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The three quark-current of delta-isobar

Abstract. Strong decay $\Delta \to N\pi$ is one of the best modes to search for particles with spin 3/2. Δ-isobar decay consist of 99% - hadronic decay and 1% - electromagnetic decay, i.e. in this process strong decay is dominating channel which makes it important for nuclear research. The relativistic three-quark current describe all parameters of particle in quantum field theory. The three-quark current of Delta-isobar is similar with all large group of particles with quantum numbers $J^p=(3/2)^+$. Relevant interpolating three-quark current is given in great details. It is shown that, this current has a single form. If we have three-quark current, we can write the interaction Lagrangian of baryon interacting with their constituent quarks. The Lagrangian is the base of the Standard model. The generalization of the current to nonlocal case can be used to such nonlocal theory as covariant quark model. Covariant quark model has been applied to a large number of elementary particle processes.

Key words: covariant quark model, confinement, Δ-isobar, relativistic three-quark current.

Introduction

On LHCb at 14 march 2017 was published preprint about discovery of $\Omega_c(3000)^0$, $\Omega_c(3050)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$ and $\Omega_c(3119)$ that gave an impetus for study of Ω_c-baryon [1]. Therefore, we need to construct the relativistic three-quark current of particles with quantum numbers $J^p=(3/2)^+$. Since we have not any physical quantities, we started from reference decay $\Delta \to N\pi$ and wrote the corresponding three-quark current [2]. Strong decay $\Delta \to N\pi$ is one of the best modes to search for particles with spin 3/2. Δ-isobar decay consist of 99% - hadronic decay and 1% - electromagnetic decay, i.e. in this process strong decay is dominating channel which makes it important for nuclear research.

The relativistic three-quark current describe all parameters of particle in quantum field theory. The three-quark current of Delta-isobar is similar with all large group of particles with quantum numbers $J^p=(3/2)^+$.

At first we needed to construct relevant interpolating three-quark current. For this purpose we use Rarita-Schwinger equation, properties of Delta-isobar and Fierz transformation. It is shown that the current has a single form. If we know the Δ-isobar current, we can write an interaction Lagrangian of baryon with their constituent quarks. The Lagrangian is the base of the Standard model. The generalization of the current to nonlocal case can be used to such nonlocal theory as covariant quark model. Covariant quark model has been applied to a large number of elementary particle processes.

The three quark-current of delta-isobar

The relevant interpolating three-quark current with quantum numbers $J^P = (3/2)^+$ is given

$$\left(J^{\mu}\right)^{k_1k_2k_3} = \varepsilon^{a_1a_2a_3}\Gamma_1 q_{a_1}^{k_1}\left[\left(q_{a_2}^{k_2}\right)^T C\Gamma_2 q_{a_3}^{k_3}\right], \qquad (1)$$

where $a_1, a_2, a_3 = 1,2,3$ are the color indeces; C is the charge conjugation matrix; $1\, k_1, k_2, k_3 = 1,2$ are isospin indeces. We did not write the Lorenz index "μ" down clearly, because it can be either in Γ_1 or in Γ_2.

The charge conjugation matrix have following properties

$$C = C^{-1} = C^{+} = -C^{T}$$

We will use properties given by

$$C\Gamma^T C^{-1} = \begin{cases} \Gamma, & for\, \Gamma = S, P, A \\ -\Gamma, & for\, \Gamma = V, T \end{cases} . \qquad (2)$$

where $S \to I, P \to \gamma^5, A \to \gamma^\mu \gamma_5, T \to \sigma^{\mu\nu}$.

Table 1 – The two quark color state $\left(q_{a_2}^{k_2}\right)^T C\Gamma_2 q_{a_3}^{k_3} \varepsilon^{a_1 a_2 a_3}$ is diquark

The scalar diquark	$\left(q_{a_2}^{k_2}\right)^T C\gamma_5 q_{a_3}^{k_3} \varepsilon^{a_1 a_2 a_3}$	$J^P=0^+$
The pseudoscalar diquark	$\left(q_{a_2}^{k_2}\right)^T C q_{a_3}^{k_3} \varepsilon^{a_1 a_2 a_3}$	$J^P=0^-$
The vector diquark	$\left(q_{a_2}^{k_2}\right)^T C\gamma_5 \gamma^5 q_{a_3}^{k_3} \varepsilon^{a_1 a_2 a_3}$	$J^P=1^-$
The axial-vector diquark	$\left(q_{a_2}^{k_2}\right)^T C\gamma^\mu q_{a_3}^{k_3} \varepsilon^{a_1 a_2 a_3}$	$J^P=1^+$
The tensor diquark	$\left(q_{a_2}^{k_2}\right)^T C\gamma_5 \sigma^{\mu\nu} q_{a_3}^{k_3} \varepsilon^{a_1 a_2 a_3}$	$J^P=1_+$
The pseudotensor diquark	$\left(q_{a_2}^{k_2}\right)^T C\sigma^{\mu\nu} q_{a_3}^{k_3} \varepsilon^{a_1 a_2 a_3}$	$J^P=1^-$

where $\sigma^{\mu\nu} = i/2\left[\gamma^\mu, \gamma^\nu\right]$.

The multiplet of all isospin states of Delta-isobar is $\Delta^{k_1 k_2 k_3}(x)$. The spinor $\Delta^{k_1 k_2 k_3}$ is symmetric under permutation of k_1, k_2, k_3 and obey the Rarita-Schwinger equation $\Delta_\mu \gamma^\mu = 0$ and $\partial_\mu \gamma^\mu = 0$. Connection of multiplet elements to physical conditions are

$$\Delta^{111} = \Delta^{++}, \quad \Delta^{211} = \frac{1}{\sqrt{3}}\Delta^{+}, \quad \Delta^{122} = \frac{1}{\sqrt{3}}\Delta^{0}, \quad \Delta^{222} = \Delta^{-} \tag{3}$$

Since there is a symmetry in an arbitrary permutation of the indices of the isospin indices in the spinor $\Delta^{k_1 k_2 k_3}$, the corresponding three-quark current also has this property. This means that the current in Eq. (1) must be symmetric under a permutation of $k_1 \leftrightarrow k_2 \leftrightarrow k_3$. This condition imposes restrictions on the choice of matrices. First we consider a permutation of the indices in the diquark. Equity must be satisfied

$$\begin{aligned} &\varepsilon^{a_1 a_2 a_3} q_{a_2 \alpha_2}^{k_2} \left(C\Gamma_2\right)_{\alpha_2 \alpha_3} q_{a_3 \alpha_3}^{k_3} = \\ &= \varepsilon^{a_1 a_2 a_3} q_{a_2 \alpha_2}^{k_3} \left(C\Gamma_2\right)_{\alpha_2 \alpha_3} q_{a_3 \alpha_3}^{k_2}. \end{aligned} \tag{4}$$

Taking into account the anticommutativity of the fermions fields, we swap the quarks on the right-hand side of the equation. Thus we have the following equation:

$$\varepsilon^{a_1 a_2 a_3} q_{a_3 \alpha_3}^{k_3} \left(C\Gamma_2\right)_{\alpha_2 \alpha_3} q_{a_2 \alpha_2}^{k_2} = -\varepsilon^{a_1 a_2 a_3} q_{a_3 \alpha_3}^{k_2} \left(C\Gamma_2\right)_{\alpha_2 \alpha_3} q_{a_2 \alpha_2}^{k_3} = \varepsilon^{a_1 a_2 a_3} q_{a_2 \alpha_3}^{k_2} \left(C\Gamma_2\right)_{\alpha_3 \alpha_2}^T q_{a_3 \alpha_2}^{k_3}. \tag{5}$$

We see that right-hand side and left-hand side of Eq. (0.1) is equal for

$$C\Gamma_2 = \left(C\Gamma_2\right)^T = \Gamma_2^T C^T = -\Gamma_2^T C, \quad C\Gamma_2^T C^{-1} = -\Gamma_2. \tag{6}$$

From Eq. (2) Γ_2 may be either vector $\Gamma_2 = \gamma^\mu$ or tensor $\Gamma_2 = \sigma^{\mu\nu}$. It is known that Δ-isobar has positive parity. Consider the case of $\Gamma_2 = \gamma^\mu$ corresponding to axial-vector diquark (Table 1) with positive parity, hence $\Gamma_1 = I$. In the case of $\Gamma_2 = \sigma^{\mu\nu}$ it follows that $\Gamma_1 = \gamma_\nu$.

Let us consider the case of interchanging of k_1 and k_2. In this case, we have following identity

$$\begin{aligned} &\varepsilon^{a_1 a_2 a_3} \left(\Gamma_1\right)_{\alpha\alpha_1} q_{a_1 \alpha_1}^{k_1} \cdot \left[q_{a_2 \alpha_2}^{k_2} \left(C\Gamma_2\right)_{a_2 \alpha_3} q_{a_3 \alpha_3}^{k_3}\right] = \\ &= \varepsilon^{a_1 a_2 a_3} \left(\Gamma_1\right)_{\alpha\alpha_1} q_{a_1 \alpha_1}^{k_1} \cdot \left[q_{a_2 \alpha_2}^{k_2} \left(C\Gamma_2\right)_{a_2 \alpha_3} q_{a_3 \alpha_3}^{k_1}\right]. \end{aligned} \tag{7}$$

One can interchange q^{k_1} and q^{k_3} quarks in right-hand side of identity. Thus we have

$$-\varepsilon^{a_1a_2a_3}\left(\Gamma_1\right)_{\alpha\alpha_1} q^{k_1}_{a_3\alpha_3}\cdot\left[q^{k_2}_{a_2\alpha_2}\left(C\Gamma_2\right)_{\alpha_2\alpha_3} q^{k_3}_{a_1\alpha_1}\right]=$$
$$=\varepsilon^{a_3a_2a_1}\left(\Gamma_1\right)_{\alpha\alpha_1} q^{k_1}_{a_1\alpha_1}\cdot q^{k_1}_{a_3\alpha_3}\cdot\left[q^{k_2}_{a_2\alpha_2} C_{\alpha_2\beta_2}\left(\Gamma_2\right)_{\beta_2\alpha_3} q^{k_3}_{a_1\alpha_1}\right]. \quad (8)$$

Next use Fierz transformation

$$4\left(\Gamma_1\right)_{\alpha\alpha_1}\left(\Gamma_2\right)_{\beta_2\alpha_3}=\sum_D\left(\Gamma_1\Gamma^D\right)_{\alpha\alpha_3}\left(\Gamma_2\Gamma^D\right)_{\beta_2\alpha_1}. \quad (9)$$

where $\Gamma^D=\left\{I,\gamma^\mu,\sigma^{\mu\nu}\left(\mu<\nu\right),\gamma^5,i\gamma^\mu\gamma_5\right\}$ is complete set of the base Dirac matrix.

First we introduce some notation.

$$\begin{cases}\left(O_1\right)_{\alpha\alpha_1}\left(O_2\right)_{\beta_2\alpha_3}=\left(\tilde{O}_1\right)\otimes\left(\tilde{O}_2\right)\\ \left(O_1\right)_{\alpha\alpha_3}\left(O_2\right)_{\beta_2\alpha_1}=\left(O_1\right)\otimes\left(O_2\right)\end{cases} \quad (10)$$

We have two combination of gamma matrix

$$\Gamma_1\times\Gamma_2=I\times\gamma^\mu. \quad (11)$$
$$\Gamma_1\times\Gamma_2=\gamma_\nu\times\sigma^{\mu\nu}. \quad (12)$$

We will first discuss the case of $\Gamma_1\times\Gamma_2=I\times\gamma^\mu$. Thus we obtain

$$4\tilde{I}\otimes\tilde{\gamma}^\mu=I\otimes\gamma^\mu+\gamma_\alpha\otimes\gamma^\mu\gamma^\alpha+$$
$$+\frac{1}{2}\sigma_{\alpha\beta}\otimes\gamma^\mu\sigma^{\alpha\beta}+\gamma_5\otimes\gamma^\mu\gamma_5-\gamma_\alpha\gamma_5\otimes\gamma^\mu\gamma^\alpha\gamma_5 \quad (13)$$

The right-hand side of identity can be transformed to basis by using the following expression

$$\gamma_5\sigma^{\mu\nu}=-\frac{i}{2}\varepsilon^{\mu\nu\alpha\beta}\sigma_{\alpha\beta}, \quad (14)$$

where we use Levi-Civita symbol presented as $\varepsilon_{0123}=-\varepsilon^{0123}=1$.

After long transformation one can obtain

$$4\tilde{I}\otimes\tilde{\gamma}^\mu=I\otimes\gamma^\mu+\gamma^\mu\otimes I-i\gamma_\nu\otimes\sigma^{\mu\nu}+$$
$$+i\sigma^{\mu\nu}\otimes\gamma_\nu+i\sigma^{\mu\nu}\gamma_5\otimes\gamma_\nu\gamma_5+\gamma_5\otimes\gamma^\mu\gamma_5 \quad (15)$$
$$-\gamma^\mu\gamma_5\otimes\gamma_5+i\gamma_\nu\gamma_5\otimes\sigma^{\mu\nu}\gamma_5.$$

Transformations of Eq. (0.2) will also take place in a similar manner, we find that

$$4\tilde{\gamma}_\nu\otimes\tilde{\sigma}^{\mu\nu}=i\left(3I\otimes\gamma^\mu-3\gamma^\mu\otimes I+i\gamma_\nu\otimes\sigma^{\mu\nu}+i\sigma^{\mu\nu}\otimes\gamma_\nu-i\sigma^{\mu\nu}\gamma_5\otimes\gamma_\nu\gamma_5-3\gamma_5\otimes\gamma^\mu\gamma_5-3\gamma^\mu\gamma_5\otimes\gamma_5+i\gamma_\nu\gamma_5\otimes\sigma^{\mu\nu}\gamma_5\right). \quad (16)$$

The symmetry with respect to permutation of indeces $k_2\leftrightarrow k_3$, to Rarita-Schwinger equation $\bar{\Delta}^\mu\gamma_\mu=0$ and to the identity

$$i\gamma_\nu\gamma_5\otimes\sigma^{\mu\nu}\gamma_5=-i\gamma_\nu\otimes\sigma^{\mu\nu}, \quad (17)$$

leads to the relation

$$\begin{cases}4\tilde{I}\otimes\tilde{\gamma}^\mu=2I\otimes\gamma^\mu-2i\gamma_\nu\otimes\sigma^{\mu\nu}\\ 4i\tilde{\gamma}_\nu\otimes\tilde{\sigma}^{\mu\nu}=-4I\otimes\gamma^\mu\end{cases} \quad (18)$$

It is clear that combination

$$\Gamma_1\otimes\Gamma_2=I\otimes\gamma^\mu-\frac{i}{2}\gamma_\nu\otimes\sigma^{\mu\nu}, \quad (19)$$

is symmetric under Fierz transformation and transform in itself. Thus, three-quark current of Δ-isobar has only possible form

$$\left(J^\mu\right)^{k_1k_2k_3}=\varepsilon^{a_1a_2a_3}$$
$$\left[q^{k_1}_{a_1}\cdot\left[q^{k_2}_{a_2}C\gamma^\mu q^{k_3}_{a_3}\right]-\frac{i}{2}\gamma_\nu q^{k_1}_{a_1}\cdot\left[q^{k_2}_{a_2}C\sigma^{\mu\nu}q^{k_3}_{a_3}\right]\right]. \quad (20)$$

Conclusion

We construct the relativistic three-quark current of Delta-isobar. For this, we use the nontrivial way from permutation invariance and parity invariance. Rarita-Schwinger equation and Fierz transformation enabled to construct the symmetric three-quark current under isospin indices permutations.

The generalization of the Delta-isobar current to nonlocal cases enables to describe any of large group of particles, such as $\Omega_c(3000)^0$, $\Omega_c(3050)^0$, $\Omega_c(3066)^0$, $\Omega_c(3090)^0$, $\Omega_c(3119)$ and others with quantum numbers $J^p=(3/2)^+$.

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