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An inverse problem of dcis model based on nonlocal and terminal data

Abstract. Astheearliest period of breast cancer, the ductal carcinoma in situ (DCIS) model has wide applications in the diagnosis of breast cancer and has been attracted much attention in recent years. In this paper, a novel PSO method is developed for solving an inverse problem of the DCIS model from nonlocal and terminal data. The numerical simulations show that the proposed method is efficient, accurate, robust against noise and fast. Moreover, it is better than the optimization method in the literature [8].
Key words: free boundary problem; PSO method; ductal carcinoma in situ; numerical simulation.

1. Introduction

Ductal carcinoma in situ (DCIS) means a specific diagnosis of cancer that is isolated within the breast duct, and has not spread to other parts of the breast. Tumor growth is an important research focus of mathematical modeling in recent 40 years [1-7].

In this paper we study a model about tumor growth firstly proposed by Byrne and Chaplain in 1995 [1-3]. Ward and King developed a velocity field to handle local volume changes caused by cell movement under some reasonable assumptions [4-5]. Mathematical modeling for the dynamical growth of DCIS is a free boundary problem and was developed in [6-9]. To find possible steps to simulate the growth of the DCIS model with clinical data, Xu and his collaborators performed some mathematical analysis on the modified model and performed numerical calculations on some typical cases [6]. Li and Zhou studied an inverse problem of solving the control parameter with known moving boundaries [7]. According to one of the four inverse problems proposed by Xu [6], then Liu established the uniqueness theorem for determining the inverse problem with unknown parameters, deduced an optimization problem, and proposed an effective algorithm to solve the problem [8].

Due to the difficulties caused by the time varying boundary, numerical simulations are very limited. Especially, the effective numerical approaches for the inverse problems are indispensably and urgently needed.

In this paper, we shall present a novel efficient PSO method for solving the inverse free boundary problem. In section 2, a brief introduction of direct problem of DCIS would be exhibited. The novel PSO method would be proposed in section 3. And in section 4, a numerical example are demonstrated to show the effectiveness and robustness of our novel method.

2. A Brief Introduction of Direct Problem for DCIS

In this section, the forward problem of DCIS model would be stated. The DCIS problem of the one-dimensional case in Figure 1 is modeled by the following parabolic equation

$$c\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial x^2} - \lambda(x)v(x,t) + F(x,t)\text{\textcircled{}}$$
$$0 < x < \psi(t), t > 0, \tag{2.1}$$

where $0 \le c = T_{diffusion} / T_{growth} \ll 1$ (normally, $T_{\text{diffusion}} \approx 1\,\text{minute}$ $T_{\text{growth}} \approx 1\,\text{day}$) is the ratio of the nutrient diffusion time scale to the tumor growth time scale, v denotes the tumor growth

pattern which is using dimensionless nutrient concentration, $\lambda(x)v(x,t)$ means the nutrient consumption rate, $F(x,t)$ represents the transfer of nutrient from/to the neighborhood and $\psi(t)$ are the growing boundary of the tumor. Moreover, $v(x,t)$ should satisfy the following initial and free boundary conditions,

$$\begin{aligned} v(x,0) &= f(x),\ 0 < x < \psi(0), \\ v\big(0,t\big) &= g_1(t),\ 0 < t < T, \\ v\big(\psi(t),t\big) &= g_2(t),\ 0 < t < T, \end{aligned} \tag{2.3}$$

Where the final time $T < \infty$ is a constant. Furthermore, the mass conservation consideration indicates the following equality

$$\frac{d\psi(t)}{dt} = \sigma \int_0^{\psi(t)} \big(v(x,t) - s_0\big)dx, \tag{2.3}$$

where both σ and s_0 are positive constants. The term $\sigma(v - s_0)$ in (2.3) represents the cell proliferation rate inside the tumor, and the cell birth rate is denoted as σv while the death rate is provided by σs_0. The direct problem of this model is to determine $\{v(x,t),\psi(\mathrm{t})\}$ for given $\{\lambda(x), F(x,\mathrm{t}), g_1(\mathrm{t}), g_2(\mathrm{t}), \psi(0), \sigma, s_0\}$. The direct problem can be solved by the finite difference method, we refer to the literature [8].

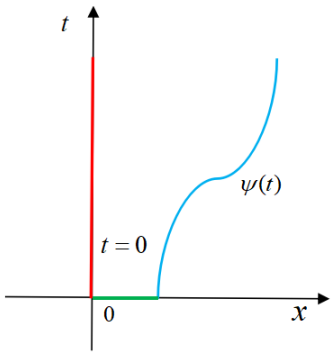


Figure 1 – The demonstration of the free boundary problem DCIS in the one dimension

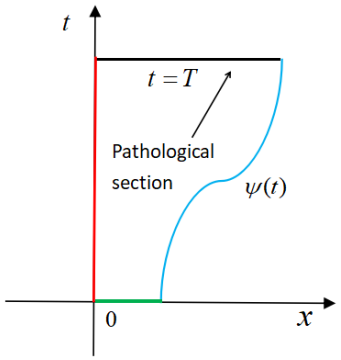


Figure 2 – The demonstration of pathological sections at time t = T

3. Inverse Problem of DCIS

In this section, the inverse problem of DCIS model would be investigated. In a routine physical examination, a possible breast tumor would be noticed, and it may be benign. The tumor would be growing bigger and bigger in the following days.

Therefore, the patient have to do an incisional biopsy to determine the DCIS pattern along with the changing rate at a fixed period (e.g. a couple of weeks). In this case, the initial data is not available, and only the information of set $\{v(x,t),\psi(\mathrm{t}),\mathrm{W}\}$ is provided by the incisional biopsy at the examine time $\mathrm{t} = T$, see Figure 2 for the demonstration.

The inverse problem of our interest is to determine the rate $\lambda(x)$ for $0 < x < \psi(T)$ from the examined data set $\{v(x,t),\psi(\mathrm{t}),\mathrm{W}\}$ and the given data set $\{\mathrm{c},\sigma,s_0 F(x,T),g_1(T),\mathrm{g}_2(T)\}$, where the illustration of $\mathrm{W}(\xi)$ is provided in (3.1). With the recovered $\lambda(x)$ for $0 < x < \psi(T)$ and the given data set, the process to approximate $\psi(t)\big|_{t>T}$ and $v(x,t)\big|_{t>T}$ becomes the direct problem. Finally, we are able to diagnose the breast tumor is benign or not from the information of estimate $\{\lambda(x),\psi(t)\big|_{t>T}, v(x,t)\big|_{t>T}\}$. Consequently, the inverse problem comes down to determine $\lambda(x)$ from the examined and given data. Moreover, the uniqueness of inverse problem is equivalent to the uniqueness of $\lambda(x)$.

We consider the DCIS model as follows

$$\begin{cases} c\dfrac{\partial v}{\partial t}=\dfrac{\partial^2 v}{\partial x^2}-\lambda(x)v(x,t)+F(x,t)\,,\ \ 0<x<\psi(t),\,0<t<T,\\ v\big(0,t\big)=g_1(t),\quad 0<t<T,\\ v\big(\psi(t),t\big)=g_2(t)\,,\quad 0<t<T,\\ \psi'(t)=\sigma\displaystyle\int_0^{\psi(t)}\big(v(x,t)-s_0\big)dx,\quad 0<t<T,\\ \displaystyle\int_0^{\psi_T} v_t(x,T)\varrho(x,\xi)dx=W(\xi)\,,\qquad \xi\in D, \end{cases} \tag{3.1}$$

where D is a parameter set, and $\{\rho(\cdot,\xi)\big|\xi\in D\}$ is assumed to be complete in $L^2([0,1])$. In a clinical aspect, the function represents the obtained data for the growth rate of tumor cells.

By the variables substitutions[8], the above problem (3.1) is equivalent to determine $\mu(\varsigma)$ such that

$$\begin{cases} \dfrac{\partial u}{\partial t}=\dfrac{1}{\psi^2(t)}\dfrac{\partial^2 u}{\partial\zeta^2}+c\dfrac{-\psi'(t)/2+\zeta\psi'(t)}{\psi(t)}\dfrac{\partial u}{\partial\zeta}-\mu(\zeta)u(\zeta,t)+H(\zeta,t),\\ u(0,t)=g_1(t),\qquad 0<t<T,\\ u(1,t)=g_2(t),\qquad 0<t<T,\\ u(\zeta,T)=u_T(\zeta)\ ,0<\zeta<1,\quad \psi(T)=\psi_T,\\ \psi(t)=\psi_T e^{-\sigma\int_t^T\int_0^1(u(\zeta,t)-s_0)d\xi d\tau}\,,\\ W(\xi)=\displaystyle\int_0^1 u_t(\zeta,T)\varrho(\zeta,\xi)d\zeta,\qquad \xi\in D\,. \end{cases} \tag{3.2}$$

where $\mu(\varsigma)$ and $H(\varsigma,t)$ are respectively presented as follows

$$\mu(\zeta)=\lambda\big(\zeta\big(\psi(t)\big)\big), \tag{3.3}$$

$$H(\zeta,t)=F\big(\zeta\big(\psi(t)\big),t\big), \tag{3.4}$$

We now consider the set $\Omega=\{\rho(\varsigma,\xi)\big|\varsigma\in[0,1],\xi\in D\}$ which forms a base of $L^2([0,1])$. Without loss of generality, the set is selected as $\Omega=\{\sin(\pi\varsigma\xi)\big|\varsigma\in[0,1],\xi=0,1,\cdots\}$.

4. PSO method for the Inverse Problem of DCIS

In this section, we convert the inverse problem of estimating $\mu(\varsigma)$ into a minimization problem and obtain the solution for the optimization problem by a stochastic search method which is known as particle swarm optimization algorithm.

The inverse problem of estimating $\mu(\varsigma)$ is expressed as follows similar to the literature [8],

$$\min_{\mu\in L^\infty([0,1])} J(\mu) \tag{4.1}$$

where

$$J(\mu)=\left\|\int_0^1\mu(\zeta)u(\zeta,T)\varrho(\zeta,\xi)\mathrm{d}\zeta-\gamma(\xi)\right\|^2_{L^2(D)}. \tag{4.1}$$

There are many approaches are existing to solve the above optimization model, we refer to [8-13]. In the reference [8], the optimization problem is transformed into a solution of linear algebraic equations by direct discrete method, and

then the linear algebraic equations are solved by regularization method. In this paper, we would like to apply the PSO method which is knowns as an effective method to solve the optimization problems.

The PSO method [14-18] is an efficient technique for solving many nonlinear, nondifferentiable and multi-modal complex optimization problems. It has become very popular because its implementation is very simple and can be quickly aggregated into a good solution. It does not require any gradient information of the optimization function, and only uses the original mathematical operator. The PSO method is a stochastic algorithm, which does not depend on the initial value select, and can converge to the global optimal solution.

This group of particles is called a swarm in PSO. A swarm consists of M particles moving around in a D-dimensional search space. The position of the *i*-th particle can be represented $z_i = \left(z_{i_1}, z_{i_2}, \ldots, z_{i_D}\right)$. The velocity of the *i*-th particle can be written as $\Delta z_i = \left(\Delta z_{i_1}, \Delta z_{i_2}, \ldots, \Delta z_{i_D}\right)$. The optimal position so far found by particle *i*-th is denoted as $z_i^p = \left(z_{i_1}^p, z_{i_2}^p, \ldots, z_{i_D}^p\right)$ called p_i^{best}. The best value of the all individual p_i^{best} values is denoted as the global best position $z_i^g = \left(z_{i_1}^g, z_{i_2}^g, \ldots, z_{i_D}^g\right)$ and called g_i^{best}. In each iteration, the particle updates its speed and position according to the following formula:

$$\Delta z_i^{\text{new}} = w \times \Delta z_i^{\text{old}} + c_1 r_1 \left(p_i^{\text{best}} - z_i^{\text{old}}\right) + c_2 r_2 \left(g_i^{\text{best}} - z_i^{\text{old}}\right) \tag{4.2}$$

$$z_i^{\text{new}} = z_i^{\text{old}} + \Delta z_i^{\text{new}}. \tag{4.2}$$

where r_1 and r_2 are random numbers between $[0,1]$, c_1 and c_2 are acceleration constants which control how far particles move in a single generation. Velocities Δz_i^{new} and Δz_i^{old} denote the velocities of the new and old particle respectively. z_i^{old} is the current particle position, and z_i^{new} is updated particle position. The Inertial factor w controls the impact of the previous velocity of a particle on its current one.

The algorithm only requires the fitness function of each particle, without continuity, differentiability and other assumptions, which is very useful for discontinuous functions.

5. Numerical Simulations

In this section, we would like to state a numerical example to exhibit the feasibility and effectiveness of our methods. And we would compare the reconstructions of PSO method and Liu's method of the literature [8] in the following numerical experiments.

We investigate the above DSCI model (2.1)–(2.3) with $\lambda(x) = x$, $f(x) = (1+x)\mathrm{e}^{2x}$, $F(x,t) = \left[(1+x)(2-t)^2 - 2t + 4\right]\mathrm{e}^{(2-t)x}$, $g_1(t) = 1$, $g_2(t) = \left(1 + \frac{1}{4-2t}\right)\mathrm{e}^{1/2}$, $\psi(t) = \frac{1}{4-2t}$, then the solution can be represented as $v(x,t) = (1+x)\mathrm{e}^{(2-t)x}$. And the parameters $c=1$, $s_0 = \mathrm{e}^{1/2} - \mathrm{e}^{-1/2}$, $\sigma = 1/(\frac{1}{2}\mathrm{e}^{1/2} + \frac{3}{2}\mathrm{e}^{-1/2})$. The mesh sizes of x and t variables are respectively selected as $h = 0.01$ and $\tau = 0.001$, and the time interval is chosen to be [0,1].

The parameters in PSO are set as $M = 300, c_1 = c_2 = 1.4962, w = 0.7298.$

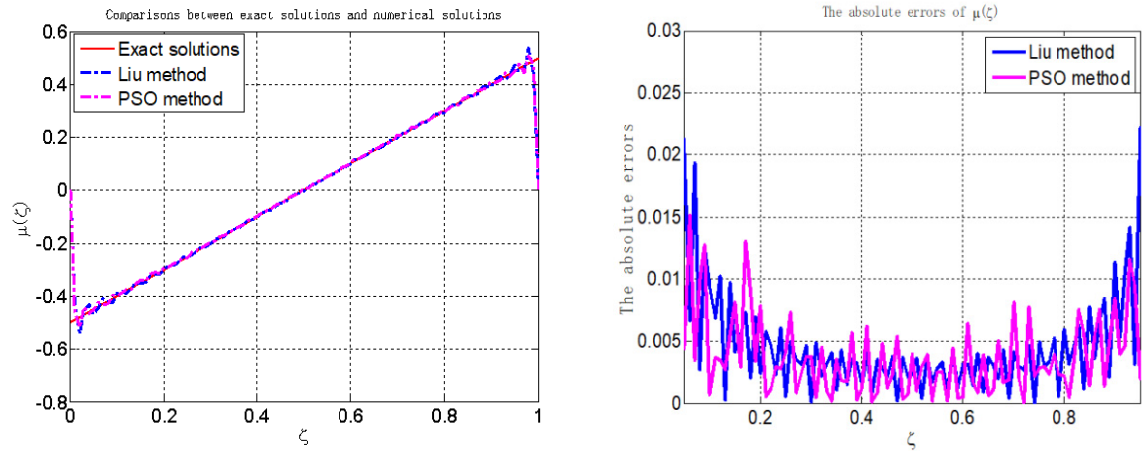
In order to compare the results involving random measurement noise, we add a uniform distribution uncorrelated errors. The simulated inexact measurement data can be expressed as

$$u(\zeta, T) = u(\zeta, T)[1 + \delta K(\zeta)], \quad \zeta \in [0,1] \tag{5.1}$$

where δ=1% or δ=3% means the noise level and $K(\zeta)$ is a random number which varies from -1 to 1 and is uniformly distributed.

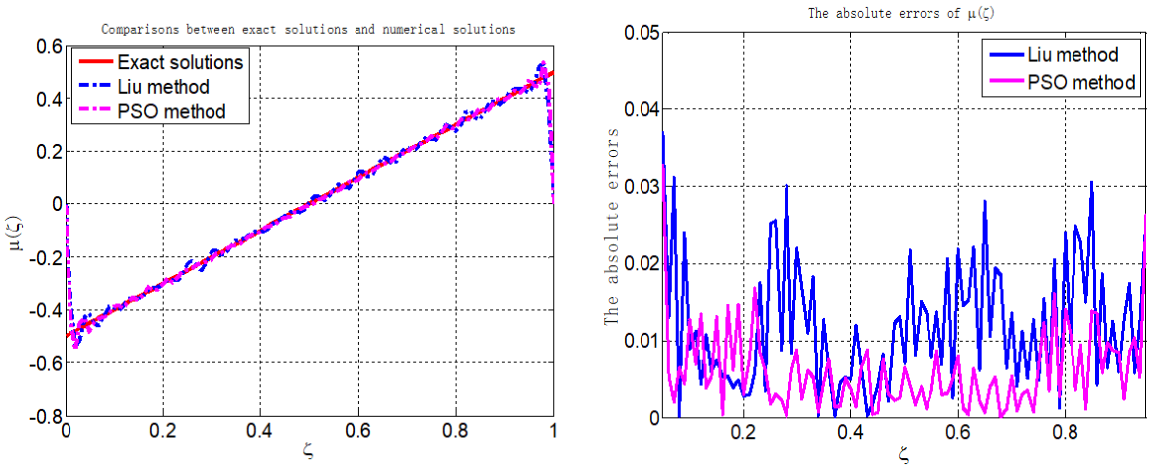
Given that the accurate measured data, the inversion results of the two methods are close to each other and the relative errors are not much different in Figure 3. If the measurements contain perturbations, PSO method gives better results than the method in [8] from Figures 4 and 5.

The inversion results of exact measurement data are better than the results of data containing noisy from Figure 3-5. When noisy measurements δ=1% , the results of PSO are have smaller relative errors than the results of Liu's method. The same results can be obtained with noisy measurement δ=3%.



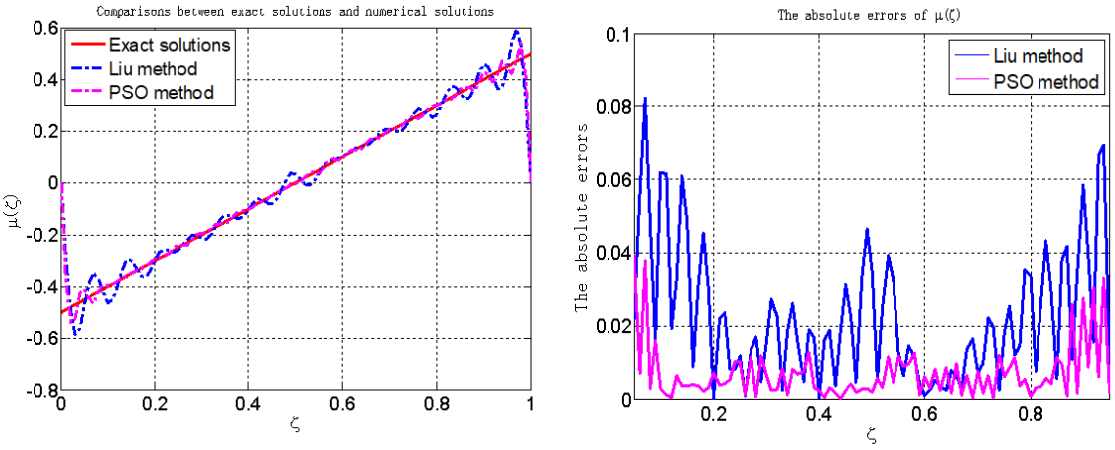
(a) Comparison results (b) Absolute errors

Figure 3 – Comparisons results and absolute errors between exact solutions and numerical solutions with exact measurements



(a) Comparison results (b) Absolute errors

Figure 4 – Comparisons results and absolute errors between exact solutions and numerical solutions with noisy measurements δ=1%



(a) Comparison results (b) Absolute errors

Figure 5 – Comparisons results and absolute errors between exact solutions and numerical solutions with noisy measurements δ=3%

6. Conclusion remarks

We adopt PSO algorithm to solve the inverse problem of DCIS model, which is converted into an optimization problem. The advantage of the characteristic of this random search method is that it does not need gradient calculation and the choice of initial guess. Therefore, even if there is a small noise, the PSO algorithm is still stable when dealing with this inverse problem. It can be observed from the numerical results that PSO method is effective and robust to solve the inverse problem of DCIS model.

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