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Arbitrary amplitude nonlinear waves in a four-component quantum plasma

Abstract: Based on the approach of arbitrary amplitude Sagdeev pseudo-potential, the fully nonlinear structures, such as soliton and double-layer waves, of a four-component quantum plasma system have been extensively investigated. The plasma system under consideration consists of inertial positive and negative dust species and inertialess quantum ions and electrons. For simplicity, the dust grains are considered to have a constant surface charge. For typical plasma parameters, like Mach number, temperatures ratio, densities ratios and quantum parameter, the bifurcation analysis enables us to distinguish three types of nonlinear waves, namely compressive soliton, rarefactive soliton and double layer depending on the magnitude of the configurational plasma parameters. The formations and the existence ranges of both solitons and double layers are studied and found to be highly sensitive to the values of the chosen plasma parameters and the quantum indices as well. The implications of the obtained results have a wide relevance in the study of space and laboratory quantum dusty plasmas where the positive and negative dust particulates are presented, and quantum effects are taken into account.
Key words: Four-component quantum dusty plasma; Sagdeev pseudo-potential; Solitary waves and double-layers

Introduction

The topic of propagating linear and nonlinear waves in quantum dusty plasma had a great deal of interest due to its wide application in quantum plasma media such as in high-intensity laser-produced plasmas, dense astrophysical objects and ultra-small electronic devices [1-5]. It is well known that dusty plasma is the most general form of plasmas in most of space and astrophysical bodies which comprising with electrons, positive ions, neutral molecules and massive micrometer-sized charged dust grains. In addition to plasma laboratory, dusty plasmas are also observed in pulsar magnetospheres, active galactic nuclei, planetary rings, solar atmosphere, interstellar medium, comet tails and noctilucent clouds [6-14]. Dusty plasma may have dust grains with negative charge, dust grains with positive charge or may have both types of dust grains depending on the charging processes [15-17]. Due to the presence of large number of charged particles in dusty plasma media, many reports indicate that there are two sorts of low-frequency waves in dusty plasmas, called dust acoustic (DA) waves and dust-ion acoustic (DIA) waves [18, 19]. Consequently, investigation of nonlinear wave characteristics in such media has been studied by many researchers [18, 20, 21]. Mannan and Mamun [18] studied the DA solitary waves associated with general and realistic self-gravitating dusty plasma medium composed of positive and negative charged warm dust grains as well as non-thermal ions. The population effect of non-thermal ions is examined and found that it modifies the basic properties of the obtained DA solitary waves. Khaled et al. [20] studied the presence of DA solitary waves in a dusty plasma medium with opposite polarity of dust grains and Maxwellian distributed electrons and ions taking into account the polarization force effect. They observed that the propagation characteristics of the obtained DA solitary waves are significantly modified due to the polarization force effect. In Farooq et al. [21] work, they investigated the polarization force effect on the nonlinear properties of DA solitary waves in a four component dusty plasma composed of positive and negative dust grains, non-Maxwellian electrons follows hybrid

Cairns–Tsallis distribution and Maxwellian ions. As a result of the presence of non-Maxwellian electrons, it is found that the polarization force acting on negatively charged dust is different from that acting on positively charged dust.

In quantum plasma media, the plasma behaves like degenerate plasma when its components are at higher densities and low temperature as well as the thermal de Broglie wavelength of charged species becomes same or larger than the inter-particle distance [22-24]. Quantum effects of plasma species are found to play an important role in many plasma physics applications such as in ultra-cold plasmas, ultra-small electronic devices, nanostructure devices, semiconductors, laser produced plasmas as well as in astrophysical plasmas such as interiors of white dwarfs and neutron stars [25-30]. In quantum plasma, the classical hydrodynamic equations are no longer suitably at all and should be modified [31], where the Bohm potential term may be added in the momentum equation. This of course makes the procedure of finding an explicit form for the pseudo-potential function very difficult. This leads some authors [32, 33] to restrict their studies only on small amplitude nonlinear wave's approximations, accordingly. The presence of different types of nonlinear waves such as solitary waves, shock structure and double layers in quantum dusty plasma had been investigated by many researchers in the framework of KdV-type equations. For instance, El-Hanbaly et al. [32] investigated the propagation characteristics of DA solitary waves interaction in four-component quantum dusty plasma and the quantum effects of plasma species are discussed. They found that the quantum parameters of electrons and ions played an important role on the features of the DA solitary waves such as phase shifts in trajectories due to collision. Nonlinear properties of DA solitary waves considering dust polarity effects as well as electrons and ions quantum effects in magnetized quantum dusty plasma were analyzed by Gao [33]. By performing numerical analysis, he found that the nature of solitary waves was modified depending on the effects of quantum electrons and ions. However, most of these studies have only focused on the use of reductive perturbation technique.

In order to investigate the fully nonlinear structures in quantum plasma, Sagdeev pseudo-potential technique and bifurcation analysis have been widely used [34, 35]. For example, Abulwafa et al. [34] have used the Sagdeev pseudo-potential technique and bifurcation analysis to study the properties of DA double-layers in four-component dusty plasma with q-non-extensive distributed electrons and ions. They investigated that the behavior of the double-layers solution is extremely sensitive to the non-extensive parameters of electrons and ions as well as the Mach number strength. El-Monier and Atteya [35] used the bifurcation analysis to investigate the DA wave propagation in four-component dusty plasma. Abulwafa et al. [36] investigated the formation and propagation of small amplitude nonlinear waves in a four-component quantum dusty plasma. Therefore, we plan to extend the previous our study by investigating the fully nonlinear arbitrary amplitude waves in quantum dusty plasma by employing the Sagdeev pseudo-potential technique and bifurcation analysis. The effect of physical parameters such as Mach number M, dust charge-mass ratio R, temperature ratio σ, electron density ratio μ_e, ion density ratio μ_i and electron quantum parameter H_e are studied and found that they play vital role on the formation of both compressive and rarefactive solitary waves and on the creation of compressive double layers.

The quantum dusty plasma basic equations are presented in Sect. 2. Sagdeev Pseudo-potential and phase portrait analysis are derived in Sect. 3. Nonlinear analysis are explained and discussed in Sect. 4 while in Sect. 5, the conclusion is provided.

Basic Equations

Quantum dusty plasma system has four components comprising of inertial positive and negative dust species and inertialess quantum ions and electrons is considered. Such dusty plasma medium can be existed in many astrophysical situations such as Jupiter's magnetosphere, upper and lower mesosphere and comet tails [37-41]. For simplicity, the dust grains are considered to have a constant surface charge. The normalized basic equations of the considered system are described as follows:

The dynamic equations for quantum electrons and ions are described by

$$\begin{gathered}\frac{\partial \varphi}{\partial x}-\sigma_j n_j \frac{\partial n_j}{\partial x}+ \\ +\frac{\delta_j}{2} H_j^2 \frac{\partial}{\partial x}\left(\frac{1}{\sqrt{n_j}} \frac{\partial^2}{\partial x^2} \sqrt{n_j}\right)=0\end{gathered}, \qquad (1)$$

where j = e and i refer to electrons and ions, respectively. n_j represents the density number, $H_j = (\hbar\omega_{pd}/C_d^2)\sqrt{Z_-/(m_j m_-)} = \hbar\omega_{pd}/(k_B T_{Fi})\sqrt{m_-/(m_j Z_-)}$ is the quantum factor that is defined as the ratio of the quantum energy to the thermal energy of the particle, $C_d = (k_B T_{Fi} Z_-/m_-)^{1/2}$ is the DA speed, $\omega_{pd} = [4\pi n_{0-}(Z_-)^2 e^2/m_-]^{1/2}$ is the dust plasma frequency and the temperature ratios are $\sigma_e = \sigma = T_{Fe}/T_{Fi}$, $\sigma_i = -1$ and $\delta_e = 1$, $\delta_i = -1$.

Both continuity and momentum equations for the two dust species are given by

$$\frac{\partial n_k}{\partial t} + \frac{\partial (n_k u_k)}{\partial x} = 0, \tag{2a}$$

$$\frac{\partial u_k}{\partial t} + u_k \frac{\partial u_k}{\partial x} + R_k \frac{\partial \phi}{\partial x} = 0, \tag{2b}$$

where k = + and − refer to positive dust and negative dust, respectively, n_k is the particle density number, u_k is the particle fluid velocity and the dust charge-mass ratios are $R_+ = R = \alpha/\beta$ where $\alpha = Z_+/Z_-$ and $\beta = m_+/m_-$ while $R_- = -1$.

Finally, the Poisson's equation of the given quantum dusty plasma system can be written as

$$\partial^2\phi/\partial x^2 - n_- + \mu_+ n_+ - \mu_e n_e + \mu_i n_i = 0. \tag{3}$$

The following normalizations $x \to x/\lambda_{Dd}$, $t \to t\omega_{pd}$, $n_{j(k)} \to n_{j(k)}/n_{0j(k)}$, $u_k \to u_k/C_d$, $\phi \to e\phi/(2k_B T_{Fi})$ have been applied into (1)-(3). In the above equations, ϕ is the electrostatic potential, $n_{0j(k)}$ refers to the unperturbed plasma species density number. Here, $\lambda_{Dd} = [2k_B T_{Fi}/(4\pi n_{0-} Z_- e^2)]^{1/2}$ is the dust Debye length and the species density ratio is $\mu_j = Z_j n_{0j}/(Z_- n_{0-})$. In addition, k_B, T_{Fj}, Z_k, $m_{j(k)}$ and e refer to the Boltzmann constant, the species type Fermi temperature, the number of electronic charge residing on the surface of the dust particle, the particle mass and electronic charge, respectively.

The quantum effects due to dust grains are ignored in the considered model as they have large inertia in comparison with both electrons and ions. In addition, the two quantum factors H_e and H_i are governed by the relation $H_e/H_i = \sqrt{m_i/m_e}$.

Sagdeev Pseudo-potential Analysis

To investigate the existence and propagation of the arbitrary amplitude localized electrostatic waves in the plasma system under consideration, we consider moving coordinate ansatz

$$\zeta = x - Mt, \tag{4}$$

that moves parallel to the mentioned waves with Mach number M.

Inserting the travelling wave transformation (4) in the system of equations (1) and (2), one gets a system of nonlinear coupled ordinary differential equations, viz.

$$\frac{d\phi(\zeta)}{d\zeta} - \sigma_j n_j(\zeta)\frac{dn_j(\zeta)}{d\zeta} + \frac{\delta_j}{2} H_j^2 \frac{d}{d\zeta}\left[\frac{1}{\sqrt{n_j(\zeta)}}\frac{d^2}{d\zeta^2}\sqrt{n_j}(\zeta)\right] = 0, \tag{5}$$

$$-M\frac{dn_k(\zeta)}{d\zeta} + \frac{d[n_k(\zeta)u_k(\zeta)]}{d\zeta} = 0, \tag{6a}$$

$$-M\frac{du_k(\zeta)}{d\zeta} + u_k(\zeta)\frac{du_k(\zeta)}{d\zeta} + R_k\frac{d\phi(\zeta)}{d\zeta} = 0, \tag{6b}$$

while the Poisson's equation can be rewritten as

$$d^2\varphi(\zeta)/d\zeta^2 = n_-(\zeta) - \\ -\mu_+ n_+(\zeta) + \mu_e n_e(\zeta) - \mu_i n_i(\zeta) . \tag{7}$$

where all physical quantities in (5) and (6) become only functions of a single variable ζ .

With appropriate boundary conditions, viz. $n \to 1$, $dn/d\zeta \to 0$, $u \to 1$ and $\phi \to 0$ as $\zeta \to \pm\infty$, the forms of density functions n_j of electrons and ions and n_k of dust particles in terms of electrostatic potential ϕ can be explicitly obtained after integrating (5) and (6) as

$$n_j(\zeta) = \sqrt{[1+2\phi(\zeta)/\sigma_j] + \delta_j(H_j^2/\sigma_j)[1+2\phi(\zeta)/\sigma_j]^{-\frac{1}{4}} d^2[1+2\phi(\zeta)/\sigma_j]^{\frac{1}{4}}/d\zeta^2} , \tag{8}$$

$$n_k(\zeta) = M/\sqrt{M^2 - 2R_k\phi} . \tag{9}$$

By looking upon (8), one may see that the density function of both electrons and ions $n_j(\zeta)$ depend obviously on the corresponding quantum factors H_j, whereas (9) shows that the dependence of dust densities $n_k(\zeta)$ on H_j is not exist. In addition, in order to guaranty the plasma state variables $n_k(\zeta)$ are analytical functions (i. e; real) we have to recall the well-known classical inequality, $\phi \le \phi_{\max} = M^2/(2R_k)$ where $\phi_{\max}$ refers to the maximum value of ϕ (the amplitude of nonlinear waves in the plasma). However, it provides us a useful check for high amplitude limits in terms of the dust charge-mass ratio R and Mach number M. Inserting the forms of $n_j(\zeta)$ and $n_k(\zeta)$, (8) and (9), into the reduced Poisson's equation (7), one obtains

$$d^2\phi/d\zeta^2 = -\rho(\phi) , \tag{10}$$

where the charge density $\rho(\phi)$ is represented by

$$\begin{aligned} \rho(\phi) = {} & -M/\sqrt{M^2+2\phi} + \mu_+ M/\sqrt{M^2-2R\phi} \\ & - \mu_e\sqrt{(1+2\phi/\sigma)+(H_e^2/\sigma)(1+2\phi/\sigma)^{-1/4} d^2(1+2\phi/\sigma)^{1/4}/d\zeta^2} \\ & + \mu_i\sqrt{(1-2\phi)+H_i^2(1-2\phi)^{-1/4} d^2(1-2\phi)^{1/4}/d\zeta^2} , \end{aligned} \tag{11}$$

Now, the advantage of using Sagdeev pseudo-potential technique is that the four-component quantum system equations are reduced to a single equation (10). This implies that studying the fully nonlinear structures of the plasma system is equivalent to study the solution of (10).

Multiplying (10) by $d\phi/d\zeta$ and integrating once, the order of (10) is reduced by one and can be written in the form of an energy balance integral equation

$$(d\phi/d\zeta)^2/2 + V(\phi) = E , \tag{12}$$

where

$$V(\phi) = -\int^{\phi} d\phi'\rho(\phi') , \tag{13}$$

and E is the integration constant. The first term of (12) refers to the kinetic energy whereas $V(\phi)$ is known as Sagdeev pseudo-potential energy function. This procedure is called Sagdeev pseudo-potential formalism. Using appropriate boundary conditions $\phi = d\phi/d\zeta = d^2\phi/d\zeta^2 \to 0$, at $\zeta \to \pm\infty$, the right side of (12) goes to zero.

Having integrated (13), $\phi = +\int^{\phi} d\phi'\sqrt{-V(\phi')}$, the existence and propagation of the fully nonlinear structures in our plasma system can be identified exactly.

It is noted that knowing the explicit form of the Sagdeev potential $V(\phi)$ enables us to obtain the maximum and minimum values of the Mach number (M_{max}, M_{min}) at which the permitted nonlinear waves exist. The maximum value of Mach number M_{max} can be defined, in terms of plasma parameters, by solving the algebraic equation $V(\phi_{\max}) = 0$ for M where $\phi_{\max} = M^2/(2R_k)$, the maximum values of the electrostatic potential ϕ for compressive and rarefactive waves. But on the other side, the minimum value of the Mach number M_{min} can be determined by solving the inequality $d^2V/d\phi^2 < 0$

at $\phi = 0$. The interval of Mach number ($M_{min} - M_{max}$) leads us to identify the existence range at which the formula of nonlinear waves exists and propagates in the plasma model. It is anticipated from the above analysis that the existence and propagation of the nonlinear waves in the plasma system requires the values of Mach number M included in $M_{\min} < M < M_{\max}$, otherwise, the existence of nonlinear waves is not possible.

With the knowledge of the explicit form of Sagdeev potential function $V(\phi)$, the existence conditions for the localized nonlinear waves are readily examined. However, the conditions for solitary waves to exist are:

(i) $V(\phi) = dV/d\phi = 0$ at $\phi = 0$, which implies that the associated electric field ($E(\phi) = -d\phi/d\zeta$) and the charge density $\rho(\phi)$ equal zero at the origin ($\phi = 0$). Also, at the origin the potential condition $d^2V/d\phi^2 < 0$ should be satisfied, implying that the curve has maximum at $(0,0)$ and hence the origin can be viewed as fixed unstable point.

(ii) The final condition $V(\phi) < 0$, is necessary for obtaining the configurational plasma parameters for which this condition holds. This means that the electrostatic potential ϕ follow the interval $0 < \phi < \phi_{\max}$ for compressive waves or $\phi_{\max} < \phi < 0$ for rarefactive solitary waves. Here, $\phi_{\max}$ denotes the first non-zero root of $V(\phi) = 0$.

On the other side, the existence of the double-layer requires the following two conditions:

(i) $V(\phi) = dV/d\phi = 0$ at $\phi = 0$, and $\phi = \phi_{\max}$.

(ii) $d^2V/d\phi^2 < 0$ at $\phi = 0$, and $\phi = \phi_{\max}$.

Therefore, we are interested to specify the fully nonlinear structures propagating in the plasma system. The crucial point of this methodology is that the form of the Sagdeev potential function should be known explicitly by integrating (13). Since the integration procedure is very difficult task, it is convenient to deal with such equation numerically. Then, we recall the bifurcation analysis to investigate the behavior and properties of the fully nonlinear structures graphically. In addition, the impact of some relevant plasma parameters on the nonlinear waves can be also examined.

Nonlinear Analysis

First, we shall investigate the influence of the physical plasma parameters such as (Mach number M, dust charge-mass ratio R, temperature ratio σ, electron density ratio μ_e, ion density ratio μ_i and electron quantum parameter H_e) on the profile of the nonlinear structures. The obtained results can be summarized in Figures (1) – (5), where the physical plasma parameters follow $n_{e0} \approx 2\times10^{30} m^{-3}$, $T_{Fe} \approx 1.0\times10^2 K$, $n_{i0} \approx 1\times10^{30} m^{-3}$, $T_{Fi} \approx 1/6\times10^2 K$, $n_{-0} \approx 5\times10^{26} m^{-3}$, $Z_- \approx 10^3$, $m_- \approx 10^{-17} kg$, $n_{+0} \approx 1.5\times10^{27} m^{-3}$, $Z_+ \approx 10^3$, $m_+ \approx 10^{-17} kg$ [2, 42, 43].

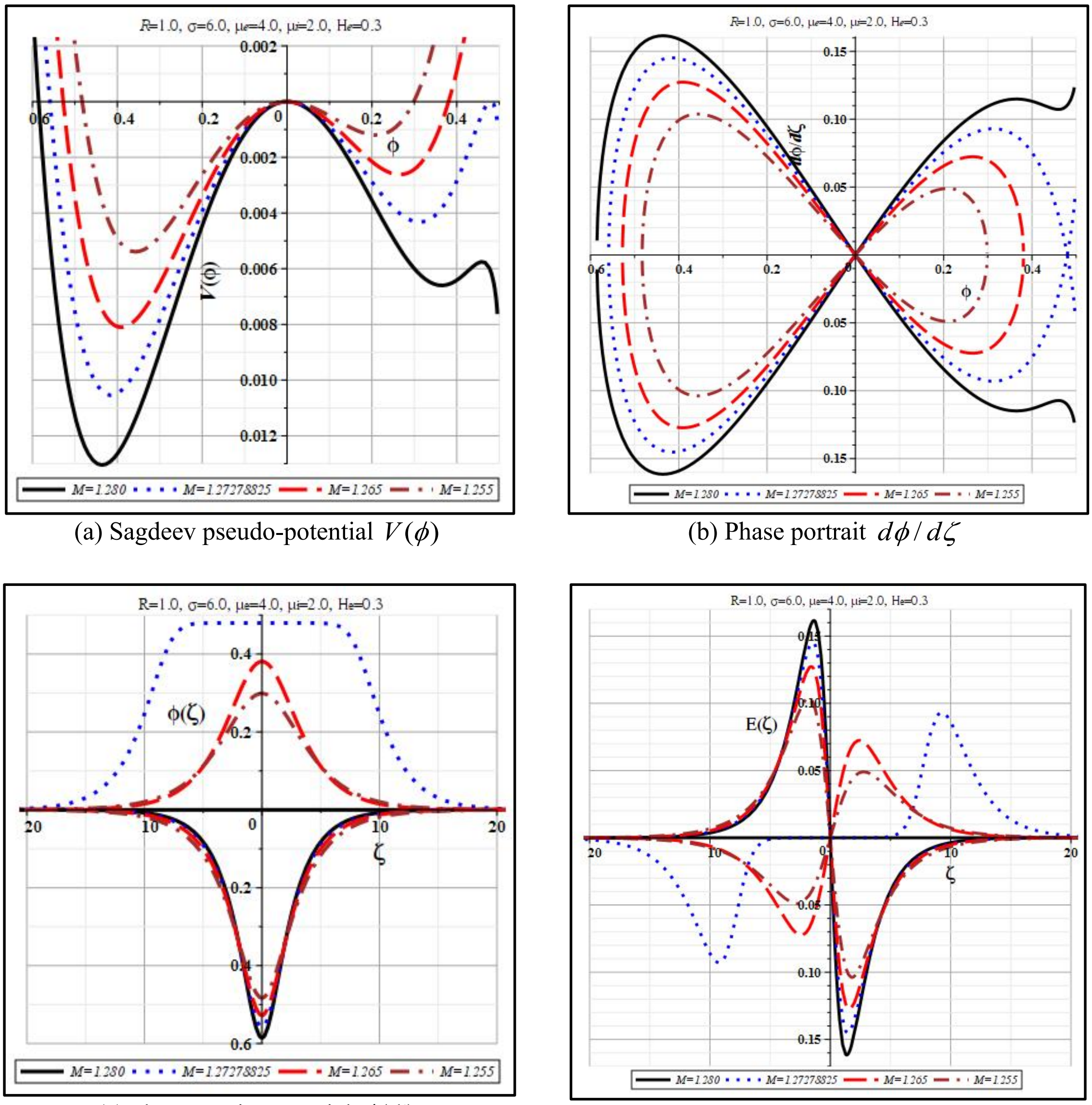
In Figs (1), the Sagdeev potential function $V(\phi)$, phase portrait ($\phi, d\phi/d\zeta$), electrostatic potential $\phi(\zeta)$ and associated electric field $E(\zeta)$ are shown graphically for different values of the Mach number M. The chosen values of the Mach number M are taken in our calculations according to the allowed interval ($M_{min} - M_{max}$), where the other physical parameters are $R = 1.0$, $\sigma = 6.0$, $\mu_e = 4.0$, $\mu_i = 2.0$, $H_e = 0.3$. Note that the width of the Sagdeev potential well means the amplitude of the solitary wave and its depth refers to the slop of the solitary pulse. In Fig (1a), it is seen that the dash-dotted potential curve (M = 1.255) and dashed curve (M = 1.265) refer to the coexistence of both compressive and rarefactive localized pulses. Obviously, increasing the Mach number, the potential well gets wider and deeper. This means that faster electrostatic pulse gets taller and narrower as the Mach number increases. Also, it can be seen that the amplitude and width of the rarefactive solitary waves are clearly greater than those of the compressive waves. The corresponding trajectories in the phase portrait diagram ($\phi, d\phi/d\zeta$), Fig (1b), have obviously two stable centers at $(\phi_1, 0)$ and

$(\phi_2,0)$ beside one unstable saddle point at $(0,0)$, where ϕ_1 and ϕ_2 are the values of the electrostatic potential ϕ at which the Sagdeev potential $V(\phi)$ is minimum. The trajectories enclosing the two centers points and passing through the saddle point is called homoclinic orbits, which refer to stable solitary waves. The profiles and properties of the solitary waves $\phi(\zeta)$ and the associated electric field $E(\zeta)$ are shown graphically in Figs (1c) and (1d). Now, it is concluded that our plasma system supports the formation of both compressive and rarefactive to coexist in this plasma configuration. Moving to higher value of the Mach number, the dashed line curve in the potential diagram, Fig (1a), shows another feature of the nonlinear waves. The dotted curve at $M = 1.27278825$ denotes to the coexistence of both compressive double-layer and rarefactive nonlinear solitary wave. The existence of double-layer can be seen as a sudden change of the electrostatic potential pulse $\phi(\zeta)$ due to the presence of space charge. The corresponding trajectory in the phase-portrait, Fig (1b), has two unstable saddle points at $(0,0)$ and $(\phi_{\max},0)$ beside a stable center point at $(\phi_1,0)$. The trajectory passing through two unstable saddle points is called heteroclinic orbit and refers to double layer. On the other side of ϕ ($\phi < 0$), there exist one stable center point at $(\phi_2,0)$ and one unstable saddle point at $(0,0)$. The orbit going from saddle point confirms the existence of rarefactive nonlinear waves. The behavior of the electrostatic potential $\phi(\zeta)$ and the associated electric field $E(\zeta)$ (dotted line curve) can be graphically shown as in Figs (1c) and (1d). Further increasing the Mach number, the solid curve at $M = 1.28$ shows that the positive amplitude nonlinear wave is no longer exist and we are only left with the rarefactive solitary wave in this plasma configuration.

In addition to the Mach number *M*, the effect of dust charge-mass ratio *R* on the formation and existence range of nonlinear waves can be also shown by introducing the bifurcation diagrams, Figs (2). In Figs (2a) and (2b), the dashed-dot curve at *R* = 1.03 and the dashed curve at *R* = 1.0 reveal that the coexistence of both compressive and rarefactive solitary waves can be observed. On the other side, when *R* – 0.96159 (dotted curve) another feature of special interest arises, where compressive double-layer and rarefactive solitary wave coexist. Decreasing further slightly the value of *R* (*R* = 0.95), only rarefactive solitary waves may be observed. The bifurcation curves also illustrate that the amplitude of the rarefactive solitary waves is greater than those of compressive solitary waves. From above, one may conclude that the dust charge-mass ratio *R* plays a significant role in formation of different types of nonlinear waves in this plasma system. The profiles of the different types of electrostatic solitary waves and the associated electric field are graphically shown in Figs (2c) and (2d), where the amplitude (width) decreases (increases) as *R* increases. This implies that the dust charge-mass ratio *R* would shrink the formation of nonlinear solitary waves.

Furthermore, in the same manner the effects of other configurational plasma parameters can be also achieved. However, the temperature ratio σ and the electron density ratio μ_e have the same behavior as that of the dust charge-mass ratio *R*, where the amplitude (width) of the nonlinear solitary waves decreases (increases) as σ and μ_e increases as shown in Figs (3) and (4), respectively. In the contrary, the effect of the ion density ratio μ_i has an opposite property, where the amplitude (width) of the nonlinear waves increases (decreases) with an increasing in μ_i. This means that the nonlinear electrostatic solitary waves and the strength of the associated bipolar electric field get stronger by increasing μ_i, as shown clearly in Fig (5).

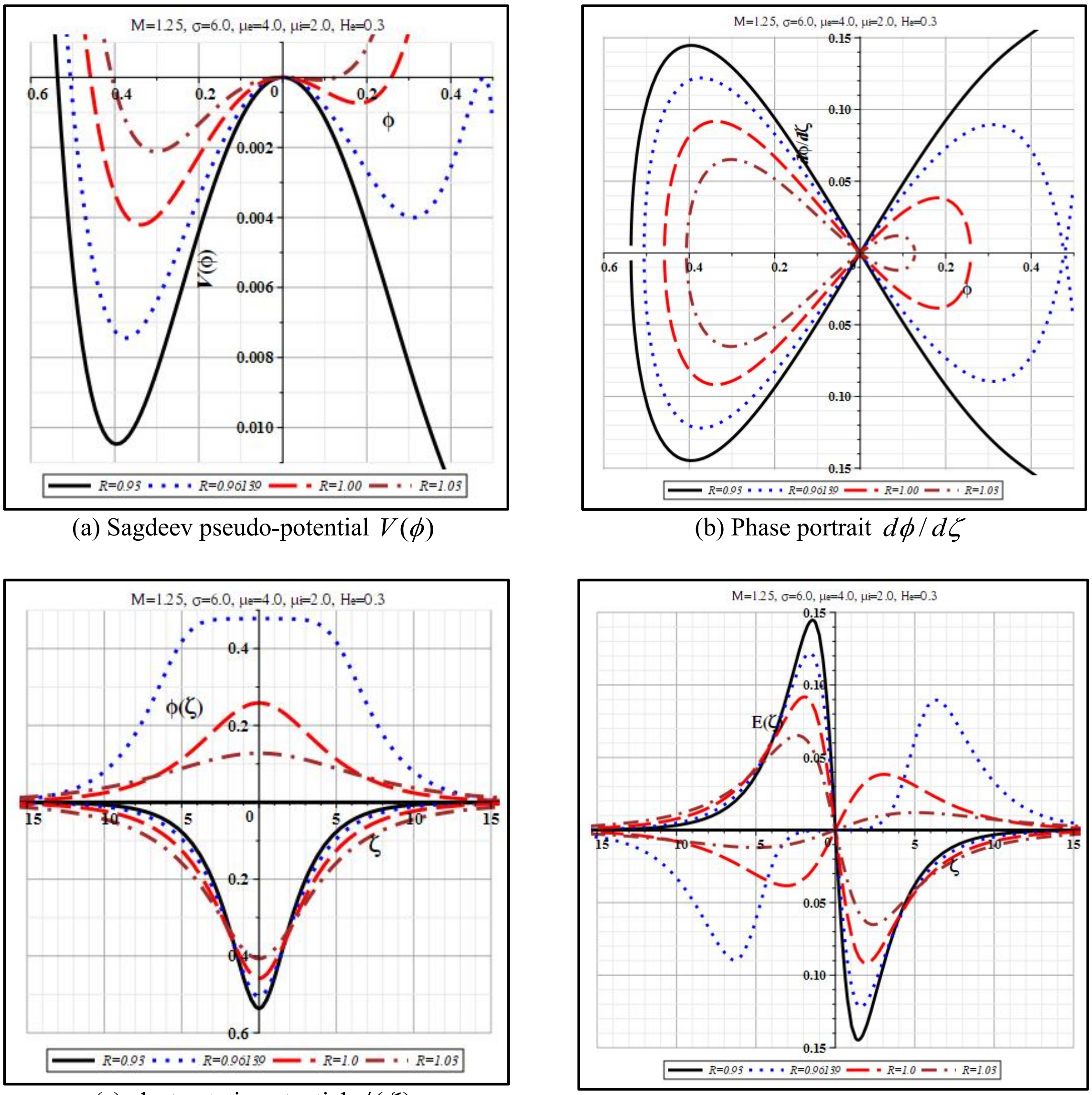
Finally, in order to see the impact of quantum index H_e, the bifurcation analysis is graphically plotted, Figs (6), for different values of H_e (H_e = 0.0, 1.0, 2.0, 3.0) where the other chosen parameters are M = 1.25, R = 1.0, $\sigma = 6.0$, $\mu_e = 4.0$, $\mu_i = 2.0$. Obviously, this analysis supports the coexistence of both compressive and rarefactive solitary waves, where the amplitudes are nearly the same while the depth of the soliton decreases as H_e increases, as evidently in Figs (6). In addition, the strength of the associated electric field increases slightly as H_e increases.



(a) Sagdeev pseudo-potential $V(\phi)$ (b) Phase portrait $d\phi/d\zeta$

(c) electrostatic potential $\phi(\zeta)$ (d) electrostatic field $\mathrm{E}(\zeta)$

Figure 1 – Nonlinear structure functions at different values of M for $R = 1.0$, $\sigma = 6.0$, $\mu_e = 4.0$, $\mu_i = 2.0$, $H_e = 0.3$



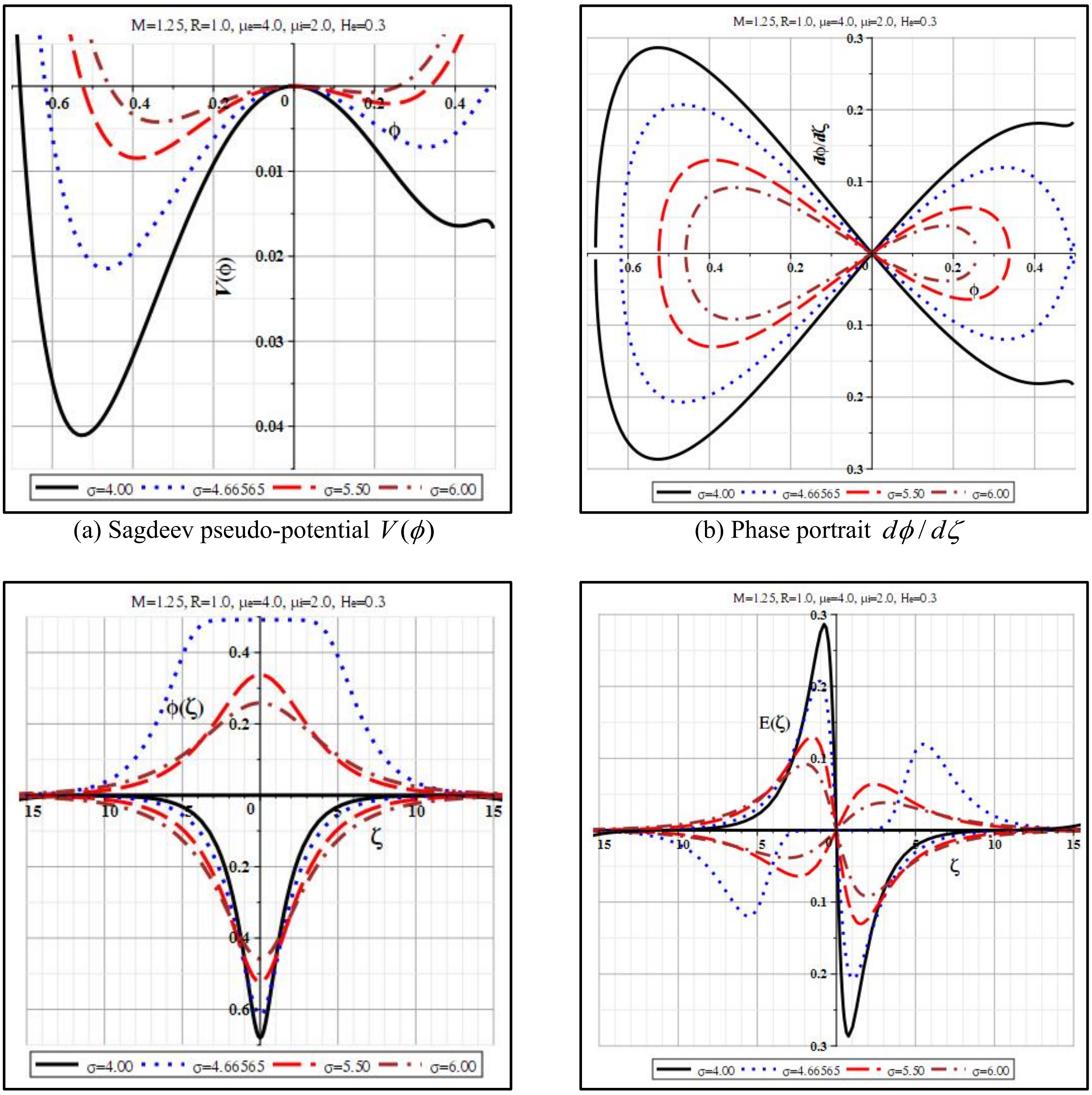
(a) Sagdeev pseudo-potential $V(\phi)$

(b) Phase portrait $d\phi / d\zeta$

(c) electrostatic potential $\phi(\zeta)$

(d) electrostatic field $\mathrm{E}(\zeta)$

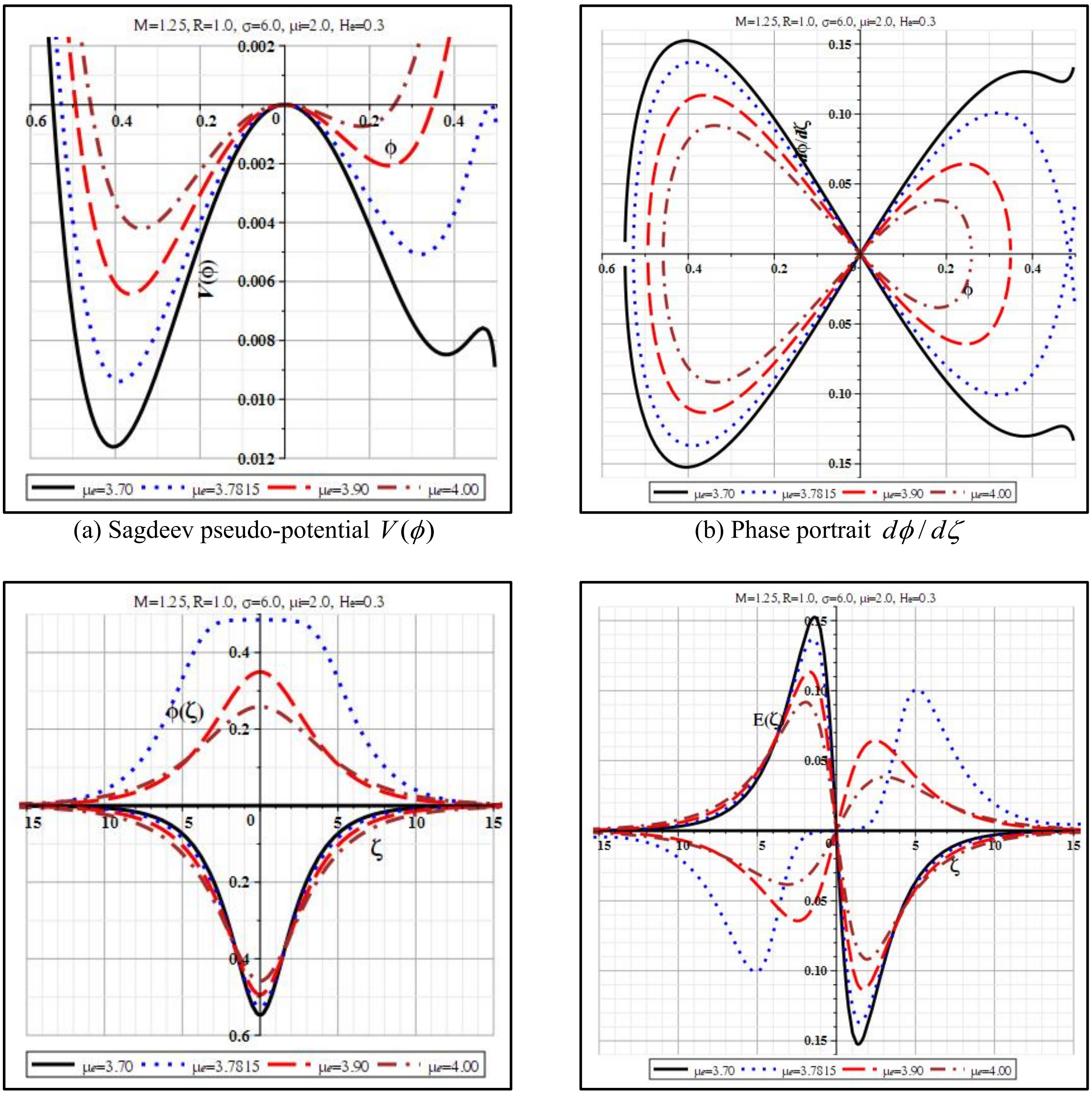
Figure 2 – Nonlinear structure functions at different values of R for $M = 1.25$, $\sigma = 6.0$, $\mu_e = 4.0$, $\mu_i = 2.0$, $H_e = 0.3$



(a) Sagdeev pseudo-potential $V(\phi)$ (b) Phase portrait $d\phi/d\zeta$

(c) electrostatic potential $\phi(\zeta)$ (d) electrostatic field $\mathrm{E}(\zeta)$

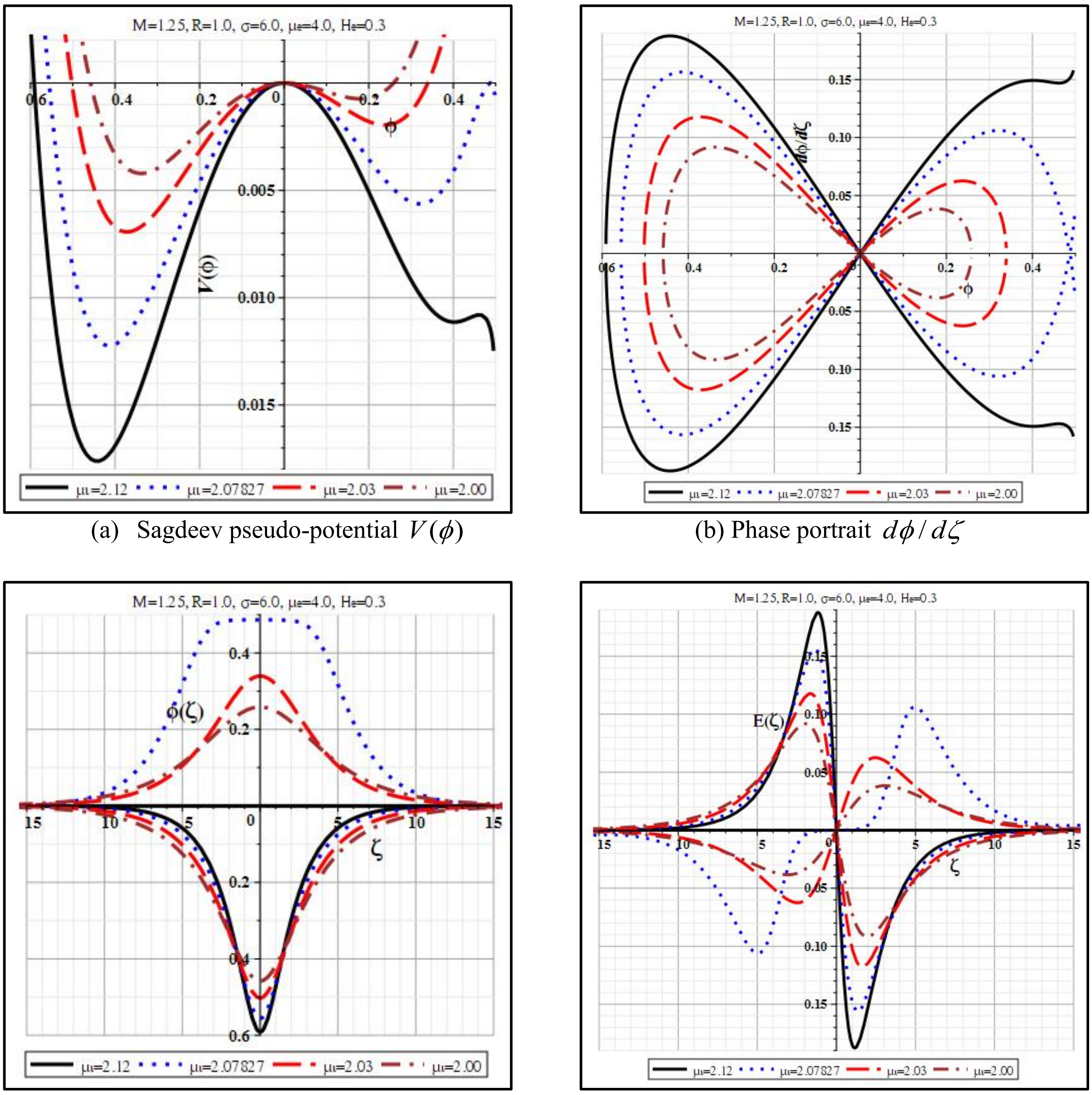
Figure 3 – Nonlinear structure functions at different values of σ for $M = 1.25$, $R = 1.0$, $\mu_e = 4.0$, $\mu_i = 2.0$, $H_e = 0.3$



(a) Sagdeev pseudo-potential $V(\phi)$ (b) Phase portrait $d\phi/d\zeta$

(c) electrostatic potential $\phi(\zeta)$ (d) electrostatic field $\mathrm{E}(\zeta)$

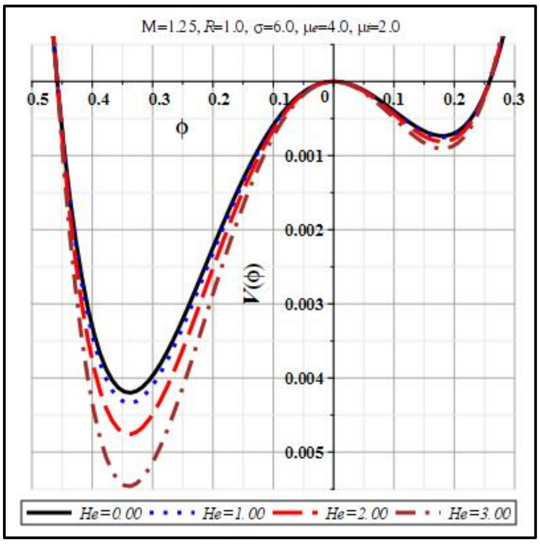
Figure 4 – Nonlinear structure functions at different values of μ_e for $M=1.25$, $R=1.0$, $\sigma=6.0$, $\mu_i=2.0$, $H_e=0.3$



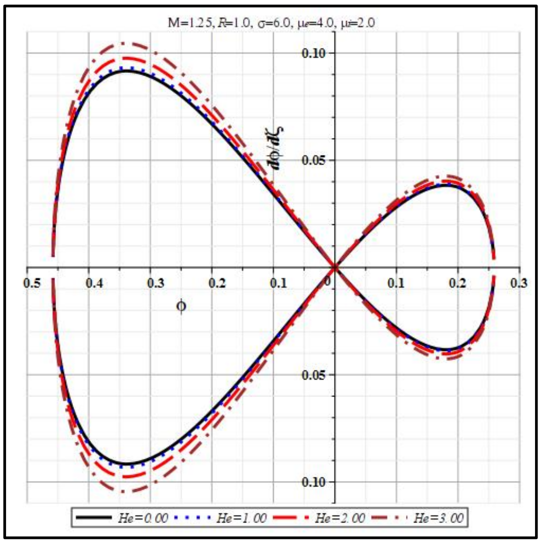
(a) Sagdeev pseudo-potential $V(\phi)$ (b) Phase portrait $d\phi/d\zeta$

(c) electrostatic potential $\phi(\zeta)$ (d) electrostatic field $\mathrm{E}(\zeta)$

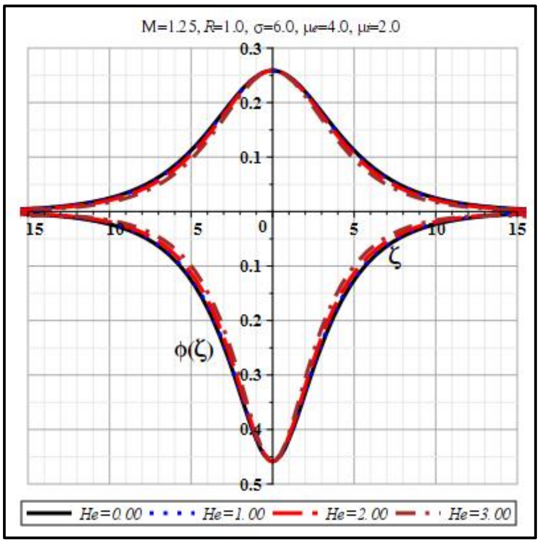
Figure 5 – Nonlinear structure functions at different values of μ_i for $M = 1.25$, $R = 1.0$, $\sigma = 6.0$, $\mu_e = 4.0$, $H_e = 0.3$



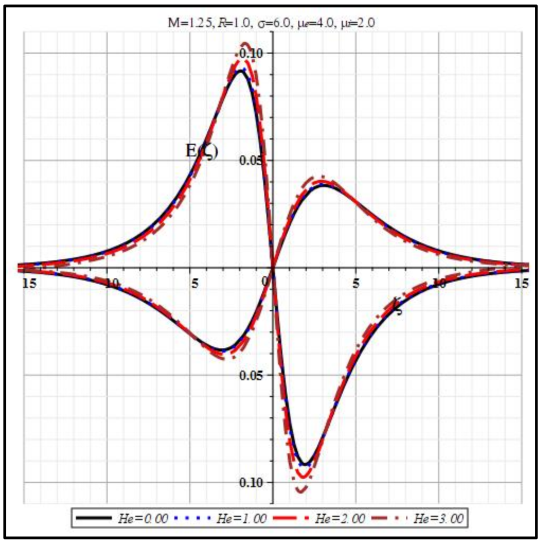
(a) Sagdeev pseudo-potential $V(\phi)$



(b) Phase portrait $d\phi/d\zeta$



(c) electrostatic potential $\phi(\zeta)$



(d) electrostatic field $\mathrm{E}(\zeta)$

Figure 6 – Nonlinear structure functions at different values of H_e for $M=1.25$, $R=1.0$, $\sigma=6.0$, $\mu_e=4.0$, $\mu_i=2.0$

Conclusions

In this work, we have considered a four-component quantum dusty plasma consisting of positive and negative dust particulates, electrons and ions that are assumed to be quantum mechanical. In order to examine the fully arbitrary nonlinear waves in such plasma model, Sagdeev pseudo-potential method has been first employed. The inclusion of the quantum effect makes the mathematical manipulations difficult task. Therefore, we recall the bifurcation analysis to achieve this task graphically rather than analytically. Based on bifurcation analysis, one may be able to distinguish three types of nonlinear arbitrary amplitude waves, namely compressive solitons, rarefactive solitons and double-layers depending on the magnitude of the configurational plasma parameters. The effects of some relevant plasma parameters on the profile of electrostatic potential $\phi(\zeta)$ and the associated bipolar electric field $E(\zeta)$ have been extensively studied. The nonlinear arbitrary solitary waves get stronger as the Mach number M and ion density ratio μ_i increase, while the dust charge-mass ratio R, temperature ratio σ and electron density ratio μ_e have opposite properties. Additionally, the analysis reveal that, in a certain plasma configuration, the coexistence of both compressive and rarefactive solitary waves occurs, while double-layer and rarefactive soliton are found to coexist also. The existence of double-layer provides us the existence range of compressive waves, for example Mach number M value of double-layer can be viewed as the maximum range of compressive solitons. In other plasma configuration, the analysis shows that the double-layer is no longer exist and we have left only with arbitrary amplitude rarefactive solitary waves. It is also found that the quantum index H_e influence the width of electrostatic potential profile only, while the amplitude remains nearly the same.

The present results of this work are applicable to investigate the dynamical characteristics of nonlinear waves in quantum space dusty plasmas.

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