous to the previous case, we define the Hubble parameter for the Starobinsky model

$$H=-\frac{2}{3}\cdot\frac{-12+12t+\sqrt{24t^4-152t^3+342t^2-384t+160}}{\left(3t^2-16t+8\right)t} \tag{20}$$

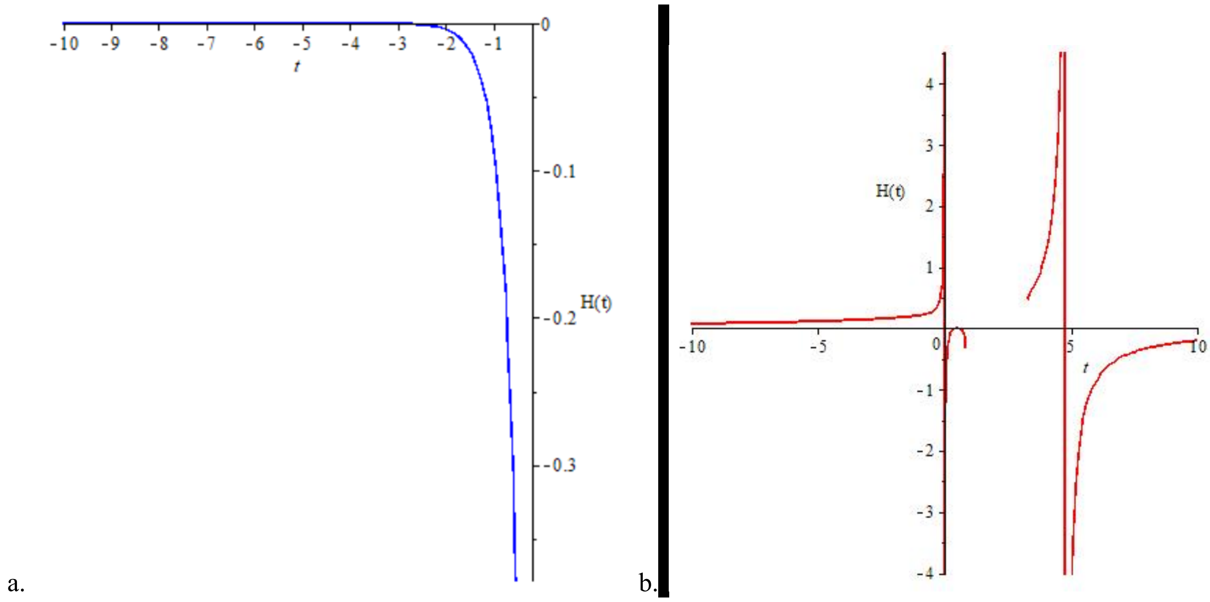

Figure 1 – Hubble parameter dynamics for cases: a. $\xi=\xi_0$, b. $\xi=3H$

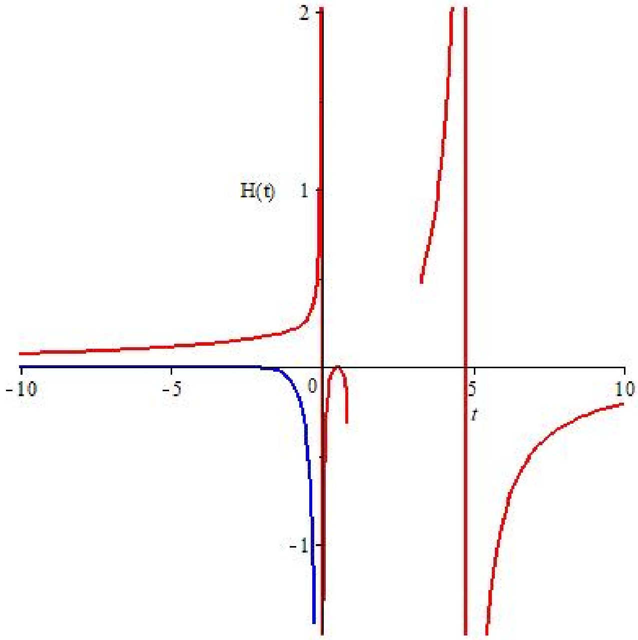

Figure 2 – Dynamics of the Hubble parameter for different solutions for $\xi=\xi_0$ indicated by a blue line, for $\xi=3H$ red line.

Conclusion

Thus, in this work, we have considered some cosmological solutions of the Starobinsky model for the flat and homogeneous Universe. In the first section we give a brief introduction to the theory of gravitation. For the FRW metric, the Lagrange function is defined, and the corresponding equations of motion are determined using the Euler-Lagrange equations and the Hamilton energy condition. As you can see, these equations are non-linear differential equations of high order, the solution of which is a difficult task. Next, using this result, we determined the Hubble parameter H and the equation of motion R.

Finally, as you can see in Fig. 2, the Hubble parameter is negative for the $\xi = \xi_0$ condition and decreases with time along the hyperbola, while in the $\xi = 3H$ state it decreases to a positive value along the hyperbola. If we compare with the well-known de Sitter solution describing accelerated expansion that time is only positive according to the law of physics, then in the case of $\xi = 3H$ the Hubble change occurs later than the de Sitter solution, and the viscosity is $\xi \neq \xi_0$ variable. If we analyze the solution for a viscosity proportional to the Hubble parameter, then at a certain moment in time the acceleration of the Universe passes into the process of instantaneous compression. But as a result, the de Sitter solution seems to be infinitely close to zero, that is, the Universe stops accelerating.

Acknowledgements

The research work is supported by the grant project AP08052197 (2020-2022) from the Ministry of Science and Education of the Republic of Kazakhstan.

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