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Modeling of the potential
of the gravitational field at the upper boundary
of the region with the existence of a subterranean anomaly

Abstract. All of humanity's activities are tied to fuel. Previously, it was coal, now it is oil. In order to find oil or coal in the earth's interior, preliminary analysis and calculations are necessary to confirm their existence, their size, location, type of mineral and much more. The aim of the study is to improve existing models of calculations for the search for minerals. To date, there are many methods and approaches to solving the problem, but they are not yet accurate enough. We solve the inverse problem of gravimetry, based on measurements of the Earth's gravitational field. At this stage, we carefully investigate the direct problem, without affecting the inverse. In the course of the study, we found interesting facts. Namely, the gravitational field on the Earth's surface with a different location of the anomaly has a very unpredictable result. The results of this study showed that lateral boundaries should be 30-50 times farther from the anomaly, with respect to the diameter of the anomaly. In addition, it reveals that the anomaly should be located in the middle of the area. This gives us the most plausible result.

Keywords: gravimetry, inverse problem, gravitational field, Poisson equation.

Introduction

One of the most important ways of intelligence and analysis of mineral deposits associated with the detection and identification of gravitational anomaly [1]. Thus based on gravimetric methods identifies various deviations of the gravitational potential, to indicate the presence of any heterogeneities in the considered crustal thickness. Interpretation of the obtained results implies the solution of any inverse problems of gravimetry (see, for example [2, 3, 4, 5, 6, 7, 8]. As is known, inverse problems of gravimetry are essentially incorrect. They not only lack the stability of the resulting solution to the input data, but often there is no uniqueness of the solution of the problem. One of the known optimization methods for their solution is assembling suggested by Strakhov [7, 9], recently from the statistical form this method has been modified to a mixed one with a deterministic approach [3]. In this paper, a gradient-type method is used to solve the problem under consideration, which is regularizing, i.e. conditionally stable. Many studies of methods of the Newton type for the solution of the restoration of the medium boundary in inverse problems of gravimetry were carried out in [5, 6, 7, 10]. The purpose of this paper is to find out exactly what information and with what degree of accuracy can be restored by measuring the potential of the gravitational and its gradient on the surface of the earth.

In this paper we consider the Poisson equation for the potential of the gravitational field in a certain region. The inverse problem is to determine the density of the medium on the basis of measuring the gravitational potential and its derivative on the surface of the earth. In this case, the remaining part of the boundary of the region under consideration is given the potential value, which would be observed in the same region in the absence of a gravitational anomaly.

Statement of the problem

We will not complicate our task. We consider the structure of the earth in a certain section, i.e. in two-dimensional measurements. We will also consider at the initial stage that our investigated area has a homogeneous structure and the density is the same everywhere, and we know the significance of this density. We assume that there is only one anomaly in the given region. Let this anomaly also have the form of a rectangle. Let us know the location and its form of anomaly. We do not know its density.

It is necessary to determine the density of the anomaly. To find out what kind of material it is.

There are gravimetric values (values of the gravitational field) at the boundaries of the region.

Let Ω (a × b) be a rectangular domain of size a × b. The anomaly has the shape of a rectangle and it is located in the some interval $x_m \leq x \leq x_n$ (the length at the surface of the earth), $z_k \leq z \leq z_l$ (the depth of the considered earth cut, with the positive direction of the OZ axis pointing downwards, so that it is convenient to count with positive values of z). The basic gravitational field equation described by the following formula:

$$\Delta \varphi = -4\rho\pi G,$$

where φ is the field potential; G, π are constants; ρ is the density of matter; Δ is the Laplace operator.

We denote by $\Delta \varphi_0 = -4\rho_0\pi G$ the potential of a gravitational field without anomaly. Here ρ_0 is the density of the considered region without anomaly.

The size of the considered region must be much larger than the size of the anomaly, in order that the potential difference on the boundary of the region be zero. The potential of the anomaly surface has the form of a "cap", and completely covers the anomaly. Thus, the potential of the anomaly field extends beyond the boundary of the anomaly. If the potential difference is not zero at the boundary of the region under consideration, then we artificially expand the area under investigation, so that the dimensions are sufficiently large in relation to the anomaly under investigation. We can expand in depth and width. The upper part of the earth's boundary remains unchanged. We denote the extensible boundary by Γ.

Thus, we calculate the difference

$$-\begin{cases} \Delta\varphi(x,z) = -4\pi\rho(x,z)G, \\ \Delta\varphi_0(x,z) = -4\pi\rho_0(x,z)G. \end{cases}$$

We obtain

$$\Delta\eta(x,z) = -4\pi\psi(x,z)G,$$

there

$$\begin{cases} \eta(x,z) = \varphi(x,z) - \varphi_0(x,z), \\ \psi(x,z) = \rho(x,z) - \rho_0(x,z). \end{cases}$$

The value of the boundary conditions along the boundary Γ is zero

$$\eta(x,z)|\Gamma = 0.$$

Hence, we have obtained that

$$\psi(x,z) = \begin{cases} 0, & outside \ \Omega, \\ \psi_0, & in \quad \Omega. \end{cases}$$

Thus, we reduced the original problem with nonzero boundary conditions to an equivalent problem with zero boundary conditions (this method of reducing the problem is called the perturbation method). This greatly facilitates the process of solving the problem.

Now the statement of the problem has the form:

$$\Delta \eta(x,z) = -4\pi\psi(x,z)G, \tag{1}$$

$$\eta(x,z)|\Gamma = 0, \tag{2}$$

$$\eta(x,z_0)|z_0 = \eta_1 \neq 0, \tag{3}$$

$$\frac{\partial\eta(x,z)}{\partial z}|z_0 = \eta_2 \neq 0, \tag{4}$$

$$\psi(x,z) = \begin{cases} 0, outside \ \Omega, \\ \psi_0, \ in \quad \Omega. \end{cases} \tag{5}$$

The problem (1) – (3) is direct problem for the Poisson equation with boundary conditions. The function η (x, z) is the potential of the gravitational field is differentiated twice in x, and twice in z. Therefore, to solve this Poisson equation, we need four boundary conditions. The equations (2) and (3) provide us with four boundary conditions (along four boundaries). That is, we can solve a direct problem. Find the potential of the field at any point in the region under consideration, with given

conditions on the boundary, we know the value of ψ (x, z) is the difference in the densities of the inhomogeneity and the region under consideration. Unfortunately, we do not know the value of the density of the inhomogeneity! Therefore, it is necessary to insert into the equation (1) the value of ψ (x, z) from equation (5). If ψ (x, z) = 0 outside the Ω, then η (x, z) = 0 outside Ω.

We still have equation (4). The value of the gradient from the potential function of the gravitational field. To solve a direct problem this condition is superfluous, that is, we solve a direct problem without it. To solve the inverse problem, we need condition (4).

We reduce the inverse problem to the optimization problem. We add the functional. Since we do not know the value of ψ_0, we will select it artificially. It must be chosen in such a way that for the solutions of the direct problem the obtained answer η (x, z), when substituted into equation (4), turns it into an identity. Then the chosen ψ_0 is the solution of the inverse problem. Such a method is laborious, perhaps even not feasible, because we may never be able to pick up ψ_0 so as to accurately "get" into η_2. In such cases, the optimization one replaces the original task.

The optimization problem obtained as follows: Instead of checking condition (4), we will minimize the functional

$$I(\psi_0) = \int_0^L \left(\frac{\partial \eta(x,z)}{\partial z}\bigg|_{z=0} - \eta_2 \right)^2 dx \to \min. \quad (6)$$

That is, we will select ψ_0 in such a way that the difference between the desired value of η (x, z) and η_2 is minimal.

We will solve the problem with the aid of the gradient method. A gradient method involves the use of a derived functional. It is necessary to calculate the derivative of the functional. We use the definition of the Gato derivative for the functional. The Gato derivative has a sufficient set of properties in order to use it in the gradient method when

calculating the inverse gravimetric problem for the model statement.

The Gato derivative has the following form:

$$I'(\psi_0) = 4\pi G \iint\limits_{\Omega} p\, dx\, dz.$$

Results and discussion

The formulation of the problem reduced to the solution of the direct and inverse problem. At the first stage, we must study the direct problem well, check for the presence of deviations and inaccuracies in the various locations of the anomaly within the investigated region. We analyze the change in density to the upper value of the gravitational field. All calculations made on the software product Comsol Multiphysics 5.2. This software product contains all the necessary numerical calculations. The obtained solutions based on the finite element method. This method is the most accurate and universal. The advantage of Comsol Multiphysics 5.2 is in the speed of solving and providing a visual solution of the problem. Unfortunately, it is difficult to solve the inverse problem on Comsol Multiphysics 5.2, but it is quite suitable for solving a direct problem.

The figures below show the last upper layer of the location of the anomaly. In Figure 1 a) you can see a graphic representation of the location of the anomaly, b) a graph of the distribution of the potential of the gravitational field on the earth's surface, c) the distribution of the derivative of the gravitational potential for a given location of the anomaly. Since the graphs are symmetrical about the central location of the anomaly, we will consider only three variants of the location of the anomaly (the anomaly shift occurs from left to right to the central location). Since we need to give three figures for one arrangement, we decided to design them as Figure1 (under one number).
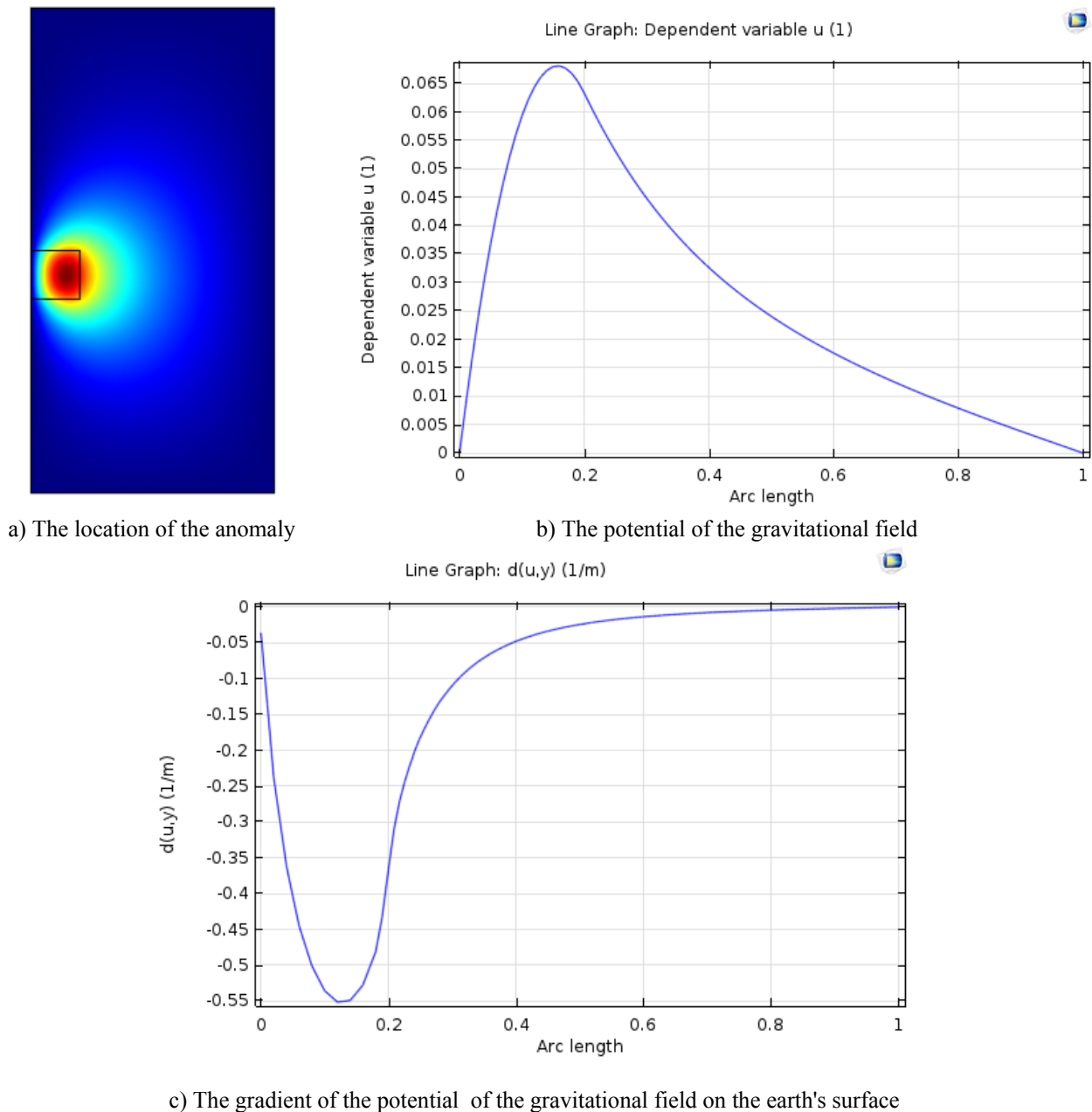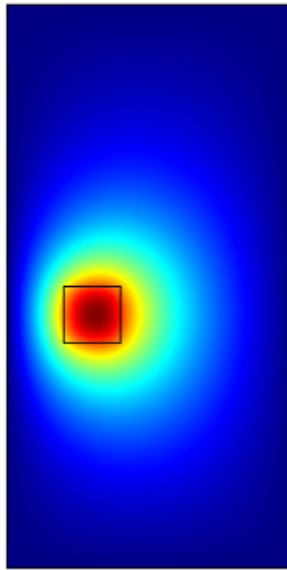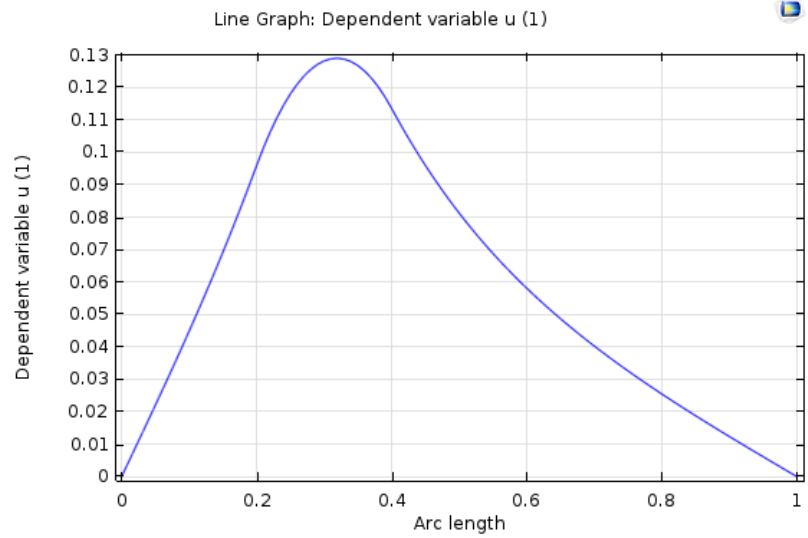
a) The location of the anomaly

b) The potential of the gravitational field

c) The gradient of the potential of the gravitational field on the earth's surface

Figure 1 – Last upper layer of the location of the anomaly

The results of solving a direct problem showed the following. With a different location of the anomaly in the study area, the upper boundary of the gravitational field did not change as expected. Namely, if the anomaly was located closer to the lateral boundaries, then the symmetry of the resulting parabola was greatly changed, even arched. In addition, the peak of the parabola did not lie exactly above the center of the anomaly, but shifted. Thus, introducing distortion and complexity for the reverse restoration of the location of the anomaly. It is connected with the zero boundary conditions, which we asked beforehand, expanding the region. Thus, the compiled mathematical model does not fully describe the process. That is, initially we did not expect an anomaly to be located near the border. The anomaly should be located strictly in the center of the region. The greatest curvature obtained closer to the surface of the earth. The deeper the anomaly, the less the curvature.
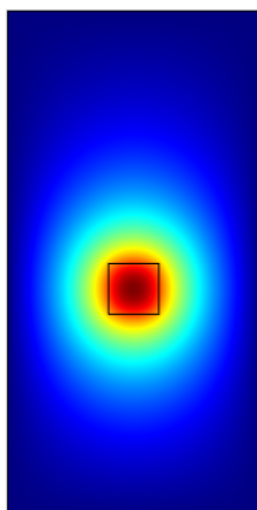
a) The location of the anomaly

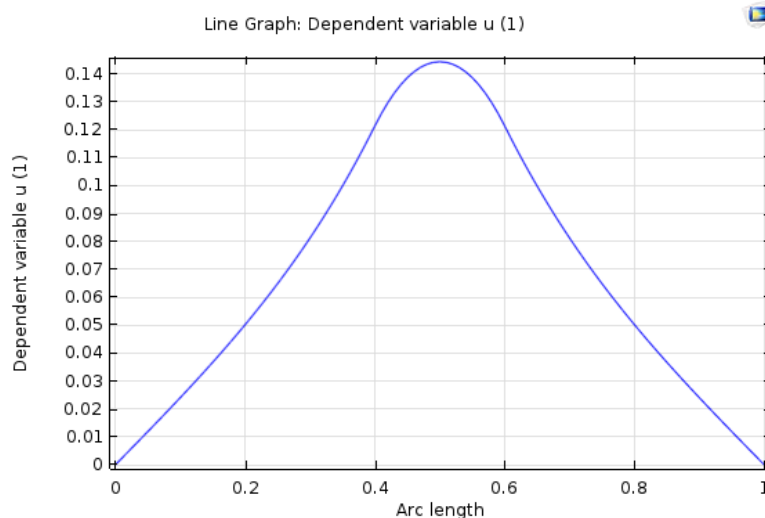b) The potential of the gravitational field
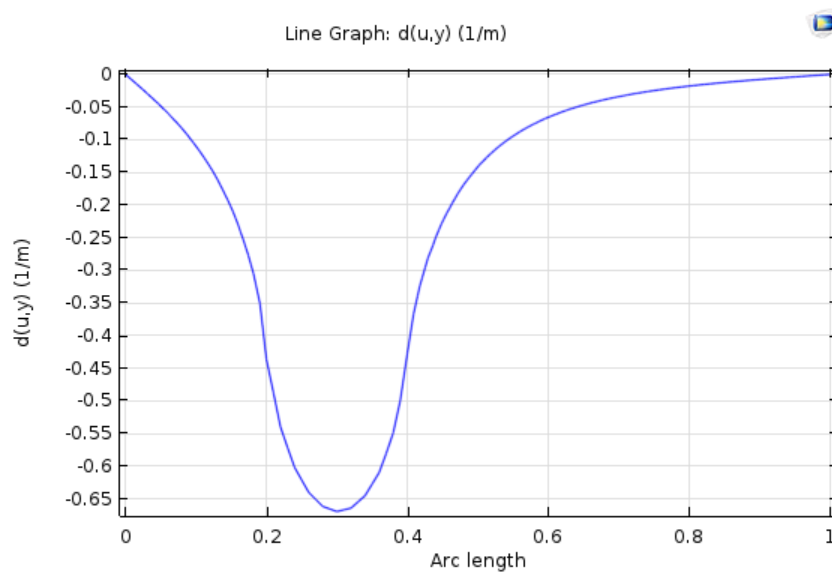
c) The gradient of the potential of the gravitational field on the earth's surface.

Figure 2

a) The location of the anomaly

b) The potential of the gravitational field

c) The gradient of the potential of the gravitational field on the earth's surface.
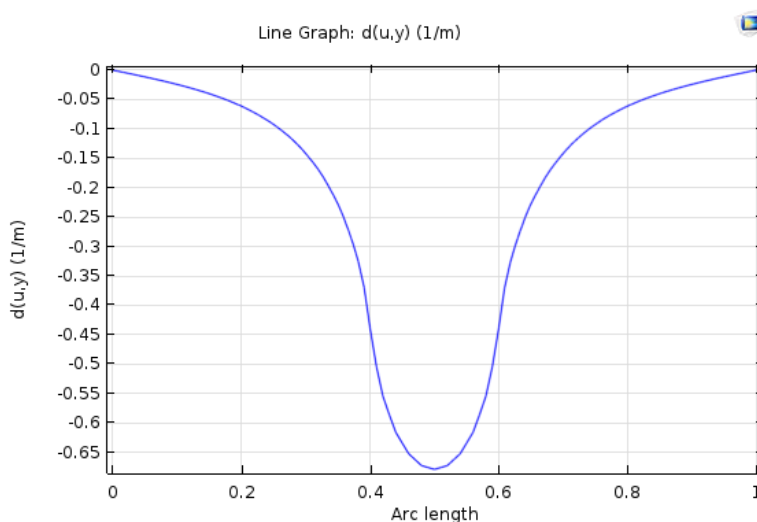
Figure 3

Conclusion

The purpose of this article was to show that when studying a region with a subterranean anomaly, one should not place an anomaly close to the lateral boundaries. This gives incorrect results associated with zeroing the boundary conditions. These results will be very useful in the further solution of the inverse problem, since in solving inverse problems we will need to solve the direct problem many times. If there are inaccuracies in the direct problem, then this will necessarily affect the results of the inverse problem. We need to most accurately investigate the direct problem in order to exclude in advance the errors of the direct problem.

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