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CONTROL OF VIBRATIONS OF ELASTICALLY FIXED OBJECTS USING ANALYSIS OF EIGENFREQUENCIES

Abstract. In this paper, a mathematical model of a controlled system is investigated, created on the basis of a fourth-order differential equation widely used in various fields of science and technology. The problem of managing the behavior of structural elements has been solved. The mechanism of transition from one system to another is considered using the analysis of natural frequencies. The rod can be fixed in different ways (termination, hinge locking, elastic termination, floating termination, free end) [1]. If the ends of the rod are fixed so that resonant vibration frequencies are generated, then the question arises: is it possible to change the fastening of the rod so as to indicate a safe range for controlling the natural frequencies. The question posed by us gives rise to many others, more specific. For example, is it possible to determine how the ends of the bar are fixed by the natural vibration frequencies of the bar? Are they springs, sealed or loose? Such applications are very important especially when the first natural frequency generates a resonance situation. It is necessary to control the natural frequencies so that the system does not receive the first natural frequency for safe operation. The main result is formulated as a theorem. The stress-strain state (SSS) control has been developed for rods with elastic fastening on the left and hinged on the right. The uniqueness theorem for boundary conditions is proved using the analysis of natural frequencies.
Key words: elastically fixed objects, natural frequencies, spectral problem.

Introduction

During the construction of technical structures, along with strength management, the issues of stress-strain state (SSS) management of its individual key elements are also important [7], [2], [8], [9]. These controls significantly affect the technical condition of the entire structure. In this paper, we have developed a SSS control for bars with elastic fixing on the left and hinged on the right. These systems are used in bridge and aircraft structures as parts of superstructure beams and floor slabs. Since the control of SSS is influenced by the natural frequencies of vibrations of the rods, in this work the methods of perturbation of the spectral theory of differential operators [3], [4-5] are used.

The need to calculate natural frequencies and the corresponding vibration modes often arises when analyzing the dynamic behavior of a structure under the influence of variable loads. The most common situation is when, when designing, it is required to make sure that there is a low probability of occurrence of such a mechanical phenomenon as resonance under operating conditions. As you know, the essence of resonance is in a significant (tens of times or more) amplification of the amplitudes of forced oscillations at certain frequencies of external influences – the so-called resonance frequencies. In most cases, the occurrence of resonance is extremely undesirable in terms of ensuring product reliability. A multiple increase in vibration amplitudes at resonance and the resulting high stress levels are one of the main reasons for the failure of products operated under vibration loads. To protect against resonance influences, you can use various mechanical devices that fundamentally change the spectral characteristics of the structure and absorb vibration energy (for example, vibration isolators). However, there is another effective way to counter resonances. It is known that resonances are observed at frequencies close to the frequencies of natural vibrations of the structure. If, when designing a product, it is possible to estimate the spectrum of natural frequencies of a structure, then it is possible with a significant degree of probability to predict the risk of resonances in a known frequency range of external influences. In order to avoid or to significantly reduce the likelihood of resonances, it is necessary that most of the lower natural frequencies of the structure do not lie in the frequency range of external influences.

Statement of the problem

We put the inverse to this spectral problem: the problem of the natural frequencies of the bending vibrations of the bar to find the unknown boundary conditions: $U_1(y)=0$, $U_2(y)=0$. Denote the matrix composed of the coefficients a_{ij} of the forms $U_1(y)$ and $U_2(y)$ through A and its second-order minors – through M_{ij}:

$$A=\begin{Vmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \end{Vmatrix},$$

$$M_{ij}=\begin{vmatrix} a_{1i} & a_{1j} \\ a_{2i} & a_{2j} \end{vmatrix}.$$

Finding forms $U_1(y)$, $U_2(y)$ is equivalent to finding the matrix A up to linear equivalence . The rod can be fixed in different ways (termination, hinge locking, elastic termination, floating termination, free end) [1].

There are various known cases of fixing the end of the rod. [13,14]

Rigid pinching

$$A=\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{Vmatrix};$$

Free support

$$A=\begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{Vmatrix};$$

Free end,

$$A=\begin{Vmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{Vmatrix};$$

Floating termination,

$$A=\begin{Vmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{Vmatrix};$$

Five different types of elastic fastening:

$$A=\begin{Vmatrix} c_1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{Vmatrix}, \begin{Vmatrix} 1 & 0 & 0 & 0 \\ 0 & -c_2 & 1 & 0 \end{Vmatrix},$$

$$\begin{Vmatrix} c_1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{Vmatrix}, \begin{Vmatrix} 0 & 0 & 0 & 1 \\ 0 & -c_2 & 1 & 0 \end{Vmatrix},$$

$$\begin{Vmatrix} c_1 & 0 & 0 & 1 \\ 0 & -c_2 & 1 & 0 \end{Vmatrix}$$

If the ends of the rod are fixed in such a way that resonant vibration frequencies are generated, then the question arises: is it possible to change the fastening of the rod so as to indicate a safe range for controlling natural frequencies.

Before presenting the main results, we recall that the equation of bending vibrations of a homogeneous rod of length l at $0<x<l,\ t>0$ with constant bending stiffness has the form

$$\rho A\frac{\partial^2 \mathrm{w}(x,t)}{\partial t^2}+EJ\frac{\partial^4 \mathrm{w}(x,t)}{\partial x^4}=0,$$

where $\mathrm{w}(\mathrm{x},\mathrm{t})$ – deflection of the current point of the bar axis; ρ – material density; A – cross-sectional area; EJ – bending stiffness of the bar.

We denote $\lambda=\dfrac{\omega^2\rho A}{EJ}$ As known [14], the frequency of bending vibrations of the beam does not depend on the initial shape of the beam, but depends only on the method of fixing its ends. In the new notation, the problem of bending vibrations of a bar with elastic fixation on the left and hinge on the right by replacement $\mathrm{w}(x,t)=y(x)\sin(\omega t)$ reduces to the following spectral problem:

$$y^{IV}(x)=\lambda y(x),\ 0<x<l, \qquad (1.1)$$

$$c_1 y(x)\big|_{x=0}=y'''(x)\big|_{x=0},$$
$$y(x)\big|_{x=l}=0,$$
$$y''(x)\big|_{x=0}=0,\ y''(x)\big|_{x=l}=0. \qquad (1.2)$$

Here c_1 spring coefficient of elasticity.

The operator corresponding to problem (1.1) – (1.2) is denoted by $Ky(x)=\lambda y(x)$. Operator eigenvalues K can be numbered in non-decreasing order

$$\lambda_1\le\lambda_2\le\lambda_3\le\ldots$$

Smallest eigenvalue λ_1 is positive, which we will show below. Moreover, the choice of the coefficient of elasticity of the spring significantly affects the behavior of natural frequencies. The system of eigenfunctions $\{y_n(x)\}_{n=1}^{\infty}$ operator K forms an orthonormal basis of the space $L_2(0,l)$.

Problem 1.1: Consider the spectral problem of the operator B corresponding to the following problem:

$$u^{IV}(x) = \mu u(x),\ 0 < x < l, \tag{1.3}$$

$$c_1 u(x)\big|_{x=0} = u'''(x)\big|_{x=0} + \alpha\int_0^l u(x) y_1(x)dx,$$

$$u(x)\big|_{x=l} = 0,\ u''(x)\big|_{x=0} = 0,\ u''(x)\big|_{x=l} = 0. \tag{1.4}$$

Here $y_1(x)$ – the first eigenfunction of the operator K. The boundary parameter α can take complex values.

Select parameter α so that the eigenvalues of the operator B was out of range $(-\lambda_2, \lambda_2)$.

Operator B can be considered a perturbation of the operator K, since only the scope has changed $D(K)$ operator K. Such applications are very important especially when the first natural frequency generates a resonance situation. Another can formulate this problem as follows: It is necessary to control natural frequencies so that the system does not receive the first natural frequency for safe operation. Let us state the main result as a theorem.

Theorem 1.1. If you choose α so that the inequality

$$\lambda_2 - \lambda_1 < \frac{\alpha}{c_1} y_1'''(0) \tag{1.5}$$

then the eigenvalues $\{\mu_n\}_{n=1}^{\infty}$ operator B determined by the formula $\mu_n = \lambda_n$ at $n \geq 2$ and μ_1 is the only real root of the equation $c_1 = \dfrac{\alpha y_1'''(0)}{\mu - \lambda_1}$.

To prove the theorem, we need the following lemma.

Lemma 1.1. Identity is valid

$$(\mu - \lambda_1)\int_0^l u(x) y_1(x)dx = \\ = \left(-\frac{1}{c_1}u'''(0) + u(0)\right) y_1'''(0).$$

Proof of Lemma 1.1. The right side of the identity can be written in the following form

$$(\mu - \lambda_1)\int_0^l u(x) y_1(x)dx = \\ = \int_0^l \mu u(x) y_1(x)dx - \int_0^l u(x)\lambda_1 y_1(x)dx = \\ = \int_0^l u^{IV}(x) y_1(x)dx - \int_0^l u(x) y_1^{IV}(x)dx.$$

Direct calculation shows that the first term is equal to

$$\int_0^l u^{IV}(x) y_1(x)dx = \\ = \left(-\frac{1}{c_1}u'''(0) + u(0)\right) y_1'''(0) + \int_0^l u(x) y_1^{IV}(x)dx.$$

Taking into account the last relation, we obtain the proof of Lemma 1.1.

Lemma 1.1 is proved

2. Proof of the theorem 1.1

To prove the theorem, the perturbed boundary condition, taking into account Lemma 1.1, can be written in the following form

$$(c_1 u(0) - u'''(0)) = \\ = \frac{\alpha y_1'''(0)}{c_1(\mu - \lambda_1)}(c_1 u(0) - u'''(0)). \tag{1.6}$$

By assumption λ_1 is not an eigenvalue of the problem (1.3)- (1.4). Therefore $c_1 u(0) \neq u'''(0)$.

Whence it follows that $c_1 = \dfrac{\alpha y_1'''(0)}{\mu - \lambda_1}$.

Let be $\eta_1 = \lambda_1 + \dfrac{\alpha}{c_1} y_1'''(0)$ и $\dfrac{\alpha}{c_н} y_1'''(0) > \lambda_2 - \lambda_1$.

Then $\eta_1 > \lambda_2$.

Let us calculate the characteristic determinant [6] of the operator B.

Let us calculate the characteristic determinant [6] of the operator K determined by the formula

$$\Delta(\lambda) = 2\cos\left(\sqrt[4]{\lambda}l\right)\cosh\left(\sqrt[4]{\lambda}l\right) - \\ -\frac{c_1}{\sqrt[4]{\lambda^3}}\left(\sin\left(\sqrt[4]{\lambda}l\right)\cosh\left(\sqrt[4]{\lambda}l\right) - \cos\left(\sqrt[4]{\lambda}l\right)\sinh\left(\sqrt[4]{\lambda}l\right)\right).$$

At $c_1 = 0$ The characteristic determinant was cleared in detail in [6].

For different values graphic way possible to ensure that the smallest eigenvalue λ_1 is positive.. From relations (1.6) it follows that the perturbed boundary condition takes the form

$$f(\mu)(c_1 u(0) - u'''(0)) = 0,$$

where $f(\mu) = 1 - \dfrac{\alpha y_1'''(0)}{c_1(\mu - \lambda_1)}$.

Taking into account the last relations, we calculate the explicit form of the characteristic determinant of the operator B

$$\Delta(\mu) = f(\mu)\Delta_0(\lambda).$$

where Δ_0 characteristic determinant of the operator K.

The last representation implies the proof of the theorem.

Conclusion

The study of inverse problems in the spectral theory of differential operators dates back to the fundamental works of the twenties and forties of the twentieth century. The impetus for the development of this direction was the work of V.A. Amburtzumyan and G. Borg. A significant contribution to the formation of this direction was made by A.N. Tikhonov, M.I. Gelfand, N. Levinson, M.G. Crane, B.M. Levitan, V.A. Marchenko, M.G. Gasimov, V.A. Sadovnichy, V.A. Yurko, Gladwell G.M.L. other. In the works of these authors, the coefficients of the boundary conditions (and not all) were identified from the spectra only incidentally with the coefficients of the differential equations themselves. In this case, not one, but two or more spectra, or a spectrum and additionally other spectral data, were used for identification.

In this paper, a mathematical model of a controlled system is investigated, created on the basis of a fourth-order differential equation widely used in various fields of science and technology. The problem of managing the behavior of structural elements has been solved. The mechanism of transition from one system to another is considered using the analysis of natural frequencies.

Also, in this work, the SSS control is developed for rods with elastic fastening on the left and hinged on the right. These systems are used in bridge and aircraft structures as parts of superstructure beams and floor slabs. Since the control of SSS is influenced by the natural frequencies of vibrations of the rods, in this work we use the methods of perturbation of the spectral theory of differential operators.

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