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Motion of stars near the galactic center

Abstract. We investigate the motion of stars at the central region of the Milky Way Galaxy. We consider two possible scenarios: 1) there is a supermassive black hole without dark matter distribution and 2) there is a dark matter core without any black hole in the galactic center. We remark that, with current observations, one cannot distinguish the two separate cases from each other at distances larger than 100 astronomical units. Here, we present improved analyses regarding the motion of stars and show that differences between these two cases appear at distances smaller than 30 astronomical units. We adopt here the so-called exponential density profile for dark matter distribution which is usually used to explain the rotation curve data points in the bulge of galaxies. For simplicity, all computations have been performed within the Newtonian gravity though the effects of general relativity become more pronounced close to the center of the galaxy in the presence of the supermassive black hole without dark matter. We underline our outcomes are model-dependent, i.e. they depend on the choice of the particular dark matter model employed for the analysis.

Key words: dark matter, black hole, motion of stars, exponential density (sphere) profile.

Introduction

Recently, the most relevant problems of modern astrophysics and cosmology, such as the problem of the formation of super massive black holes (SMBHs) and the nature of dark matter (DM), have often been debated in the literature [1]. The relevance of the first topic is connected with the latest results of observations of merging black holes (BHs) by the Laser Interferometer Gravitational-Wave Observatory (LIGO) [2, 3], observations of the shadow of a black hole in the galaxy M87 using the Event Horizon Telescope (EHT) [4-6], and with observations of very distant quasars — one of the brightest objects in the Universe [7], which left almost no doubt about the existence of supermassive black holes. It is believed that quasars emit energy due to the release of the potential gravitational energy of gas falling onto a supermassive black hole (usually with a mass of several billion solar masses [8]). Since quasars have been discovered no later than 650 million years after the "beginning" of the expansion of the Universe, then, probably, SMBHs must have already formed earlier. From the theoretical point of view, it is quite unclear exactly what processes/circumstances led to the formation of such compact and massive objects in a relatively short time.

On the other hand, regarding dark matter (DM), there is currently no direct evidence of its existence, but there are a plenty of indirect observations. All of them are mainly based on the specific behavior of astrophysical objects, in particular, on the abnormally high rotation speed of the outer regions of galaxies [9, 10]. In addition, the presence of DM is evidenced by numerous indirect observations, for example, the movement of clumps of hot gas in galactic clusters and gravitational lensing [11].

Studies of the motion of S-stars near the center of Milky Way Galaxy (MWG) have recently shown that the gravitational potential in which they move is determined by a massive a compact source in the center, Sagittarius A* (Sgr A*). As it was shown in the literature [12, 13] that the mass of Sgr A* is equal to $\sim 4\times10^{6}M_{\odot}$, such a mass is traditionally associated with the mass of a SMBH. However the latest observations of G2 object show that its trajectory cannot be fully explained in terms of BH geometry [14]. Instead as it was shown in Ref. [15] that the motion of G2 object can be well explained by the so called Ruffini-Arguelles-Rueda (RAR) model proposing an alternative nature of Sgr A* based on the fermionic DM model which leads to a dense core and diluted halo density profile [16]. The RAR model has been successfully applied to the motion of S-stars around the galactic center providing in some cases better results with respect to the SMBH case [17]. Thus the RAR model explicitly demonstrated that the DM nature of Sgr A* is quite viable.

In this work, we study the motion of test particles, under the form of stars, near the center of the MWG in the gravitational field of both a BH and DM core. In particular, we here consider a toy model formulated in Ref. [18] based on the exponential sphere DM profile first introduced in Ref. [19]. This approach can be used to model DM density distribution in the interior of the MWG and from the practical/technical point of view the model is easy to handle.

We remark that, with current observations, two possible scenarios are possible: 1) there is a SMBH without DM distribution and 2) there is a DM core without any BH in the galactic center. In principle, one cannot distinguish the two separate cases from each other at distances larger than 100 astronomical units (AU).

The aim of the work is to investigate the motion of stars near the galactic center at distances smaller than 100 AU and show that differences between these two cases appear at distances smaller than 30 astronomical units.

The paper is organized as follows: in Section 1 we review the exponential sphere profile and discuss about it features for the core of the MWG. We also show that Newtonian Gravity is quite sufficient for our analyses. In Section 2 we write down the equations of motion of stars near the galactic center in Newtonian Gravity. In Section 3 we present results obtained by numerical integration of the equations of motion. Finally in Conclusion we have a brief discussion of possible future observations that can help verify the validity of the proposed model. Our conclusions are summarized also there.

Exponential sphere profile

The distribution of DM in galactic halos usually is widely interpreted by the Einasto, Burkert, Navarro-Frenk-White, pseudo-isothermal etc profiles [20]. For the galactic center however there are not so many profiles in the literature, though there were some observations indicating the presence of DM in the central region of galaxies [21]. We adopt here the so-called exponential sphere profile first proposed in Ref. [19] to explain the rotation curve in the bulge (main and inner) of the MWG. In Ref. [18], the rotation curve data of the MWG close to the center using the exponential sphere profile, assuming the absence of a SMBH, has been computed, even for the case of SMBH where its gravitational field dominates in the absence of DM. The rotation curve near the galactic center was well fitted by both scenarios [18].

So, it turned out that not only the inner and main bulges, but also the core of the Galaxy can be well modelled by means of the exponential sphere profile, which is given by

$$\rho(r)=\rho_0 e^{-r/r_0}, \qquad (1)$$

where ρ_0 is the central density and r_0 is the scale radius, which are the free parameters of the model inferred from the rotation curve. It has been shown in Table 1 of Ref. [18] that the free parameters have the following values $\rho_0=5.873\times10^{19}\,M_{\odot}/\mathrm{pc}^3$ and $r_0=1.417\times10^{-5}\,\mathrm{pc}=2.923\ \mathrm{AU}$ when there were not any constraints imposed on the size and mass of the galactic core.

Knowing the form of the density profile (1) one can easily calculate the mass profile which is given by

$$M(r)=\int_0^r 4\pi r^2\rho(r)\,dr=M_0F(\xi), \qquad (2)$$

where $M_0=\lim_{r\to\infty}M(r)=8\pi r_0^3\rho_0$ is the total mass of the DM core, $\xi=r/r_0$ is the dimensionless radial coordinate and

$$F(\xi)=1-e^{-\xi}\left(1+\xi+\frac{\xi^2}{2}\right). \qquad (3)$$

It should be noted that numerical results from the fit show that the total mass of DM in the core is equal to the mass of a SMBH from an alternative scenario i.e. $M_0=M_{BH}$.

In Fig. 1 we show the density and mass profiles of the exponential sphere model given by Eqs. (1) and

(2). Close to the center the density becomes constant and far from the center it decreases exponentially. Instead the mass at distances larger than $\xi = 10$ or $r \approx 30$ AU becomes almost constant.

Before studying the motion of test particles (stars) in the vicinity of the galactic center we have analysed the linear velocity and gravitational potential of test particles both in Newtonian Gravity (NG) and General Relativity (GR) to make sure that our computations within NG do not violate slow motion and weak field regimes. In Table 5 of Ref. [22] the numerical values of linear velocity υ^2/c^2 and gravitational potential Φ/c^2 for a test particle in the fields of the SMBH and DM distribution have been confronted. As one can see from there the contribution of GR to DM is negligible. In the case of a BH (or point mass in NG) the effects of GR become dominant at distances smaller than 5 AU for both physical quantities. Thus one can safely perform all the analyses in NG (for more details see [22]).

It should be mentioned that for more precise analyses one should anyway refer to GR as tiny effects due to the corrections of GR can be accumulated over the several million or even billion years and change the whole picture of the considered system. However for a short time scale NG works sufficiently well.

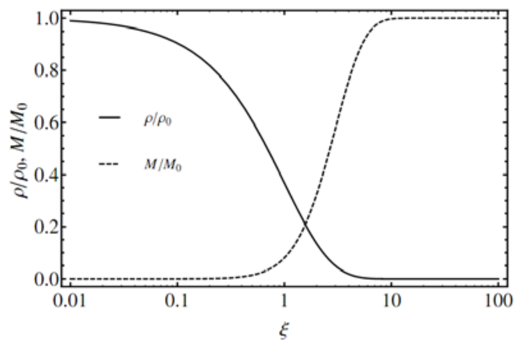

Figure 1 – Density and mass profiles of the exponential density model

Equations of motion of stars

In this article we consider two cases, similarly to Ref. [18] namely:

- It is assumed that there is a SMBH without DM at the galactic center.
- It is assumed that there is DM distribution without a SMBH at the galactic center.

These two cases are considered to be limiting cases, though there always exists possibility to have the combination of both at the same time as it was illustrated in Ref. [23].

For the above cases, we will use the following notation:

$$M = \begin{cases} M_{BH}, & \text{for BH}, \\ M(r), & \text{for DM}, \end{cases} \tag{4}$$

where M_{BH} is the BH mass and $M(r)$ is the mass of DM profile given by (2).

We consider here the motion of test particles in the gravitational fields of the SMBH and DM distribution with the aim of distinguishing the two sources from the observation of stars orbiting the galactic center. To describe the motion of a test particle in the field of DM and SMBH, it is necessary to write down the equations of motion. To this end, we use the second Newton's law and the law of universal gravitation

$$\vec{F}_N = m\frac{d^2\vec{r}}{dt^2}, \tag{5}$$

$$\vec{F}_G = -m\frac{GM}{r^3}\vec{r}, \tag{6}$$

where m is the mass of a test particle, M is the mass of the central body (BH or a point mass in NG) or DM mass profile (see Eq. 4), r is the radial coordinate (the distance from the galactic center to a considered point) as before. By equating the two forces, we obtain the equation of motion:

$$\frac{d^2\vec{r}}{dt^2} = -\frac{GM}{r^3}\vec{r}. \tag{7}$$

Since for simplicity we study the motion of test particles (stars) in xy plane then $\vec{r} = \vec{i}x + \vec{j}y$ and $r^2 = x^2 + y^2$. Correspondingly, Eq. (7) splits into two components as:

$$\begin{cases} \dfrac{d^2x}{dt^2} = -\dfrac{GMx}{\left(x^2+y^2\right)^{3/2}}, \\ \dfrac{d^2y}{dt^2} = -\dfrac{GMy}{\left(x^2+y^2\right)^{3/2}}. \end{cases} \tag{8}$$

From the definition of the gravitational potential

$$\frac{d\Phi(r)}{dr} = \frac{GM}{r^2} = \frac{\upsilon^2}{r}, \tag{9}$$

we obtain formulas for the linear velocity of a test particle in the gravitational field of a point mass and DM core

$$\upsilon=\begin{cases}\sqrt{\dfrac{GM_{BH}}{r}}, & \text{for BH,}\\ \sqrt{\dfrac{GM(r)}{r}}, & \text{for DM,}\end{cases} \quad (10)$$

Let us show for completeness the relationship between the force and potential of the gravitational field for BH and DM in NG.

$$\vec{F}_G=-m\vec{\nabla}\Phi=-m\frac{d\Phi}{dr}\frac{\vec{r}}{r}=-m\frac{GM}{r^3}\vec{r}\,, \quad (11)$$

where

$$\Phi=\begin{cases}\Phi_{ex}=-\dfrac{GM_{BH}}{r}, & \text{for BH,}\\ \Phi_{in}=-\dfrac{GM_0}{r}\left[1-e^{-\frac{r}{r_0}}\left(1+\dfrac{r}{2r_0}\right)\right], & \text{for DM,}\end{cases} \quad (12)$$

Φ_{ex} is the external gravitational potential of a spherically symmetric or point body and Φ_{in} is the internal potential describing the DM distribution in the galactic core. It is well known that the solution to the equations of motion (8) for a test particle, orbiting a point mass whose gravitational potential is given by Φ_{ex}, can be obtained analytically (The Kepler problem). However in the field of the DM core no analytic solution is available.

Numerical results

Now we consider the motion of a test particles both in the field of a DM core and BH. In a BH the whole mass is concentrated inside the event horizon, possibly in the central singularity whereas in DM the mass profile is spatially varying as a function of the radial coordinate.

Once we set r at $t = 0$ as an initial value, the velocity will be automatically calculated from Eq. (10). Thus, it is always possible to find circular geodesics/trajectories for test particles in both fields of SMBH and DM core. However to distinguish both objects we adopt as initial conditions the ones for circular orbits in the gravitational of the SMBH and use them also for the test particles in the field of the DM core. So we make sure that in the field of the SMBH the trajectories are always circular and analyze how they will look like in the field of DM core.

By numerically solving the system of equations of motion (8) taking into account the formula for the mass (4), we obtained the trajectories of stars with the same initial conditions in the gravitational field of a BH and DM core. Trajectories of stars are shown in Fig. 2 with different initial conditions. Red solid curves show circular orbits in the field of a point mass and black dashed curves show elliptical orbits in the DM distribution. Visual differences between the trajectories appear at distances less than 30 AU. The initial conditions were chosen in such a way to obtain circular orbits in the gravitational field of the point mass (SMBH), which lead to precessing elliptical orbits in the field of the DM core. Note, that circular orbits are always inside elliptical ones and the pericenters of elliptical orbits are on circular orbits.

If one chooses the initial conditions to have circular orbits in the field of the DM core only and uses them also for the test particles in the field of the SMBH, one will get an opposite picture i.e. elliptical orbits in the field of the SMBH as illustrated in Fig. 3. Here now the elliptical orbits are inside circular ones and the apocenters of the elliptical orbits are on circular orbits. However the elliptical orbits here are not precessing due to the fact that in NG the trajectories of test particles in the field of a point mass do not precess [24]. Unlike in NG, there is a precession even in the field of a point mass in GR. One should know orbital parameters of specific stars near the galactic center to estimate the pericenter shift. The differences appearing in the motion of test particles can shed some light on the nature of the central compact object in the MWG with future more accurate observations.

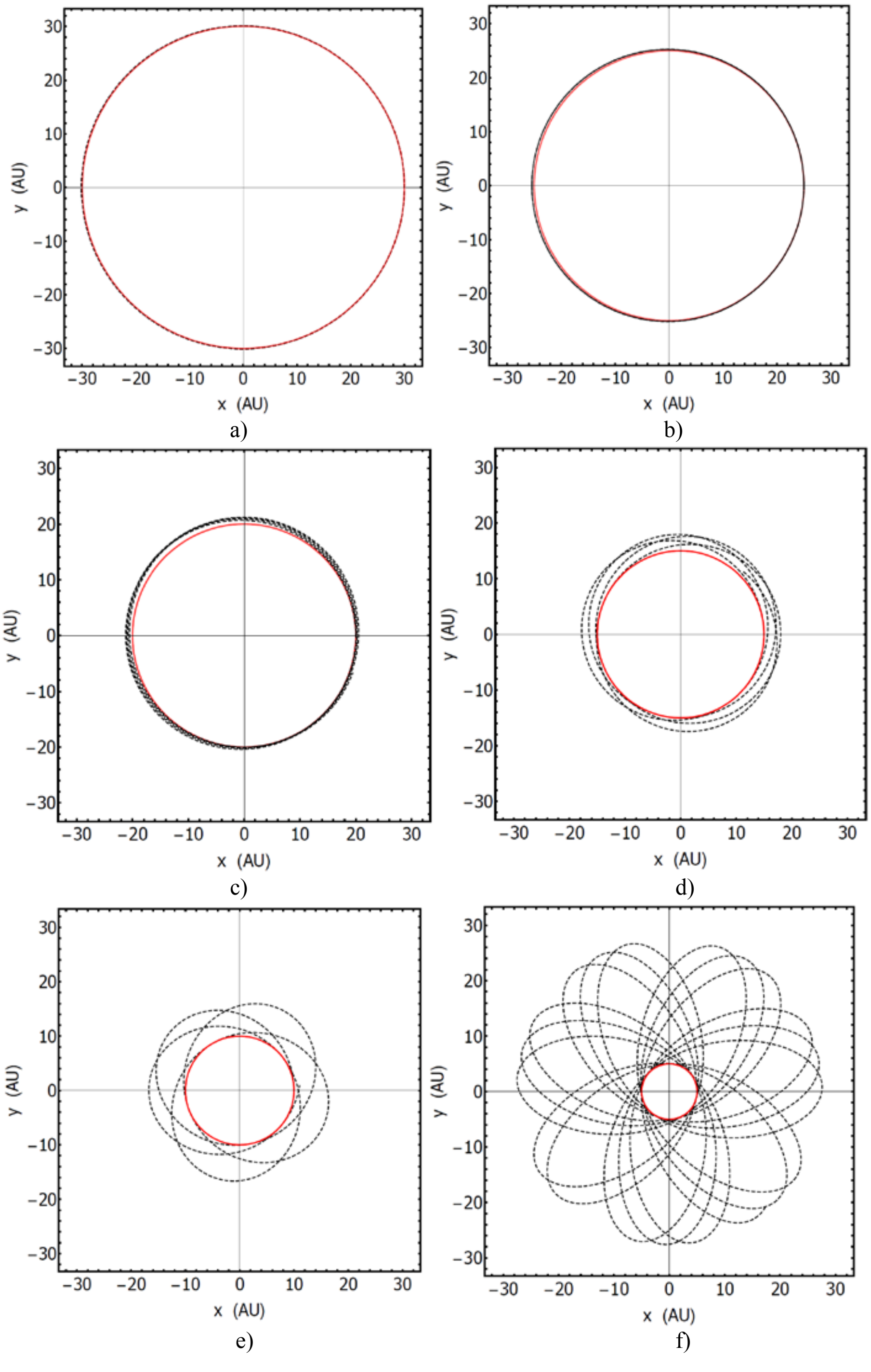

Figure 2 – Motion of test particles near the galactic center. The solid red and dashed black curves show the motion of test particles in the field of a BH and DM core, respectively.
Initial conditions are chosen in such a way to always get circular orbits in the field of a BH.
a) $x(0)=30$ AU, $y(0)=0$, $\dot{x}(0)=0$, $\dot{y}(0)=7.4\times10^{-5}$ AU/s, b) $x(0)=25$ AU, $y(0)=0$, $\dot{x}(0)=0$, $\dot{y}(0)=8.2\times10^{-5}$ AU/s, c) $x(0)=20$ AU, $y(0)=0$, $\dot{x}(0)=0$, $\dot{y}(0)=9.1\times10^{-5}$ AU/s, d) $x(0)=15$ AU, $y(0)=0$, $\dot{x}(0)=0$ $\dot{y}(0)=10.5\times10^{-5}$ AU/s, e) $x(0)=10$ AU, $y(0)=0$, $\dot{x}(0)=0$, $\dot{y}(0)=12.9\times10^{-5}$ AU/s, f) $x(0)=5$ AU, $y(0)=0$, $\dot{x}(0)=0$, $\dot{y}(0)=18.2\times10^{-5}$ AU/s.

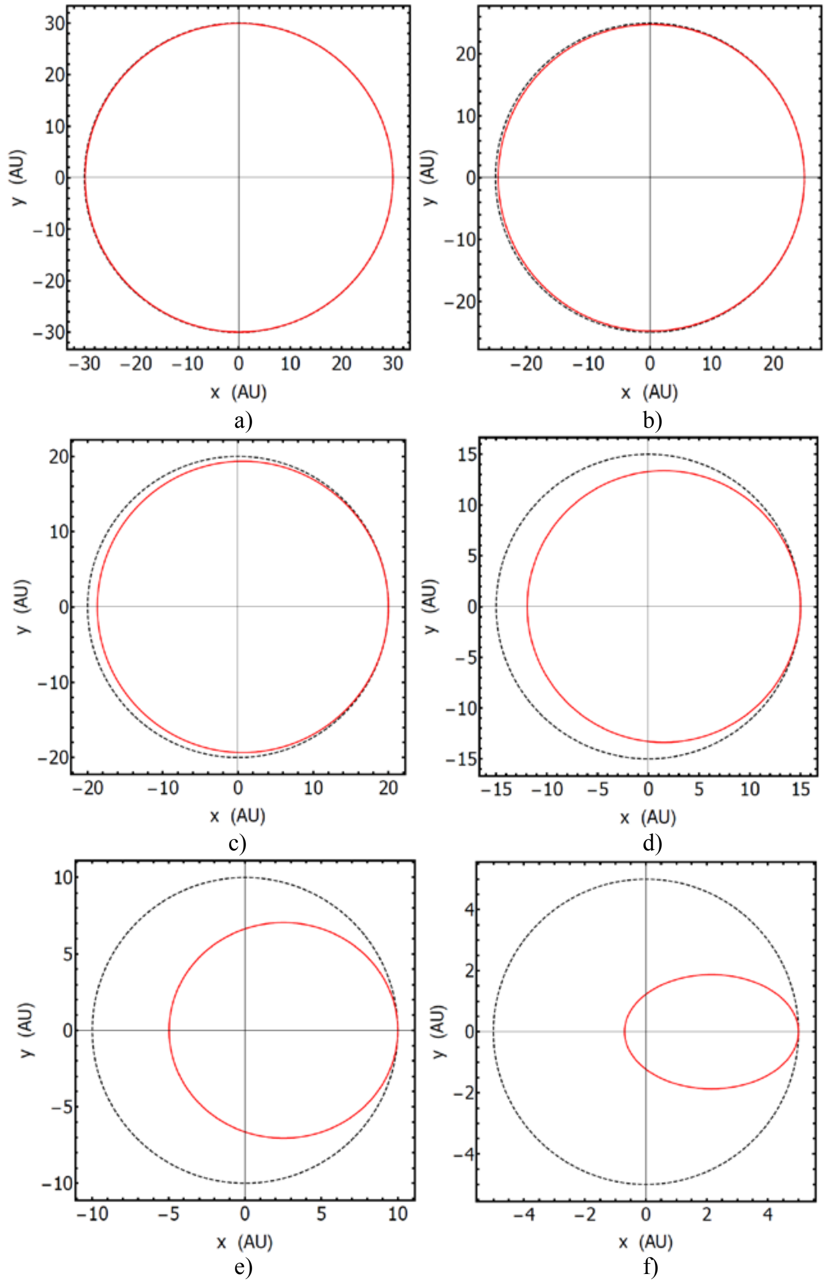

Figure 3 – Motion of test particles near the galactic center. The solid red and dashed black curves show the motion of test particles in the field of a BH and DM core, respectively.
Initial conditions are chosen in such a way to always get circular orbits in the field of the DM.
a) $x(0)=30$ AU, $y(0)=0$, $\dot{x}(0)=0$, $\dot{y}(0)=7.4\times10^{-5}$ AU/s, b) $x(0)=25$ AU, $y(0)=0$, $\dot{x}(0)=0$, $\dot{y}(0)=8.1\times10^{-5}$ AU/s, c) $x(0)=20$ AU, $y(0)=0$, $\dot{x}(0)=0$, $\dot{y}(0)=9.0\times10^{-5}$ AU/s, d) $x(0)=15$ AU, $y(0)=0$, $\dot{x}(0)=0$ $\dot{y}(0)=9.9\times10^{-5}$ AU/s, e) $x(0)=10$ AU, $y(0)=0$, $\dot{x}(0)=0$, $\dot{y}(0)=10.5\times10^{-5}$ AU/s, f) $x(0)=5$ AU, $y(0)=0$, $\dot{x}(0)=0$, $\dot{y}(0)=9.0\times10^{-5}$ AU/s.

Conclusion

We analyzed the trajectories of stars near the galactic center in the gravitational field of the SMBH and the DM core. For this purpose, the equations of motion of test particles in NG were solved numerically.

As expected, a significant divergence in the motion appears at distances less than 30 AU, and it increases as we approach the center of the galaxy.

This means that current observations of the movements of stars, such as S2, which makes close to Sgr A* at 120 AU, it is impossible to distinguish the proposed DM core from the SMBH.

It would be interesting to test the current model for G2 object in analogy to Ref. [15]. However this would the issue of future studies.

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