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Structure of resonance states in the simple schematic model

Abstract. The complex scaling method is one of the powerful tool in wide areas of physics, particularly in nuclear physics. In the first stage, its advantage was mainly pointed out for description of the resonance states in the composite systems. In the last decide, the usage of this method has increased not only to obtain information on resonance states but also to determine scattering quantities in the observables. To determine the presence of many resonant states at the wave is not easy and complex scaling method can be used to determine the obtain many resonant states. The simple schematic two-body model is applied for study of many resonant states. Applying the complex scaling method, we can easily obtain several resonance states even with a wide and a sharp decay widths simultaneously. In this work, one bound and five resonance states for $J^\pi = 0^+$ wave and one bound and four resonance states for $J^\pi = 1^-$ wave are reported.

Key words: Complex scaling method, resonance and continuum states, cluster model.

Introduction

Since the cluster structures of light nuclei are often observed as resonance states, it is important to develop the study of resonance states in multi-cluster systems. In theoretical studies for light nuclei, the complex scaling method (CSM) is a useful tool to get information unbound states as well as bound states within the same treatment. The detailed information about the structure of light nuclei has been carried out using different methods, one of them is the CSM [1-2]. The complex scaling was first proposed mathematically and it has been widely used in all fields of physics, especially in resonance and scattering studies in nuclear physics.

In the first stage, its advantage was mainly pointed out for description of the resonance states in the composite systems. Developing the CSM to give us a possibility to obtain information of continuum and non-continuum states for three-, four-, and five body systems. In the last decade, the usage of this method has increased not only to obtain information on resonance states but also to determine scattering quantities in the observables [3-5]. Furthermore, to understand structure of neutron rich nuclei, it is required to study structure of continuum and bound states as well. Recently, it has attracted much attention that the CSM has applied for observation of wide resonant and virtual states considering continuum states in light nuclei [6-10].

In this work, we apply the CSM to a simple schematic two-body model [8] for obtaining many resonance states. Applying the simple schematic potential for $J^\pi = 0^+$ and 1^- waves, we calculate five resonance states in each waves.

Method

We take up two-body systems, which are described by the *Schrödinger* equation

$$\hat{H}|\Psi\rangle = E|\Psi\rangle,$$

where the Hamiltonian H consists of kinetic energy T and potential V for the relative motion between two bodies. The eigenvalue problem is generally solved under a boundary condition of asymptotic outgoing waves for bound states and resonances. The outgoing boundary condition directly enables us to solve bound states in an L^2 functional basis set because the states have negative energies and a damping behavior in the asymptotic region. Resonant states are unbound and defined as the eigenstates belonging to the complex eigenenergy, which corresponds to a complex momentum value in the lower half plane [3]. The resonant states cannot be solved in the L^2 functional space due to asymptotically divergent behavior. Furthermore, continuum states of arbitrary positive energies cannot also be obtained under the outgoing condition. The complex scaling has been introduced to solve resonant states within L^2 basis functions and is defined by the following complex-dilatation transformation for relative coordinate r and momentum k is rewritten as $r \longrightarrow re^{i\theta}$. Where θ is a scaling angle given by a real number and $0 < \theta < \theta_{\max}$. The maximum value $\theta_{\max}$ is determined to keep analyticity of the transformed potential. For example, $\theta_{\max} = \pi/4$ for a Gaussian potential. In a many-body system, this transformation makes every branch cut rotated by -2θ from the real axis on the complex energy plane. In the wedge region pinched by the rotated branch cut and the positive energy axis, resonance eigenstates are obtained by solving the following eigenvalue problem:

$$\widehat{H}(\theta)|\Psi^{\theta}\rangle = E^{\theta}|\Psi^{\theta}\rangle, \quad (2)$$

$$\Psi^{\theta} = \sum_{i=1}^{N} c_i^{\theta}\varphi_i, \quad (3)$$

within an appropriate non-orthogonal L^2 basis set $\{\varphi_i,\ i = 1, 2, \ldots, N\}$. Where $\widehat{H}(\theta)$ and Ψ^{θ} are the complex scaled Hamiltonian and wave function, respectively. The bound states are obtained on the negative-energy axis independently from θ as well as the ordinary bound states. Because of a finite number of basis states, the continuum states are discretized with complex energies distributed on the rotated branch cut (2θ line).

The eigenvalues and eigenstates of the complex scaled *Schrödinger* equation (2) are classified as

$$[E, \Psi^{\theta}] = \begin{cases} (E_b, \Psi^b) & b = 1, \cdots, N_b; \quad \text{bound states} \\ (E_r, \Psi^r) & r = 1, \cdots, N_r^{\theta}; \quad \text{resonant states}, \\ (E_c(\theta), \Psi^c) & c = 1, \cdots, N - N_b - N_r^{\theta}; \quad \text{continuum states} \end{cases} \quad (4)$$

where N_b and N_r^{θ} are the number of bound states and the number of resonant states which depend on θ, respectively. The complex energies of resonant states are obtained as $E^{\theta} = E_r - i\Gamma_r/2$, where E_r is resonance energy and Γ_r is width of the resonant state. The discretized energies $E_c(\theta)$ of continuum states are θ dependent and expressed as $E_c(\theta) = \epsilon_c^r - i\epsilon_c^i$.

These three-kind solutions of the complex-scaled *Schrödinger* equation construct the extended completeness relation:

$$\sum_{b=1}^{N_b} |\Psi^b\rangle\langle\widetilde{\Psi}^b| + \sum_{r=1}^{N_r^{\theta}} |\Psi^r\rangle\langle\widetilde{\Psi}^r| + \int_{L_c} dE_c |\Psi^c\rangle\langle\widetilde{\Psi}^c| = 1, \quad (5)$$

where the tilde (˜) in bra states means the biorthogonal states with respect to the ket states due to non-Hermitian property of H^{θ}. The integration of the third term is taken along the rotated branch cut L_c. In the case of eigenstates within a finite number of L^2 basis states, the integration for continuum states is approximated by the summation of discretized states as [6]

$$\sum_{b=1}^{N_b} |\Psi^b\rangle\langle\widetilde{\Psi}^b| + \sum_{r=1}^{N_r^{\theta}} |\Psi^r\rangle\langle\widetilde{\Psi}^r| + \sum_{c=1}^{N-N_b-N_r^{\theta}} |\Psi^c\rangle\langle\widetilde{\Psi}^c| \approx 1. \quad (6)$$

It has been investigated that the reliability of the approximation of the continuum states are confirmed by using a sufficiently large basis number of N in the CSM [2].

Results and discussion

The Hamiltonian of the present model is given as

$$H = -\frac{\hbar^2}{2\mu}\nabla^2 + V(r), \quad (7)$$

where

$$V(r) = -8.0\exp(-0.16r^2) + +4.0\exp(-0.04r^2). \quad (8)$$

For simplicity, we put $\frac{\hbar^2}{\mu} = 1$ (MeV fm^2). This potential introduced in Ref. [8] has an attractive pocket in a short range but a repulsive barrier at a large distance. To solve the Eq. (2), we employ the Gaussian basis functions given as

$$u_i(\hat{r}) = N_l(b_i)r^l \exp\left(-\frac{1}{2b_i^2}r^2\right)Y_{lm}(\hat{r}), \quad (9)$$

where the range parameters are given by a geometric progression as $b_i = b_0\gamma^{i-1}$, $i = 1, 2, \ldots, N$.

In this calculation, we apply $N = 20$ and employ the optimal values of b_0 and γ to obtain stationary resonance solutions.

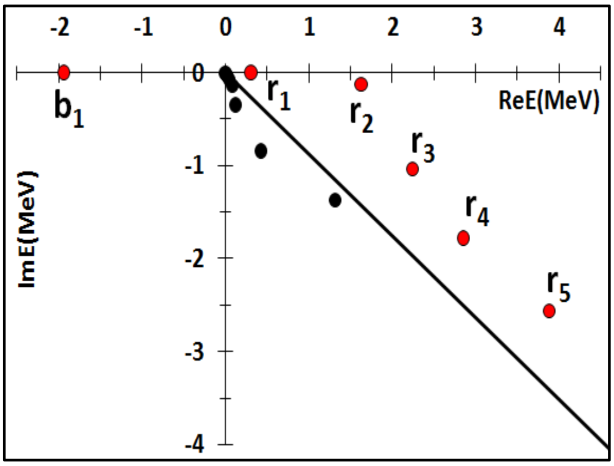

Figure 1 – Schematic scenario of energy eigenvalues for $J^{\pi} = 0^{+}$ wave on the complex energy plane. Here bound state is expressed as *b*. The resonant states are displayed *r* and indexes of *r* are corresponding the number of resonant states. $\theta = 15^{0}$ is applied as a scaling angle.

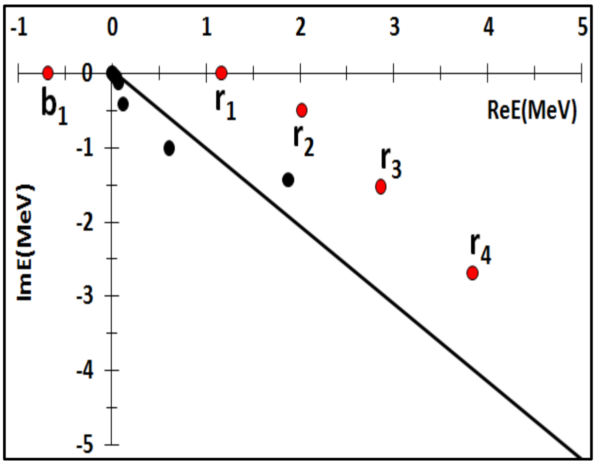

Figure 2 – Schematic scenario of energy eigenvalues for $J^{\pi} = 1^{-}$ wave on the complex energy plane. The same analysis is performed as done in Fig.1. $\theta = 20^{0}$ is applied as a scaling angle.

In Figs. 1-2, the energy eigenvalue distributions on the complex energy plane for $J^{\pi}= 0^{+}$ and 1^{-} waves are shown. When we apply $\theta = 15^{0}$, we observe isolated energy points as shown in Fig.1. In Fig.1, we find one bound and five resonance states for $J^{\pi} = 0^{+}$ wave. The bound state is expressed as a red filled circle on the negative energy axis and it expressed as *b*. The resonant states are displayed by red filled circles and given by *r* and indexes of *r* are corresponding the number of resonant states.

In Fig.2, we obtained one bound and four resonance states for $J^{\pi} = 1^{-}$ wave by applying $\theta = 20^{0}$. We calculate one bound and four resonance states for $J^{\pi} = 1^{-}$ wave. In Fig. 2, the filled red circles are expressed bound and resonant states. The bound state is expressed *b* and obtained on the negative energy axis. *r* is implies the resonant states and its indexes are corresponding the number of resonant states.

As can be seen from Figs. 1-2, energy eigenvalues are located on the 2θ lines except bound and resonance states. The resonant poles with narrow and wide decay widths appear above the threshold as shown with the filled red circles in Figs. 1-2.

The results of the calculated resonance energies and widths for $J^{\pi}=0^{+}$ and 1^{-} partial waves applying the schematic potential model are summarized in Table I. Table I contains the bound and resonance energies with decay widths at each partial waves.

Table I – The calculated resonance energies and decay widths for $J^{\pi}= 0^{+}$ and 1^{-} waves.

0^{+} wave		1^{-} wave	
E (MeV)	**state**	**E (MeV)**	**state**
-1.922782	bound	-0.674647	bound
$0.3101\text{-}i10^{-6}$	resonance	$1.1710\text{-}i4.948\text{x}10^{-3}$	resonance
1.6322-*i*0.1228	resonance	$2.0309\text{-}i4.8944\text{x}10^{-1}$	resonance
2.2493-*i*0.9367	resonance	2.8318-*i*1.7186	resonance
2.7667- *i*1.3169	resonance	3.7834-*i*2.5148	resonance
3.8433- *i*1.8445	resonance		

Conclusions

In this work we discussed many resonant states for $J^{\pi} = 0^{+}$ and 1^{-} waves applying a simple schematic potential, in the framework of the CSM to investigate the bound and unbound states. The present method is very useful to get structure information of many resonant states with bound state in the same manner. We calculated one bound and five resonance states for $J^{\pi} = 0^{+}$ wave and one bound and four resonance states for $J^{\pi} = 1^{-}$ wave, respectively. For getting the structure of many resonant states give us a possibility to develop a method for determining a broad resonance state or a virtual state calculating scattering quantities using continuum and non-continuum states.

Acknowledgments

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