IRSTI 27.31.15; 27.31.17; 27.33.15 https://doi.org/10.26577/ijmph.2020.v11.i1.04

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AN EXISTENCE SOLUTION TO AN IDENTIFICATION PARAMETER PROBLEM FOR HIGHER-ORDER PARTIAL DIFFERENTIAL EQUATIONS

Abstract. The initial-boundary value problem with parameter for higher-order partial differential equations is considered. We study the existence of its solution and also propose a method for finding approximate solutions. We are established a sufficient conditions for the existence and uniqueness of the solution to the identification parameter problem under consideration. Introducing new unknown functions, we reduce the considered problem to an equivalent problem consisting of a nonlocal problem for second-order hyperbolic equations with functional parameters and integral relations. An algorithm for finding an approximate solution to the problem under study is proposed and its convergence is proved. Sufficient conditions for the existence of a unique solution to an equivalent problem with parameters are established. The conditions for the unique solvability of the initial-boundary value problem with parameter for higher-order partial differential equations are obtained in terms of the initial data. Unique solvability to the initial-boundary value problem with parameter for higher-order partial differential equations is interconnected with unique solvability to the nonlocal problem with parameter for second-order hyperbolic equations.

Key words: higher-order partial differential equations, identification parameter problem, nonlocal problem with parameters, hyperbolic equations of second order, solvability.

Introduction

An initial-boundary value problems with and without parameters for higher-order partial differential equations belong to one of the most important classes of problems in mathematical physics [1-14]. For studying of various problems with and without parameters for higher-order partial differential equations, along with classical methods of mathematical physics, such as the Fourier method, the Green's function method, the Poincare metric concept, the method of differential inequalities, and other methods of the qualitative theory of ordinary differential equations are also often applied. Based on these methods, the solvability conditions of the considered problems with and without parameters were established and ways to solve them were offered in [15-33]. However, the search for effective criteria of the unique solvability of initial-boundary value problems with parameters still remains relevant.

It is known that an ordinary differential equation of higher order can be reduced to a system of ordinary differential equations of the first order by special substitution. Using the methods of the qualitative theory of ordinary differential equations, the solvability conditions for the obtaining system can be formulated in the terms of the fundamental matrix of the differential part or the right side of the system. An analogous approach can be applied to higher-order hyperbolic equations with two independent variables and their can be reduced to a system of second order hyperbolic equations with mixed derivatives by replacement. Further, using well-known methods for solving problems for systems of hyperbolic equations with mixed derivatives, the solvability conditions can be established in different terms.

Mathematical modeling of many problems of physics, mechanics, chemistry, biology, and other sciences has resulted into the necessity of studying initial-boundary value problems with parameter for higher-order partial differential equations of hyperbolic type. Applying the methods of the qualitative theory of differential equations directly to these problems, we can establish the conditions for their solvability [1, 7, 8, 14, 23, 27-30]. Nonlocal problems with parameter for higher-order partial differential equations of hyperbolic type by replacement are reduced to nonlocal problems with parameter for system of second-order hyperbolic equations. The theory of nonlocal problems with parameter for system of second-order hyperbolic equations has been developed in many papers. To date, various solvability conditions for nonlocal problems with parameter for hyperbolic equations have been obtained.

The criteria for the unique solvability of some classes of linear nonlocal problems for hyperbolic equations with variable coefficients were obtained relatively recently [34-36]. In [34], a nonlocal problem with an integral condition for systems of hyperbolic equations by introducing new unknown functions is reduced to a problem consisting of a family of boundary value problems with an integral condition for systems of ordinary differential equations and functional relations. It is established that the well-posedness of a nonlocal problem with an integral condition for systems of hyperbolic equations is equivalent to the well-posedness of a family of two-point boundary value problems for a system of ordinary differential equations. In terms of the initial data, a criterion is established for the well-posedness of a nonlocal problem with an integral condition for systems of hyperbolic equations.

In present paper, we consider a higher-order partial differential equation defined in a rectangular domain. The boundary conditions for the time variable are specified as a combination of values from the partial derivatives of the desired solution in rows t = 0, t = T and $t=\theta$. We also study the existence and uniqueness of the solution to the initial-boundary value problem with parameter for a higher-order partial differential equation and its applications.

To solve the problem under consideration, we use the method of introducing additional functional parameters [34-36] and reduce the original problem to an equivalent problem consisting of a nonlocal problem with parameter for a second-order hyperbolic equation with functional parameters and integral relations. We establish sufficient conditions for the unique solvability of the considered problem in the terms of unique solvability of nonlocal problem with parameter for a second-order hyperbolic equation. Algorithms for finding a solution to an equivalent problem are constructed. The conditions for the unique solvability of the initial-boundary-value problem with parameter for the higher-order partial differential equations are established in the terms of the coefficients of the system and the boundary matrices.

Statement of problem and scheme of method introduction functional parameters

At the domain $\Omega=[0,T]\times[0,\omega]$, we consider the initial-boundary value problem with parameter for the higher-order partial differential equation of the following form:

$$\frac{\partial^{m+1}u}{\partial t\partial x^m}=\sum_{i=1}^{m}\left\{A_i(t,x)\frac{\partial^i u}{\partial x^i}+B_i(t,x)\frac{\partial^i u}{\partial t\partial x^{i-1}}\right\}+C(t,x)u+D(t,x)\mu(x)+f(t,x),\ (t,x)\in\Omega, \tag{1}$$

$$\sum_{j=0}^{1}\sum_{i=1}^{m}\left\{P_{ij}(x)\frac{\partial^i u(t,x)}{\partial x^i}+S_{ij}(x)\frac{\partial^i u(t,x)}{\partial t\partial x^{i-1}}\right\}\Big|_{t=t_j}=\varphi(x),\ x\in[0,\omega], \tag{2}$$

$$u(t,0)=\psi_0(t),\ \frac{\partial u(t,x)}{\partial x}\Big|_{x=0}=\psi_1(t),\ \ldots,\ \frac{\partial^{m-1}u(t,x)}{\partial x^{m-1}}\Big|_{x=0}=\psi_{m-1}(t),\ t\in[0,T], \tag{3}$$

$$L(x)\frac{\partial^m u(\theta,x)}{\partial x^m}+M(x)\mu(x)=\varphi_1(x),\ x\in[0,\omega], \tag{4}$$

where $u(t,x)$ and $\mu(x)$ are an unknown functions, the functions $A_i(t,x)$, $B_i(t,x)$, $i=\overline{1,m}$, $C(t,x)$, and $f(t,x)$ are continuous on Ω, the functions $P_{ij}(x)$, $S_{ij}(x)$, $i=\overline{1,m}$, $j=0,1$, and $\varphi(x)$ are continuous on $[0,\omega]$, $0=t_0<t_1=T$, the functions $\psi_s(t)$, $s=\overline{0,m-1}$, are continuously

differentiable on $[0,T]$, the functions $L(x)$, $M(x)$, and $\varphi_1(x)$ are continuous on $[0,\omega]$, and $0<\theta<T$. Relation (4) is additional condition for determining unknown functional parameter $\mu(x)$. The initial data satisfy the matching condition.

A pair of functions $(u(t,x),\mu(x))$, with component $u(t,x)\in C(\Omega,R)$, $\mu(x)\in C([0,\omega],R)$ having partial derivatives $\frac{\partial^{p+r}u(t,x)}{\partial t^r\partial x^p}\in C(\Omega,R)$, $p=\overline{1,m}$, $r=0,1$, is called a solution to problem with parameter (1) – (4) if it satisfies equation (1) for all $(t,x)\in\Omega$, the initial-boundary conditions (2), (3) and additional condition (4).

We will investigate the questions of the existence and uniqueness of solutions to the initial-boundary value problem with parameter for a higher-order partial differential equation (1) – (4) and the construction of its approximate solutions. For these purposes, we apply the method of introducing additional functional parameters proposed in [34–36] for solving various nonlocal problems for systems of hyperbolic equations with mixed derivatives. The considered problem is reduced to a nonlocal problem with parameter for second-order hyperbolic equations, including additional functions, and integral relations. An algorithm for finding an approximate solution to the problem under study is proposed and its convergence is proved. Sufficient conditions for the existence of a unique solution to problem with parameter (1) – (4) are obtained in terms of the initial data.

Scheme of the method and reduction to equivalent problem.

We introduce new unknown functions

$$v(t,x)=\frac{\partial^{m-1}u(t,x)}{\partial x^{m-1}},\ v_1(t,x)=u(t,x),$$
$$v_2(t,x)=\frac{\partial u(t,x)}{\partial x},\ \ldots,\ v_{m-1}(t,x)=\frac{\partial^{m-2}u(t,x)}{\partial x^{m-2}} \quad (5)$$

and re-write problem with parameter (1)-(4) in the following form:

$$\frac{\partial^2 v}{\partial t\partial x}=A_m(t,x)\frac{\partial v}{\partial x}+B_m(t,x)\frac{\partial v}{\partial t}+A_{m-1}(t,x)v+f(t,x)+D(t,x)\mu(x)+$$
$$+\sum_{r=1}^{m-2}A_r(t,x)v_{r+1}(t,x)+\sum_{s=1}^{m-1}B_r(t,x)\frac{\partial v_r(t,x)}{\partial t}+C(t,x)v_1(t,x),\quad (t,x)\in\Omega \quad (6)$$

$$\sum_{j=0}^{1}\left\{P_{m,j}(x)\frac{\partial v(t,x)}{\partial x}+S_{m,j}(x)\frac{\partial v(t,x)}{\partial t}+P_{m-1,j}(x)v(t,x)\right\}\Big|_{t=t_j}=\phi(x)-$$
$$+\sum_{j=0}^{1}\left\{\sum_{r=1}^{m-2}P_{r,j}(x)v_{r+1}(t,x)+\sum_{s=1}^{m-1}S_{s,j}(x)\frac{\partial v_s(t,x)}{\partial t}\right\}\Big|_{t=t_j},\quad x\in[0,\omega] \quad (7)$$

$$v(t,0)=\psi_{m-1}(t),\ t\in[0,T], \quad (8)$$

$$L(x)\frac{\partial v(\theta,x)}{\partial x}+M(x)\mu(x)=\varphi_1(x),\ x\in[0,\omega], \quad (9)$$

$$v_s(t,x)=\sum_{p=0}^{s-1}\psi_k(t)\frac{x^p}{p!}+\int_0^x\frac{(x-\xi)^{m-1-s}}{(m-1-s)!}v(t,\xi)d\xi,\ s=\overline{1,m-1},\ (t,x)\in\Omega. \quad (10)$$

Here the conditions (3) are taken into account in (10).

Differentiating (10) by t, we obtain

$$\frac{\partial v_s(t,x)}{\partial t} = \sum_{p=0}^{s-1} \dot{\psi}_k(t)\frac{x^p}{p!} + \int_0^x \frac{(x-\xi)^{m-1-s}}{(m-1-s)!}\frac{\partial v(t,\xi)}{\partial t}d\xi,\ s=\overline{1,m-1},\ (t,x)\in\Omega. \quad (11)$$

A system of functions $(v(t,x),\mu(x),v_1(t,x), v_2(t,x),...,v_{m-1}(t,x))$, where the function $v(t,x)\in C(\Omega,R)$ has partial derivatives $\frac{\partial v(t,x)}{\partial x}\in C(\Omega,R)$, $\frac{\partial v(t,x)}{\partial t}\in C(\Omega,R)$, and $\frac{\partial^2 v(t,x)}{\partial t\partial x}\in C(\Omega,R^n)$, the function $\mu(x)\in C([0,\omega],R)$, the functions $v_s(t,x)\in C(\Omega,R)$ have partial derivatives $\frac{\partial v_s(t,x)}{\partial t}\in C(\Omega,R)$, $s=\overline{1,m-1}$, is called a solution to problem with parameters (6)-(10), if it satisfies the second-order hyperbolic equation (6) for all $(t,x)\in\Omega$, boundary conditions (7) and (8), additional condition (9) and integral relations (10).

For fixed $v_s(t,x)$, $s=\overline{1,m-1}$, problem (6)--(9) is a nonlocal problem with parameter for the hyperbolic equation with respect to $v(t,x)$ and $\mu(x)$ on Ω. Integral relations (10) allow us to determine unknown functions $v_s(t,x)$, $s=\overline{1,m-1}$ for all $(t,x)\in\Omega$.

Algorithm

We determine the unknown function $v(t,x)$ from the nonlocal problem with parameter for hyperbolic equations (6)-(9). Unknown functions $v_s(t,x)$, $s=\overline{1,m-1}$, will be found from integral relations (10).

If we know the functions $v_s(t,x)$, $s=\overline{1,m-1}$, then from the nonlocal problem with parameter (6)--(9) we find the functions $v(t,x)$ and $\mu(x)$. And, conversely, if we know the functions $v(t,x)$ and $\mu(x)$, then from the integral conditions (10) we find the functions $v_s(t,x)$, $s=\overline{1,m-1}$. Since both functions $v(t,x)$, $\mu(x)$, $v_s(t,x)$, $s=\overline{1,m-1}$, are unknown, then to find a solution to problem (6)--(10) we use an iterative method.

The solution to problem with parameters (6)--(10) is the system of functions $(v^*(t,x),\mu^*(x), v_1^*(t,x),v_2^*(t,x),...,v_{m-1}^*(t,x))$, which we defined as the limit of the sequence of systems $(v^{(k)}(t,x),\mu^{(k)}(x),v_1^{(k)}(t,x),v_2^{(k)}(t,x),...,v_{m-1}^{(k)}(t,x))$, $k=0,1,2,...$, according to the following algorithm:

Step 0. 1) Suppose in the right-hand side of equation (6) we have $v_s(t,x)=\sum_{p=0}^{s-1}\psi_k(t)\frac{x^p}{p!}$ and $\frac{\partial v_s(t,x)}{\partial t}=\sum_{p=0}^{s-1}\dot{\psi}_k(t)\frac{x^p}{p!}$, $s=\overline{1,m-1}$. From nonlocal problem with parameter (6)-(9) we find the initial approximations $v^{(0)}(t,x)$ and $\mu^{(0)}(x)$, and partial derivatives $\frac{\partial v^{(0)}(t,x)}{\partial x}$ and $\frac{\partial v^{(0)}(t,x)}{\partial t}$ for all $(t,x)\in\Omega$;

2) From integral relations (10) and (11) under $v(t,x)=v^{(0)}(t,x)$ and $\frac{\partial v(t,x)}{\partial t}=\frac{\partial v^{(0)}(t,x)}{\partial t}$, respectively, we find the functions $v_s^{(0)}(t,x)$ and $\frac{\partial v_s^{(0)}(t,x)}{\partial t}$, $s=\overline{1,m-1}$, for all $(t,x)\in\Omega$.

Step 1. 1) Suppose in the right-hand side of equation (6) we have $v_s(t,x)=v_s^{(0)}(t,x)$ and $\frac{\partial v_s(t,x)}{\partial t}=\frac{\partial v_s^{(0)}(t,x)}{\partial t}$, $s=\overline{1,m-1}$. From nonlocal problem with parameter (6)-(9) we find the first approximations $v^{(1)}(t,x)$ and $\mu^{(1)}(x)$, and partial derivatives $\frac{\partial v^{(1)}(t,x)}{\partial x}$ and $\frac{\partial v^{(1)}(t,x)}{\partial t}$ for all $(t,x)\in\Omega$.

2) From integral relations (9) and (10) under $v(t,x)=v^{(1)}(t,x)$ and $\frac{\partial v(t,x)}{\partial t}=\frac{\partial v^{(1)}(t,x)}{\partial t}$, respectively, we find the functions $v_s^{(1)}(t,x)$ and $\frac{\partial v_s^{(1)}(t,x)}{\partial t}$, $s=\overline{1,m-1}$, for all $(t,x)\in\Omega$.

And so on.

Step k. 1) Suppose in the right-hand side of equation (6) we have $v_s(t,x)=v_s^{(k-1)}(t,x)$ and $\frac{\partial v_s(t,x)}{\partial t}=\frac{\partial v_s^{(k-1)}(t,x)}{\partial t}$, $s=\overline{1,m-1}$. From nonlocal problem with parameter (6)-(9) we find the k-th approximations $v^{(k)}(t,x)$ and $\mu^{(k)}(x)$, and partial derivatives $\frac{\partial v^{(k)}(t,x)}{\partial x}$ and $\frac{\partial v^{(k)}(t,x)}{\partial t}$ for all $(t,x)\in\Omega$:

$$\begin{aligned}\frac{\partial^2 v^{(k)}}{\partial t\partial x}&=A_m(t,x)\frac{\partial v^{(k)}}{\partial x}+B_m(t,x)\frac{\partial v^{(k)}}{\partial t}+A_{m-1}(t,x)v^{(k)}+f(t,x)+D(t,x)\mu^{(k)}(x)+\\&+\sum_{r=1}^{m-2}A_r(t,x)v_{r+1}^{(k-1)}(t,x)+\sum_{s=1}^{m-1}B_r(t,x)\frac{\partial v_r^{(k-1)}(t,x)}{\partial t}+C(t,x)v_1^{(k-1)}(t,x),\quad (t,x)\in\Omega\end{aligned}\tag{12}$$

$$\begin{aligned}&\sum_{j=0}^{1}\left\{P_{m,j}(x)\frac{\partial v^{(k)}(t,x)}{\partial x}+S_{m,j}(x)\frac{\partial v^{(k)}(t,x)}{\partial t}+P_{m-1,j}(x)v^{(k)}(t,x)\right\}\Big|_{t=t_j}=\phi(x)-\\&+\sum_{j=0}^{1}\left\{\sum_{r=1}^{m-2}P_{r,j}(x)v_{r+1}^{(k-1)}(t,x)+\sum_{s=1}^{m-1}S_{s,j}(x)\frac{\partial v_s^{(k-1)}(t,x)}{\partial t}\right\}\Big|_{t=t_j},\quad x\in[0,\omega]\end{aligned}\tag{13}$$

$$v^{(k)}(t,0)=\psi_{m-1}(t),\ t\in[0,T],\tag{14}$$

$$L(x)\frac{\partial v^{(k)}(\theta,x)}{\partial x}+M(x)\mu^{(k)}(x)=\varphi_1(x),\ x\in[0,\omega].\tag{15}$$

2) From integral relations (10) and (11) under $v(t,x)=v^{(k)}(t,x)$ and $\frac{\partial v(t,x)}{\partial t}=\frac{\partial v^{(k)}(t,x)}{\partial t}$, respectively, we find the functions $v_s^{(k)}(t,x)$ and $\frac{\partial v_s^{(k)}(t,x)}{\partial t}$, $s=\overline{1,m-1}$, for all $(t,x)\in\Omega$:

$$v_s^{(k)}(t,x)=\sum_{p=0}^{s-1}\psi_k(t)\frac{x^p}{p!}+\int_0^x\frac{(x-\xi)^{m-1-s}}{(m-1-s)!}v^{(k)}(t,\xi)d\xi,\ s=\overline{1,m-1},\ (t,x)\in\Omega.\tag{16}$$

$$\frac{\partial v_s^{(k)}(t,x)}{\partial t}=\sum_{p=0}^{s-1}\dot\psi_k(t)\frac{x^p}{p!}+\int_0^x\frac{(x-\xi)^{m-1-s}}{(m-1-s)!}\frac{\partial v^{(k)}(t,\xi)}{\partial t}d\xi,\ s=\overline{1,m-1},\ (t,x)\in\Omega.\tag{17}$$

Here $k=1,2,3,\ldots$.

The main results

Consider auxiliary nonlocal problem with parameter

$$\frac{\partial^2 v}{\partial t\partial x}=A_m(t,x)\frac{\partial v}{\partial x}+B_m(t,x)\frac{\partial v}{\partial t}+A_{m-1}(t,x)v+f(t,x)+D(t,x)\mu(x),\ (t,x)\in\Omega,\tag{18}$$

$$\sum_{j=0}^{1}\left\{P_{m,j}(x)\frac{\partial v(t,x)}{\partial x}+S_{m,j}(x)\frac{\partial v(t,x)}{\partial t}+P_{m-1,j}(x)v(t,x)\right\}\Big|_{t=t_j}=\varphi(x),\ x\in[0,\omega], \quad (19)$$

$$v(t,0)=\psi_{m-1}(t),\ t\in[0,T], \quad (20)$$

$$L(x)\frac{\partial v(\theta,x)}{\partial x}+M(x)\mu(x)=\varphi_1(x),\ x\in[0,\omega]. \quad (21)$$

The following theorem provides conditions for the feasibility and convergence of the constructed algorithm, as well as conditions for the existence of a unique solution to problem with parameter (6)--(10). The functions functions $A_i(t,x)$, $B_i(t,x)$, $i=\overline{1,m}$, $C(t,x)$, and $f(t,x)$ are continuous on Ω, the functions $P_{ij}(x)$, $S_{ij}(x)$, $i=\overline{1,m}$, $j=0,1$, and $\varphi(x)$ are continuous on $[0,\omega]$, the functions $\psi_s(t)$, $s=\overline{0,m-1}$, are continuously differentiable on $[0,T]$, the functions $L(x)$, $M(x)$, and $\varphi_1(x)$ are continuous on $[0,\omega]$.

Theorem 1. *Let*

i) the functions $A_i(t,x)$, $B_i(t,x)$, $i=\overline{1,m}$, $C(t,x)$, *and* $f(t,x)$ *be continuous on* Ω;

ii) the functions $P_{ij}(x)$, $S_{ij}(x)$, $i=\overline{1,m}$, $j=0,1$, and $\varphi(x)$ *be continuous on* $[0,\omega]$;

iii) the functions $\psi_s(t)$, $s=\overline{0,m-1}$, *be continuously differentiable on* $[0,T]$;

iv) th nonlocal problem with parameter has a unique solution.

Then the nonlocal problem for the hyperbolic equation with parameters and integral conditions (6)--(10) has a unique solution $(v^*(t,x),\mu^*(x), v_1^*(t,x),v_2^*(t,x),...,v_{m-1}^*(t,x))$ *as a limit of sequences* $(v^{(k)}(t,x),\mu^{(k)}(x),v_1^{(k)}(t,x), v_2^{(k)}(t,x),...,v_{m-1}^{(k)}(t,x))$ *determined by the algorithm proposed above for* $k=0,1,2,...$.

The proof of Theorem 1 is similar to the proof of Theorem 1 in [35].

Then the unique solution $(u^*(t,x),\mu^*(x))$ to problem with parameter (1)-(4) determines as $u^*(t,x)=v_1^*(t,x)$ and $\mu^*(x)$.

The equivalence of problems (6)-(10) and (1)-(4) implies

Theorem 2. *Let conditions i) – iv) of Theorem 1 be fulfilled.*

Then the initial-boundary value problem with parameter for the higher-order partial differential equation (1)--(4) has a unique classical solution $(u^*(t,x),\mu^*(x))$.

Introduce the following notation:

$$\alpha(x,t,\tau)=\int_{\tau}^{t}A_m(\tau_1,x)d\tau_1,$$

$$Q(x,T,\theta)=\begin{bmatrix} P_{m,1}(x)+S_{m,1}(x)e^{\alpha(x,T,0)} & S_{m,1}(x)\int_0^T e^{\alpha(x,T,\tau)}D(\tau,x)d\tau \\ L(x)e^{\alpha(x,\theta,0)} & L(x)\int_0^\theta e^{\alpha(x,\theta,\tau)}D(\tau,x)d\tau+M(x)\end{bmatrix},\ x\in[0,\omega].$$

Theorem 3. *Let*

a) conditions i) – iii) of Theorem 1 be fulfilled;

b) the (2x2)- matrix $Q(x,T,\theta)$ *is invertible for all* $x\in[0,\omega]$.

Then the initial-boundary value problem with parameter for the higher-order partial differential equation (1)--(4) has a unique classical solution $(u^*(t,x),\mu^*(x))$.

Conclusion

Therefore, we are studied the identification parameter problem for higher-order partial differential equations with two variables. We are established the sufficient conditions for the existence and uniqueness of the solution to the considered identification parameter problem. We are reduced this problem to the equivalent problem consisting of the nonlocal problem for second-order hyperbolic equations with functional parameters and integral relations by introducing new unknown functions. An algorithm for finding an approximate solution to the equivalent problem with parameters is proposed and its convergence is proved. Sufficient conditions for the existence of the unique solution to the equivalent problem with parameters are established. The conditions for the unique solvability of the initial-boundary value problem with parameter for higher-order partial differential equations are obtained in terms of the initial data. Unique solvability to the identification parameter problem for higher-order partial differential equations is interconnected with unique solvability to the identification parameter problem for second-order hyperbolic equations. These results will be developed to various initial-boundary value problems with parameters for the higher-order system of partial differential equations and control problems for second-order system of hyperbolic equations.

Acknowledgement

The research is partially supported by grant AP05131220 of Ministry of Education and Science of the Republic of Kazakhstan (2018-2020).

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