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Mathematical problems of gravimetry and its applications

Abstract. Gravimetry is associated with analysis of the gravitational field. The gravitational field is characterized by its potential. This is described by the Poisson equation, the right side of which includes the density of the environment. There exists direct and inverse problems of gravimetry. Direct gravimetry problems involve the determination of the potential of the gravitational field in a given region. The inverse problems of gravimetry imply the restoration of the structure of a given area from the results of measuring the characteristics of the gravitational field. Such studies are needed to assess on the basis of gravimetric geodynamic events occurring in oil and gas fields. The relevance of such research is necessary, because with prolonged development of the oil and gas fields, negative consequences may occur. This paper discusses some of the features of direct and inverse gravimetry problems. A description of the mathematical model of the processes under consideration is given. Different direct and inverse gravimetry problems are posed. Describes the methods of its solving. Based on the analysis of the results of a computer experiment, appropriate conclusions are made.
Key words: gravimetry, inverse problems, mathematical model.

Introduction

Gravimetry is a science related to the study of gravitational fields. The gravitational field is potential, i.e. the work expended on movement in this field along a closed curve is zero. The main function characterizing a potential field is the potential. The potential of the gravitational field is described by the Poisson equation, the right-hand side of which includes the density distribution in a given region [1, 2].

Mathematical problems of gravimetry are divided into direct and inverse. Direct gravimetry problems involve finding the distribution of the potential of a gravitational field over a known density distribution in a given region. This is achieved by solving the corresponding boundary value problem for the Poisson equation. In the inverse problems of gravimetry, on the contrary, it is necessary to reconstruct the structure of the considered set by measuring the gravitational field.

The relevance of the inverse problems of gravimetry is due to the fact that in the process of long-term operation of deposits of different minerals (in particular, oil and gas), significant changes occur that have undesirable consequences [3–5]. In this regard, monitoring of existing fields is regularly conducted. In this case, we are interested in gravimetric monitoring. With the help of gravimeters, measurement of the acceleration of gravity, corresponding to the gradient of the potential of the gravitational field, is carried out [6, 7]. This experimental information can be used as a basis for the formulation of inverse problems of gravimetry.

It is known that the inverse problems are ill-posed, in principle [8]. However, the inverse problems of gravity are essentially ill-posed. In particular, the values of the acceleration of gravity determined during the measurement process may be due to various gravity anomalies. Thus, the solution of the inverse problem of gravimetry in the full formulation is not the only solution. Naturally, in

the numerical solution of such a problem, the algorithm outputs to one of these solutions. However, this result may not correspond to reality. Mathematically correct, it will have no practical meaning. In this connection, in practice, only partial inverse gravimetry problems are actually solved (see, for example, [9–18]). These works differ, firstly, in the volume of measured information, secondly, in the amount of identifiable information and, thirdly, in research methods.

In this paper, we discuss some peculiar properties of the formulation of direct and inverse gravimetry problems based on the available experimental information, as well as methods for solving these problems. We characterize some of the difficulties encountered in solving direct and inverse gravimetry problems and discuss ways to overcome.

Statement of the problem

At first, give the general direct gravimetry problem (see, for example, [1,2]). The gravitational field in the given volume is described by the Poisson equation

$$\Delta\psi(x,y,z) = -4\pi G\rho(x,y,z), \qquad (1)$$

where ψ is the gravitational potential, ρ is the density, G is the gravitational constant.

It is necessary to add the boundary conditions. In principle, we can have some results of measuring on the ground surface. Unfortunately, we do not, as a rule, any information about the gravitational field underground. However, it is known that the influence of the object to the gravitational field decreases with distance from the object and tends to zero with unlimited distance from it. Then we can extend the given set such that the gravitational potential on the boundary of the extended set will be zero. Thus, the general direct gravimetry problem is finding the gravitational potential $\psi = \psi(x,y,z)$ from the homogeneous Dirichlet problem for the Poisson equation (1), using known density distribution $\psi = \psi(x,y,z)$.

For formulating inverse problems of gravimetry, it is necessary to determine what specific information we can directly get into the process of gravimetric monitoring and what exactly we would like to find on the basis of this information. Note that when analyzing deposits, we have some territory S in the x, y plane. The terrain in this area is known. In addition, the maximum depth that is of interest to the research is usually specified. It is natural to choose it as a reference, i.e. the zero value of the vertical coordinate z. Then we can assume that the given function is $h = h(x, y)$, which characterizes the height of the terrain at the point x, y of the surface S with respect to the chosen system. Thus, the system is considered in three-dimensional volume

$$V = \{(x,y,z) \mid 0 < z < h(x,y), (x,y) \in S\}.$$

In practice, using gravimeters, the gravitational acceleration is measured, which, up to a sign, coincides with the vertical derivative of the potential of the gravitational field. Thus, the following condition holds

$$\frac{\partial\psi\left(x,y,h(x,y)\right)}{\partial z} = -g(x,y), \ (x,y) \in S, \quad (2)$$

where g is the experimentally measured value of the gravitational acceleration. This information, which is the result of gravimetric monitoring, can be used as the basis for the formulation of inverse gravimetry problems.

The purpose of the gravimetric monitoring of the existing field is largely to clarify the geological and tectonic structure and geological field information of the study area in order to highlight the risk of geodynamic processes. Such information can be obtained by knowing the density distribution in a given region. Thus, the object of the search in the process of solving the inverse problem of gravimetry is the density function, which is in the right-hand side of the considered equation (1). Therefore, the general gravimetry problem is finding the density distribution $\rho = \rho(x,y,z)$ in the volume V such that the solution of the homogeneous Dirichlet problem for the Poisson equation (1) in the extended set satisfies the additional condition (2).

As is known, the most natural way to solve inverse is to reduce them to optimization problems. In particular, the stated inverse problem can be reduced to the problem of minimizing the functional

$$I = I(\rho) = \int_S \left[\frac{\partial\psi\left(x,y,h(x,y)\right)}{\partial z} + g(x,y)\right]^2 dS, \quad (3)$$

where ψ is a solution of the considered direct problem corresponding to the given function ρ. Naturally, the solution of the inverse problem also turns out to be the solution of the optimization

problem, and the solution of the optimization problem under the condition of its existence will be the solution of this inverse problem. The practical solution of the obtained optimization problem is carried out using numerical optimization methods, for example, the gradient method [19–21].

Note that in reality the measurement of the gravitational acceleration is carried out not everywhere in a given area S, but only at certain points (x_i,y_i), $i = 1,2,\ldots,M$, where gravimeters are located. Thus, in fact, instead of (2), we have the condition

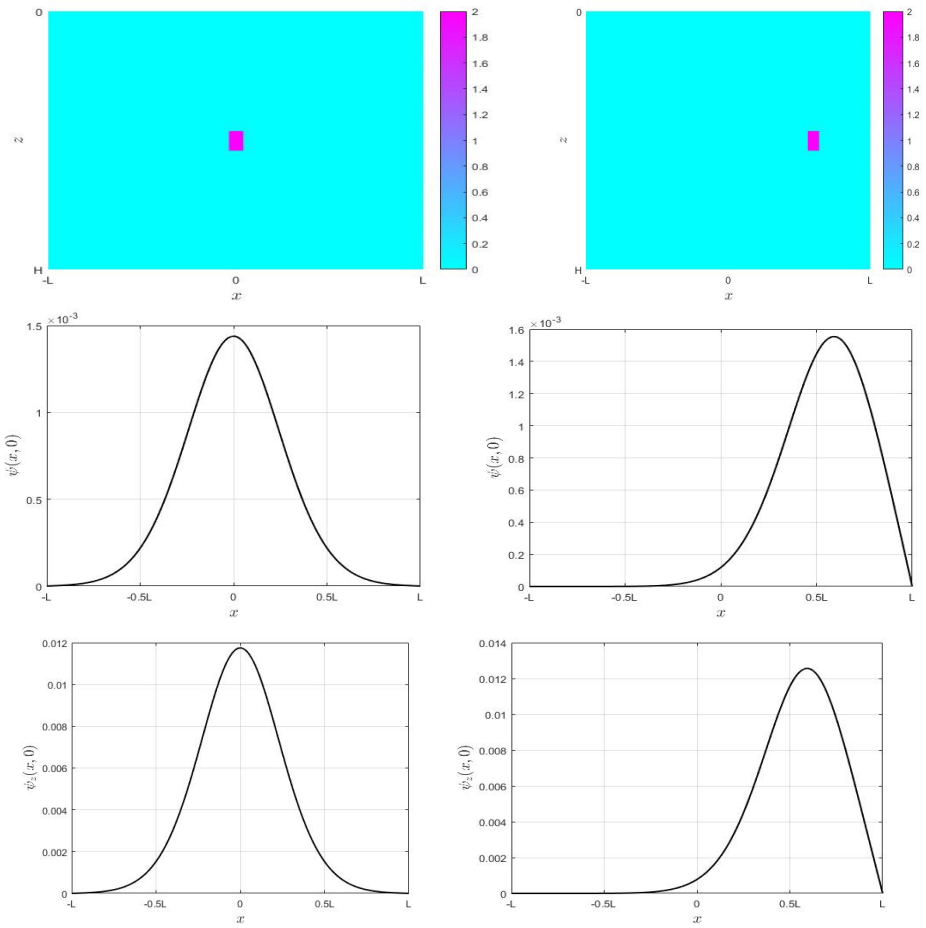
$$\frac{\partial\psi\left(x_i,y_i,h(x_i,y_i)\right)}{\partial z} = -g_i,\ i = 1,\ldots,M \qquad (4)$$

with known values g_i. Thus, in practice, either by interpolation, the transition to condition (2) is performed, followed by minimization of the functional (3), or we solve the minimization problem for the functional.

$$I = I(\rho) = \sum_{i=1}^{M}\left[\frac{\partial\psi\left(x_i,y_i,h(x_i,y_i)\right)}{\partial z} + g_i\right]^2. \qquad (5)$$

Results and discussion

For simplicity, we perform the analysis for the two-dimensional case, considering the horizontal coordinate x and the vertical coordinate z. For the direct problem in the simplest case, we consider a rectangular area in which the gravitational anomaly is located, i.e. an object significantly different in density from the environment. Figure 1 shows the density distribution over a given area, as well as the calculated distribution of the gravitational potential and its derivative in the upper part of the region corresponding to the earth's surface if the anomaly is located in the center of the considered set (Figure 1a) and near its boundary (Figure 1b).



a) anomaly is in the center b) anomaly is near the boundary

Figure 1 – Density distribution, potential, and its vertical derivative for the case of rectangular anomaly

As can be seen from Figure 1a, at the location of the anomaly (point x), the potential and its vertical derivative have their maximum value. As one moves away from the anomaly, these values decreases to zero equally in both directions, which is the corollary to zero boundary conditions. However, when the anomaly is located near the boundary of the region under consideration (Figure 1b), the potential distribution and its derivative are no longer symmetrical, which is not satisfactory. The results suggest that, in order to eliminate the influence of the boundaries, the area under consideration should be significantly extended.

At the next stage of the study, we are already repelling ourselves from the geological and lithographic section of the real field. Figure 2 depicts the density distribution in the considered area, with the yellow color indicating the area filled with clay – the predominant environmental material and relatively high density, blue – the air that inside the field corresponds to the existing voids with significantly lower density, and green – oil, more lighter than clay, but heavier than air. When extending a given area, it is assumed that air is located above the surface of the earth, and clay is located outside the initial area.

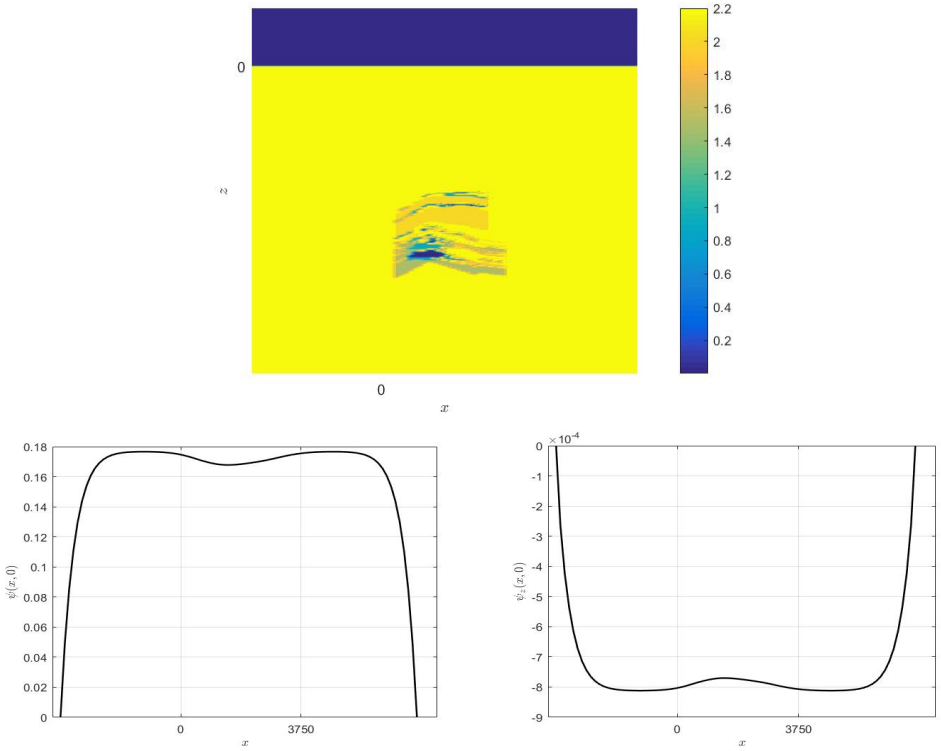


Figure 2 – Density distribution, potential, and its vertical derivative for the extended set with real parameters and usial density outside the initial set

As can be seen from the results obtained, the potential on the surface of the earth over the zone of predominance of oil and voids is lower compared to the neighboring zones where clay is predominant. This is due to the fact that the clay has a greater density. In this case, the vertical derivative of the potential is negative, since as the distance from the object increases, the potential value decreases, and the larger, the larger the potential value itself. We draw attention to the fact that outside the initial region, the potential value turns out to be rather large and decreases sharply to zero in the vicinity of the boundaries. This is explained by the fact that there is heavy clay outside the initial region, and zero potential values are rigidly set at the boundary. Besides, the potential derivative greatly increases in the neighborhood of the boundaries. Such a result cannot be considered satisfactory, and suggests that the density in the extended part of the region should be continued to zero. Indeed, we consider the gravitational field created by objects located in a given area.

At the next stage of the analysis, we carried out calculations with the extension of the set so

that the density outside the initial region is assumed to be zero. In this case, the distribution of the potential gradually decreases to zero as the boundaries approach. The value of the vertical derivative potential also tends to zero, see Figure 3.

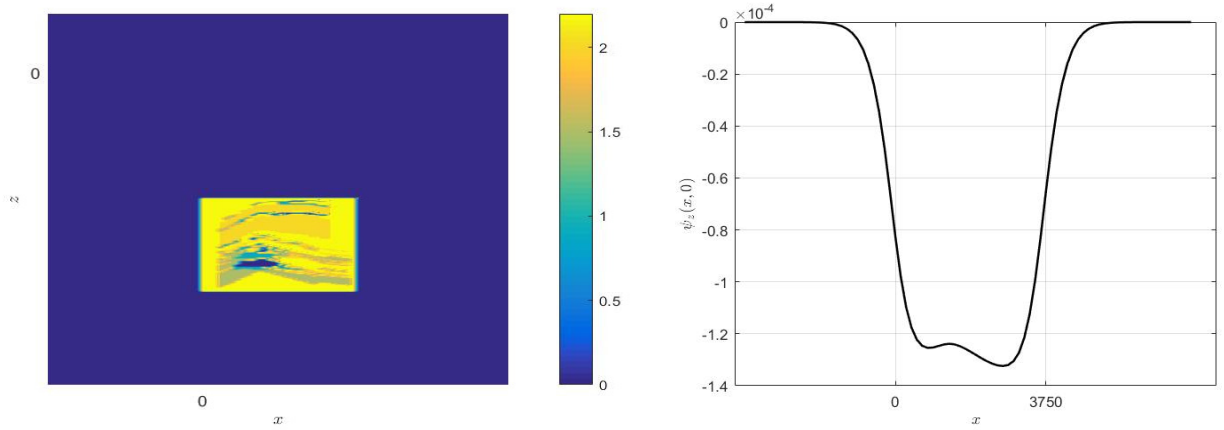
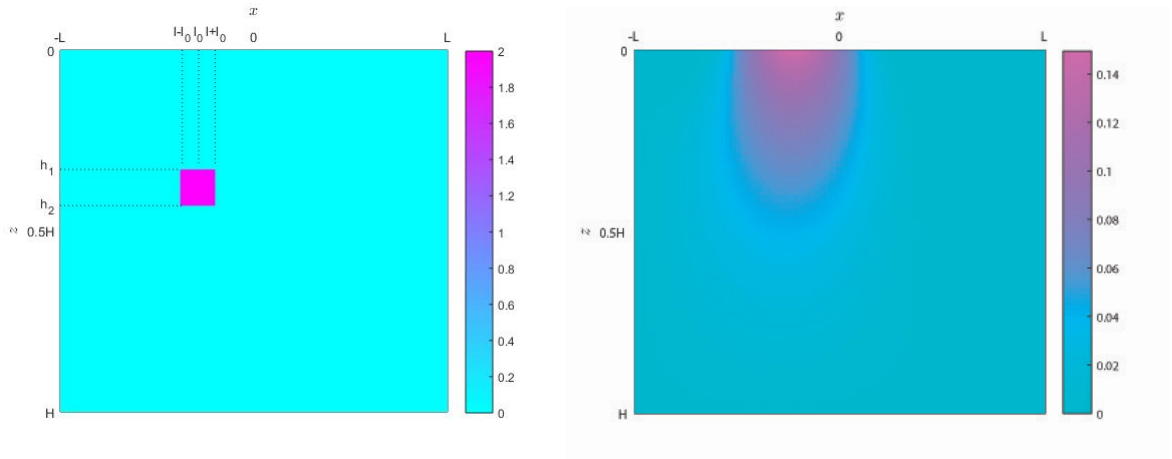


Figure 3 – Density distribution and vertical derivative of the potential for the extended set with real parameters and usial density outside the initial set

The question arises, how we can determine the size of the extended set. First, some extension is selected, and the value of the derivative potential on the earth's surface in the given region is calculated. Then the area extends again and calculations are carried out. If the newly found value of the derivative potential practically does not differ from that found earlier, then the calculations are terminated. Otherwise, a new extension is carried out.

Now consider the inverse problem. At first, we try to solve the general inverse problem for the two-dimensional case. We determine the gravitational anomaly as the square with a higher density than the density of the environment (see Figure 4a). Then we solve the Poisson equation with given boundary condition and calculate the vertical derivative of the potential at the ground surface. Now we put the result to the minimizing functional and solve the minimization problems by means of the gradient method. The iterative method converged, and the sequence of functional tends to zero. Thus, we found the solution of the optimization problem that is the solution of inverse problem too. The obtained result is shown in the Figure 4b). Unfortunately, this result is significantly different from the real. This is the corollary of the non-uniqueness of the considered inverse problem.



a) real position b) obtained position

Figure 4 – Position of the gravitational anomaly of the inverse problem

Its is clear that the general inverse problem of gravimetry is not very interesting because of its significantly non-uniqueness. We had a few data (boundary value of potential derivative) for finding the many information (density as a function of spatial variables). Then we consider two partial cases. For the first case, we suppose that we know the position of the homogeneous anomaly, its form and size, but its density is unknown. For the second case, we consider inverse situation. The density of the anomaly with given form and size is known, and its position is unknown. The first problem was be solved by gradient methods [19–21] with good enough exactness. The second (geometric) inverse problem has the peculiarity. The minimizing functional is not Gateaux differentiable. It is subdifferentiable only. Then we use the methods of non-smooth optimization, particularly, the subgradient method [22], the Nelder – Mead method [23], and genetic algorithms [24]. The exactness of the results was be good enough too. This is clear, because for both partial inverse problems, we determine one (constant density) or two (coordinates of the anomaly) parameters, using the knowledge of the function (vertical derivative of the potential at the ground surface).

Conclusion

Based on the obtained results, the following conclusions can be drawn:

1. The direct problem of gravimetry is based on the Poisson equation with respect to the potential of the gravitational field with a density included in the right-hand side of the equation.
2. To find the potential distribution in a given region, the given region should be extended by setting uniform Dirichlet boundary conditions on the extended set.
3. The density value outside the source region is assumed to be zero.
4. Experimentally measured the acceleration of gravity, which corresponds to the vertical derivative of the gravitational potential.
5. The method of choosing the size of the extended set is proposed.
6. The general inverse problem of gravimetry is to find the density distribution in a given area, using the measure of the potential derivative on the outer surface.
7. The general inverse problem of gravimetry has essentially not the only solution, as a result of which the value of the density distribution found using standard optimization methods may differ from its real value.
8. Some particular inverse problems of gravimetry have been solved, in particular, the restoration of constant density and coordinates of the location of the gravitational anomaly.
9. Optimization problems corresponding to inverse gravimetry can be characterized by a non-differentiable functional. In this case, non-smooth optimization methods can be used, in particular, the subgradient method, the Nelder – Mead method, and genetic algorithms.
10. The obtained results can be used in monitoring oil and gas fields.

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