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Numerical aspects of the adaptive computational grid in solving the problems of electrical prospecting with direct current

Abstract. This paper is devoted to the numerical aspects of the adaptive computational grid in solving the problems of electrical prospecting with direct current. The purpose of this work is to determine the acceptable parameters of various algorithms for constructing a computational grid for computing electrical tomography curves associated with the ground surface relief. Optimal algorithms for constructing a computational grid in problems of calculating of apparent resistivity curves, associated with the ground surface relief, can improve the accuracy in computation and cost-effectiveness in using computational resources. A mathematical model for calculating the field and of apparent resistivity curves, based on the theory of potentials, and the discretization of the surface of the calculated boundary are described. The problem is reduced to solving an integral equation.

We described here results of that method applied to the relief of simple 2D forms. Our calculations show that use of grid with triangulation gives the same results as a grid constructed with a refinement at the vicinity of the source electrode. However, due to the refinement at the vicinity of the source electrode and measuring line, the grid with triangulation is more efficient and allows one to calculate the function of apparent resistivity with relatively small number of nodes – approximately above 2000.

The data obtained in numerical experiments are basis for further research and for definition of the influence of relief forms on the distortion of apparent resistivity curves.

Key words: Method of integral equations, EIT, ground surface relief, apparent resistivity curves, computational grid.

Introduction

Progress in computing technologies has led to significant changes in software and hardware for geophysical methods of sounding of non-homogeneous media. Portable multi-channel systems for computerized geophysical equipments have evolved, which have changed the traditional method of field work. One of the leading methods of geoelectric research, used worldwide is the Vertical Electrical Sounding (VES) method in the modification of the electrical impedance tomography (EIT). The works that had the most influence on the development of the electrical tomography method in geophysics are the following: Edwards L.S. (1977); Barker R.D. (1981, 1992); Griffiths D.H. and Turnbull J. (1985); Zohdy A.A.R.

(1989); Dahlin T. (1993, 1996); Loke M.H. and Barker R.D. (1996); Bobachev A.A., Modin I.N. and others (1995, 1996, 2006, 2008) [1-10].

In the problems of VES the study of the influence of experimental conditions on apparent resistivity curves is of major importance, in particular, that is the impact of a relief of the sounding medium. The review of the main researches concerning the influence of the ground surface relief is reported in the article [11]. As it is shown in works [11, 12], an efficient and accurate way to calculate the influence of a shape of a ground surface relief on sounding data is the Integral Equations Method (IEM). The method is based on representation of the potential of the stationary electric field via potentials of simple layers distributed on a surface of the medium and internal contact boundaries.

In practice, construction of a geoelectric section of the medium is carried out on the basis of measurements of apparent resistivity and using of 2D and 3D inversion programs. These programs solve the inverse problem of EIT (for instance, Res2DInv, the author is M.H. Loke, 2000; and ZondRes2D of A.E. Kaminsky, last updated 26.06.10). In most cases the solution is smooth with blurred boundaries that do not always correspond to real geological situation. The sharp geoelectric borders become diffused, and the distortions of curves of apparent resistivity related with a surface relief generate pseudo anomalies. From the best of our knowledge there are no programs which accurately calculate the influence of distortions, related with a ground surface relief.

Mathematical model and discretization of the surface

As it is shown in [11], the problem of the numerical computation of a direct current field in the homogeneous medium with a ground surface relief can be reduced to the solution of the Fredholm integral equation of the second kind with a weak singularity:

$$q(P) = \iint_{\Gamma} q(M) \frac{\cos \psi_{PM}}{|PM|^2} d\Gamma(M) + F_0(P) \quad (1)$$

Here M, P are points of the boundary Γ of the medium on which the integral is taken, $q(P)$ is the density of a simple layer on boundary Γ which allows to calculate the potential of the field, ψ_{PM} is a corner between a normal vector at the point P and the vector PM , $F_0(P)$ is the given function. Actually, $F_0(P)$ is expressed via the potential of the source electrode.

In [11] the method of integral equations is realized numerically on the grid refined near the source electrode, where large gradients of the field exist. The relief is mapped on the plane surface and the calculated grid is constructed in polar coordinates. The source electrode is located at the origin point. The measuring line passes along the radius. Then the grid is adapted to the relief surface and to the position of the source electrode by the inverse mapping it to the relief surface (Figures 1, 2). Due to the integration error for coarse grid the calculated values of apparent resistivity show nonphysical oscillations when the distance from the origin of the coordinates (Figure 3, an asterisk indicates the position of the current source electrode) increases.

To avoid these oscillations in the numerical solution, we have to make a significant refinement of the mesh and a local refinement of grid cells near the measurement line. It complicates the algorithm and breaks uniformity of the calculations.

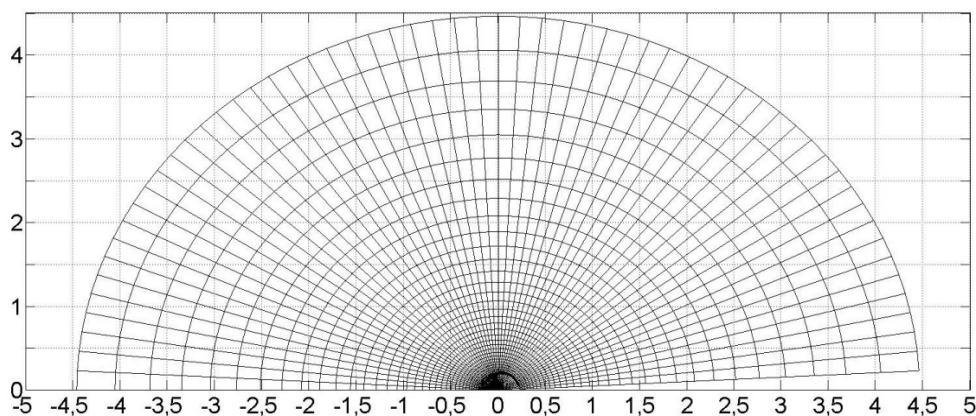


Figure 1 – Computational grid in the polar coordinate system mapped to the plane

Other source of errors is the replacement of the infinite domain of integration by a finite domain; it means that we neglect induced charges outside of the calculated area. For decrease of this error it is necessary to reach compromise between expansion of calculated domain and the number of grid nodes. The solution is the use of a coarse grid far from current sources and the measurement line, since in these

areas the potential of the field decreases inversely proportional to radius.

To avoid mentioned above nonphysical oscillation and exclude excessive refinement of the grid, an alternate computational grid is constructed. This grid is based on the triangulation and is adapted not only to the position of the source electrode, but also is condensed near measuring line. In that

algorithm the calculated area is set by some oval, its size is sufficient in order that it has been possible to neglect the field far from the source (Figures 4, 5).

Acceptable size of the grid is determined via series of numerical experiments for each considered relief form.

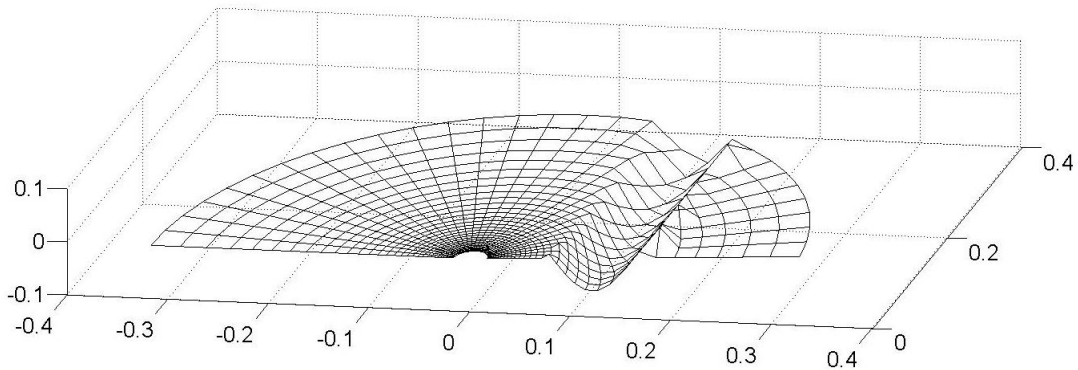


Figure 2 – Computational grid in the Cartesian coordinate system with the surface topography

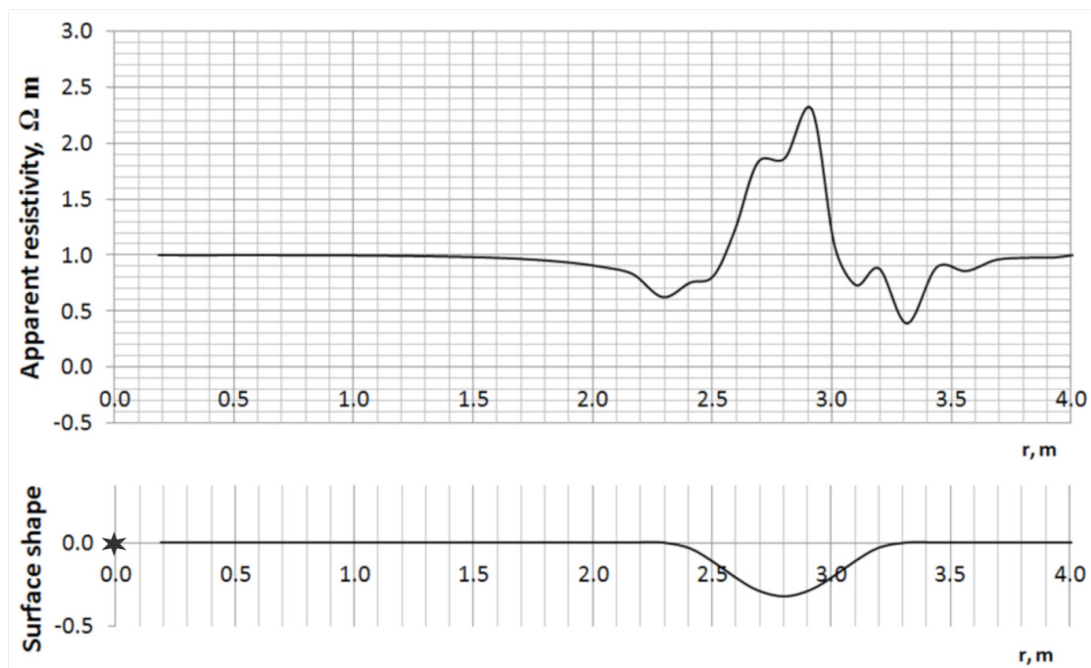


Figure 3 – The calculation of the apparent resistivity far from the current source electrode and the corresponding relief for the grid with number of layers $N = 20$

The grid construction problem is reduced to the following steps: map a relief surface of the medium on 2D domain of an oval shape on the plane. This oval is composed of two semicircles and one rectangle. Then we divide that oval into triangles with a condensation to the line connecting the centers of the semicircles. This line lies on a larger axis of an oval and corresponds to the measuring line. Due to the symmetry for 2D examples described below, only

nodes of one half of the oval are used for calculations.

Note that in the article [13] several tests of the described method are successfully performed for two-layered model of the medium

Brief description of the construction algorithm of a grid nodes. The user sets quantity of layers N , i.e. the oval is divided into N layers by the rule of concentric semi-ovals of radius $r_i = \exp(i * h_s -$

$l)/\alpha$, where i is the number of a concentric semi-oval, $hs = \ln(1 + a * \alpha) / N$ is a grid step on radius in the logarithmic coordinates, α is the coefficient of irregularity of the grid. Then thickness of the i -th layer will be $r_i - r_{i-1}$. Then the massif of nodes placed on for each layer is defined. By the triangulation method the set of triangles satisfying Delaunay condition is formed for the given set of nodes. This condition allows to generate a set of

triangles which are whenever possibly close to the set of equilateral triangles. Though in mathematical packages the functions realizing Delaunay's triangulation are described, we elaborated the algorithm which is much simpler than the common algorithm, because we takes into account features of our grid, namely, its layered structure and logarithmic expansion with distance from the axis of the calculated area.

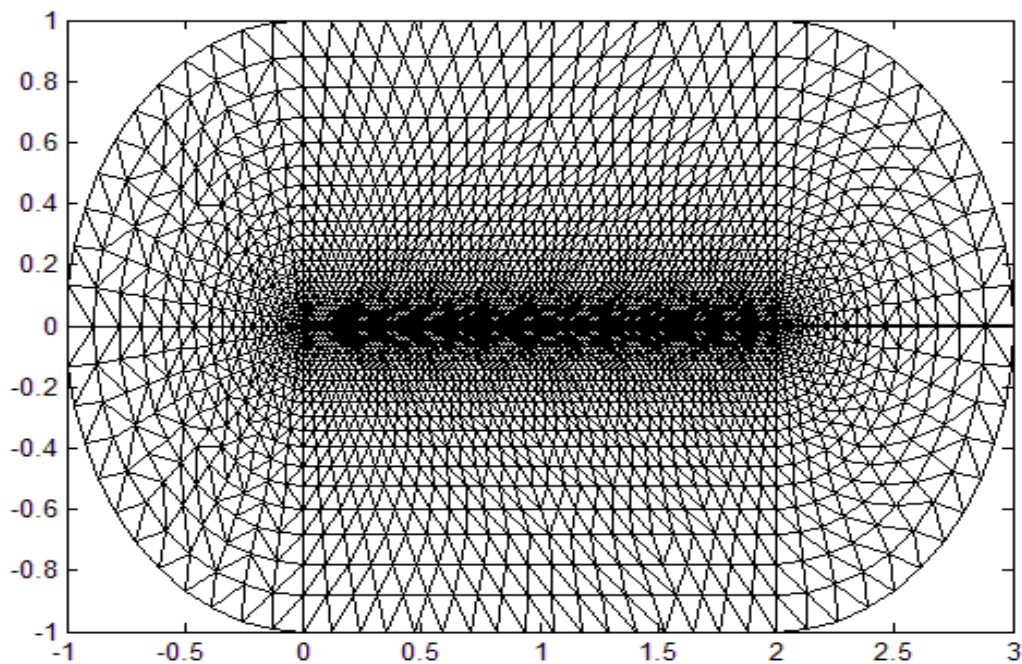


Figure 4 – The triangulation of the calculated area refined near the measuring line

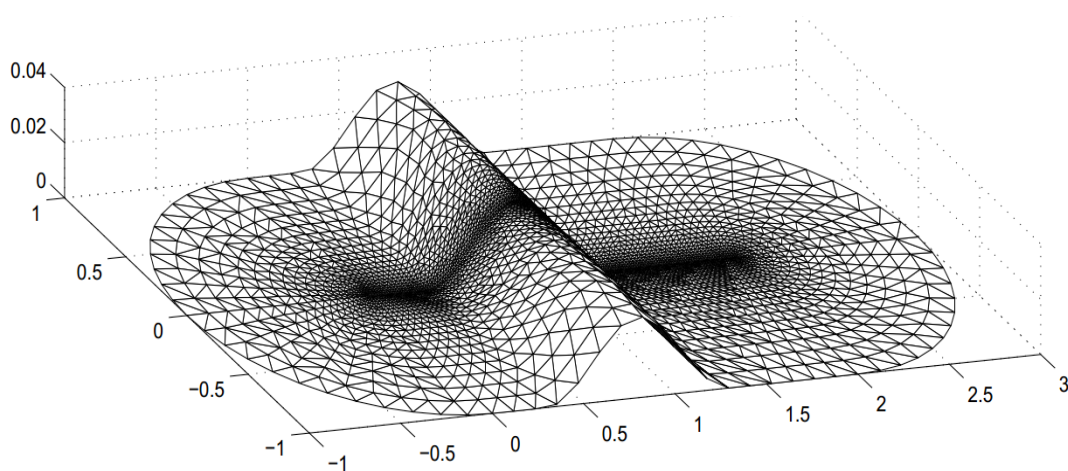


Figure 5 – Triangulation of the relief surface

Calculated parameters of the algorithm are as follows: L is the distance between center of side semicircles of an oval; a is the maximal radius of semicircles of an oval; N is a quantity of layers of the grid; α is a coefficient of irregularity of the grid. The higher the coefficient of irregularity is, the more is a difference between the size of internal and external triangles. The program generates a set of nodes and triangles which further are used to solve an integral equation.

Numerical results

Series of numerical experiments have been made to define acceptable parameters of the grid (Figures 6, 7, 8), and comparison of the results for two algorithms of grid's construction are provided (Figures 9, 10).

Impact on the triangulation of the irregularity parameter α , number of layers N , the radius of semicircles of an oval a and length L have been analyzed. Number of nodes and triangles depend on these parameters and are determined after triangulation. Calculations are made with parameters α in the range 1.0÷16.0, N changes in the range 10÷100, parameters of a and L are assigned as a in 0.5÷2.0, $L = 2 \cdot a$.

Numerical experiments are executed for models with the negative and positive relief shapes, with sharp and smooth slope angles, also for a wavy shape of a relief. Source electrode is located in the origin of the coordinates. In Figure 9 curves of apparent resistivity are constructed for a ground surface relief in the shape of hemispherical convexity with smooth slope angles for different calculation parameters. An asterisk indicates the position of the current source electrode.

Numerical experiments show that the most acceptable parameters of calculations provide a sufficient condensation of the grid at the proximity of the measuring line, and the sufficient length of this line: at $L=2 \cdot a$. The most admissible values are the following: α – not less than 8.0; N – not less than 20. At the same time the 20-layer grid is formed of triangles, with number of nodes is equal to 1834 and number of triangles is equal to 3416. Then the computational domain has been made wider by increasing parameter a in the interval [1, 2.]. It turns out that changes of curves of apparent resistivity are within 0.6%. However, for every value of relief slope angles it is recommended to determine admissible parameters anew by making refinements of the grid and comparing the results.

Calculated curves of apparent resistivity for models of a ground surface relief for the negative and positive shapes in the form of hemispherical (semicircular) concavity and convexity for different slope angles $\alpha = 30, 45, 60^\circ$ are given in Figures 10, 11, an asterisk indicates the position of the current source electrode. It follows from numerical experiments that values of maxima (minima) of the apparent resistivity curve considerably increases (decreases) with increase of a slope angle.

Numerical results obtained for the same relief form for two type of grids has been compared with number of nodes close each other. For the grid with triangulation on relief with sinusoidal shape and slope angle 60° we use values of parameters $L=5$, $a = 2.5$, $\alpha = 8.0$, $N=20$. In those parameters 20-layer grid has 2170 nodes.

The main parameters for the grid refinement only in the vicinity of the source electrode are the number of divisions along the radius and the angle [11]. Calculations on this grid are made on 20-layered grid with 2000 nodes.

Calculated apparent resistivity curves for two grid types are compared in Figures (12, 13). An asterisk indicates the position of the current source electrode. It is seen that for the grid refined only near source electrode non-physical oscillations appear, which are related with coarse grid away from the source electrode. Satisfactory results for this grid were found only with number of nodes equal to 8640. So, this kind of grid leads to the consumption of large machine resources. At the same time, results which are taken by mentioned above triangulation algorithm gives physical reliable curves of sounding with number of nodes above 2000.

Then we have made numerical simulations to check influence of other parameters. When the parameter a changes in the interval from 1 to 2, the relative change of apparent resistivity is not more than 0.6%. At the same time, maximal relative difference between apparent resistivity calculated for $N = 30$ and $N = 90$ is 2.5%. Changes of N between 90 and 100 is followed by changes of apparent resistivity not more than 0.5%. Changes of α between 8 and 16 yield to relative difference of apparent resistivity not more than 0.2%. These mean, that for this relief form and length of measurement line with slope angle $\leq 20^\circ$ admissible calculation parameters are: $a = 1$, $\alpha=8$, $N = 30$.

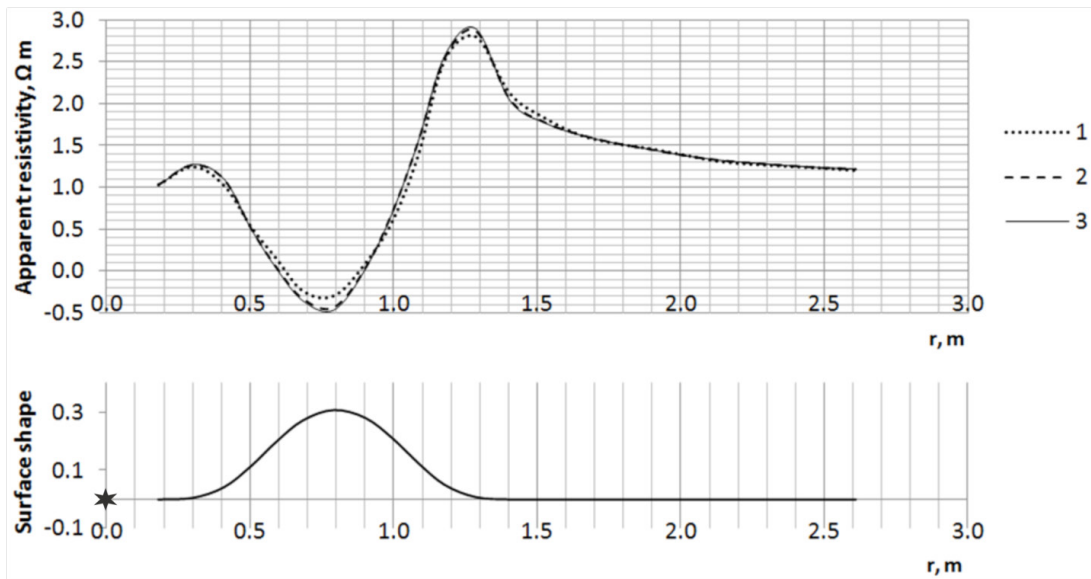
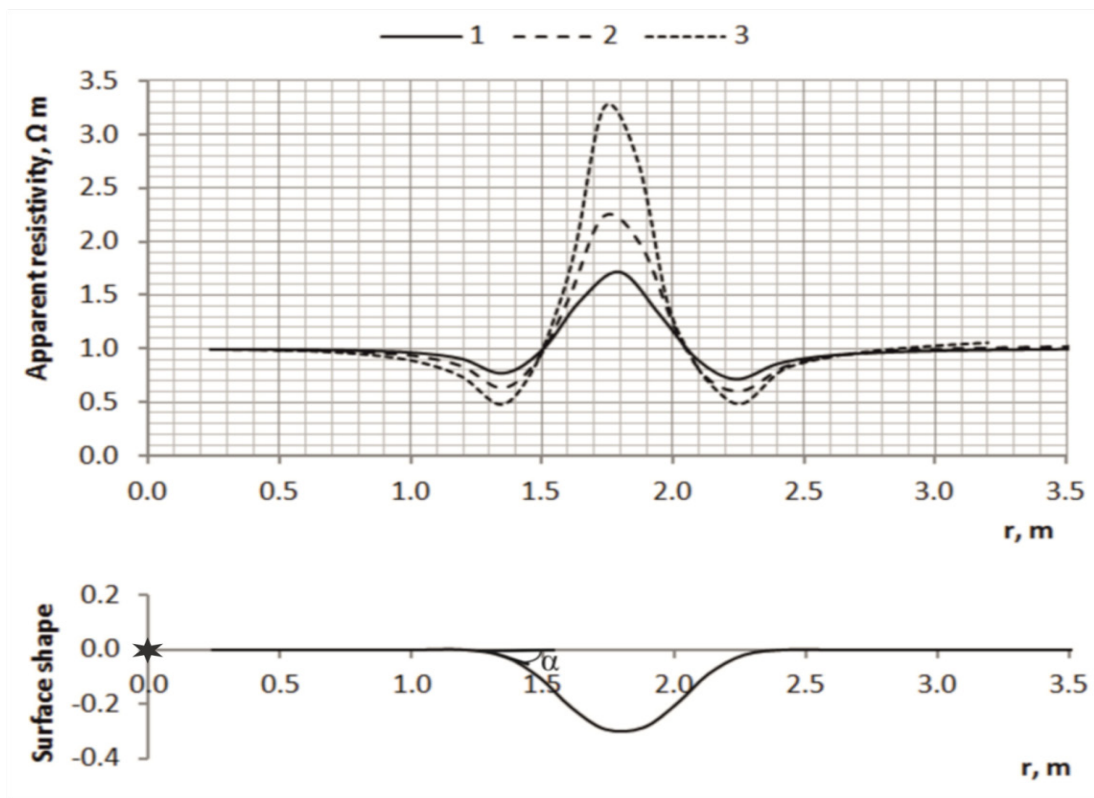


Figure 6 – The shape of the ground surface relief and curves of apparent resistivity:
 1 – the solution obtained for parameters $N=10, \alpha=1.0, f=372, k=660$;
 2 – the solution obtained for parameters $N=20, \alpha=8.0, f=1834, k=3416$;
 3 – the solution obtained for parameters $N=40, \alpha=16.0, f=8716, k=16718$



1 – slope angle $\alpha = 30^\circ$; 2 – slope angle $\alpha = 45^\circ$; 3 – slope angle $\alpha = 60^\circ$

Figure 7 – Curves of apparent resistivity for the model of the surface with negative shape

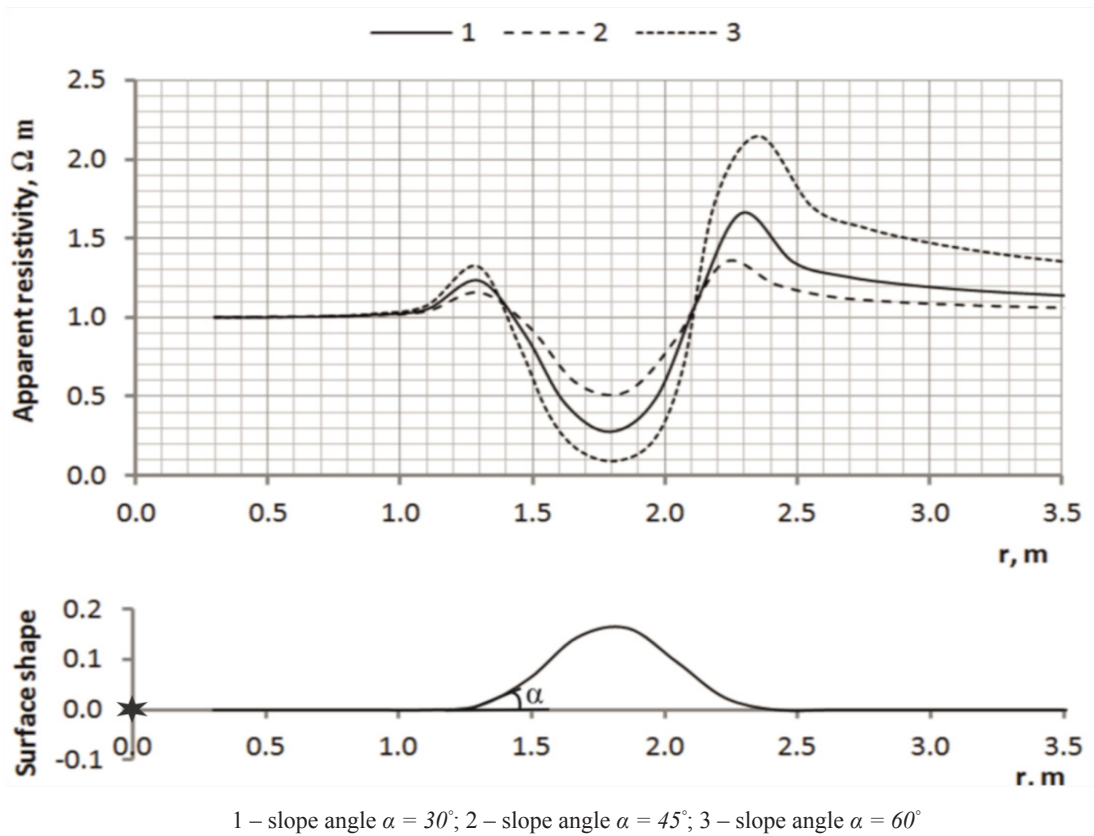


Figure 8 – Curves of apparent resistivity for the model of the surface with positive shape

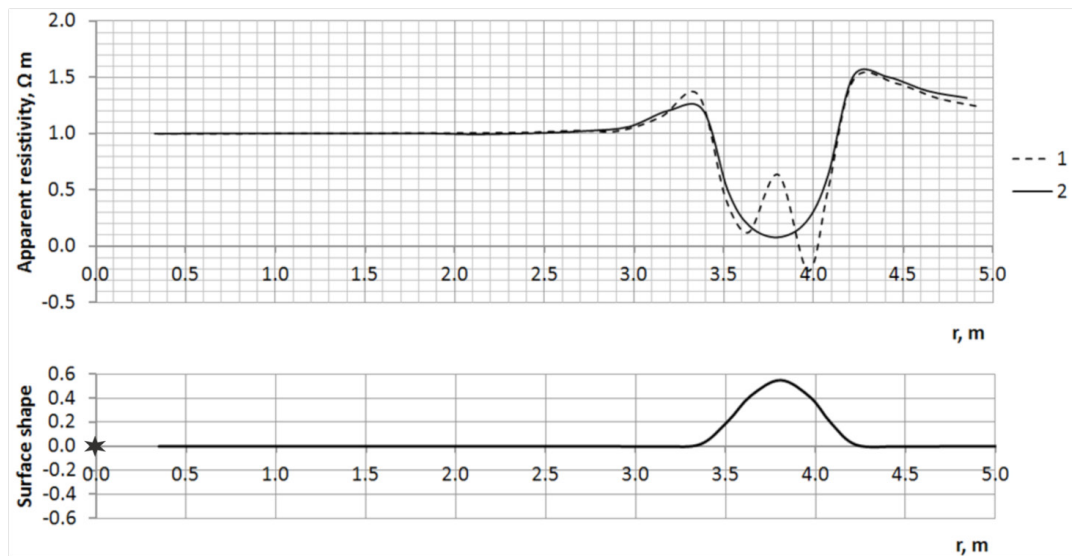


Figure 9 – The shape of the simulated ground surface and curves of apparent resistivity which are taken with different algorithms of grid's construction:
1 – by grid adapted to source electrode, 2 – by grid with triangulation adapted to measuring line

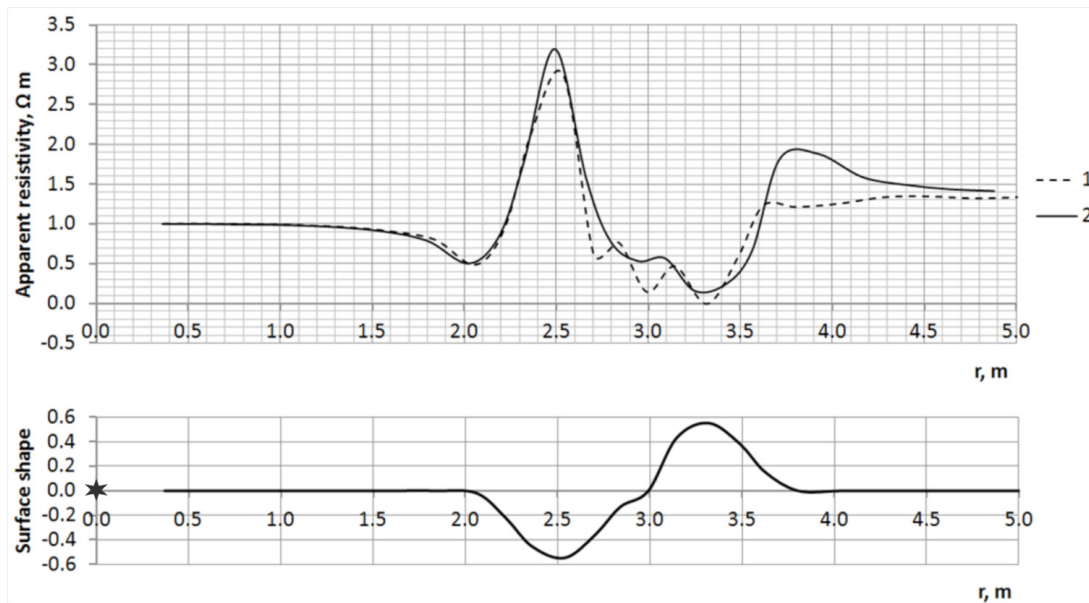


Figure 10 – The shape of the simulated ground surface and curves of apparent resistivity are taken with different algorithms of grid's construction:

1 – by grid adapted to source electrode, 2 – by grid with triangulation adapted to measuring line

Calculations are made for surfaces with given analytical form. Such method of assigning a function of the relief is very comfortable for simulations. However, in practice, the relief parameters are determined by field experiments. In electrical tomography method the relief is determined by the profiling step (distance between electrodes) and heights of measuring electrodes which are placed along sounding area. This definition of the relief allows creating a table of values of the height function. The heights correspond to the values of the function $-z_j$ ($j=1..k$), and the corresponding values of the argument $-x_j$, can be calculated through the values of the step along the profile. For approximation of that tabulated function the interpolation methods are applied.

For construction of computational grid on the arbitrary relief we considered two methods of interpolation of the relief surface: based on spline functions and on radial basis functions (RBF) [14]. Calculations are performed for different parameters of grid (number of nodes has been equal to 4147, 5222, 7155, 8044). The big advantage of RBF interpolation method is its computational efficiency compared with the spline interpolation method. For example, simulation on the grid with number of nodes $f=7155$ on a computer with processor Intel Core i7-4700, frequency 2.40 GHz, 16 GB RAM, takes 900-1000 seconds for spline interpolated functions, while calculations with RBF method take 120-140

seconds. Note that the calculation time depends on relief form also.

Conclusion

Interpretation of EIT data without taking into account influence of the relief form can give pseudo anomalies. We described here numerical method to compute the field and curves of apparent resistivity for a homogeneous medium with relief boundary based on the potential theory. Problem is reduced to the solution of an integral equation. The main feature of the method is its high accuracy and efficiency in calculations of the field for three-dimensional geometry of the relief and for medium with several inner contact boundaries [15]. We described here results of that method applied to the relief of simple 2D forms. Our calculations show that use of grid with triangulation gives the same results as a grid constructed with a refinement at the vicinity of the source electrode. However, due to the refinement at the vicinity of the source electrode and measuring line, the grid with triangulation is more efficient and allows one to calculate the function of apparent resistivity with relatively small number of nodes – approximately above 2000.

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