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Strong form factor of delta (1232)

Abstract. Our article is devoted to study of the $\Delta \to N\pi$ decay. Strong decay $\Delta \to N\pi$ is one of the best modes to search for particles with spin 3/2. Strong decay is dominating channel of decay, i.e. Δ-isobar decay consist of 99% - hadronic decay, 1% - electromagnetic decay, therefore the decay is important for nuclear research. We calculate relevant form factors in the framework of the covariant quark model with infrared confinement in the full kinematical momentum transfer region. The covariant quark model has been applied to a large number of elementary particle processes. This model can be viewed as an effective quantum field approach to hadronic interactions, which based on an interaction Lagrangian of hadrons interacting with their constituent quarks. The coupling strength is determined by the compositeness condition $Z_H = 0$ where Z_H is the renormalization constant of the hadron wave function. We compare the obtained results with available experimental data and the results from other theoretical approaches.
Key words: relativistic quark model, confinement, Δ-isobar, strong decay, strong form factor.

Introduction

In hadron physics the strong interaction dominates the decay width of a resonance if appropriate hadronic channels are open. For Δ-isobar decay 99% - hadronic decay and 1% - electromagnetic decay.

The nucleon and the Δ-isobar are investigated as three-quark systems in the quark-confinement model (QCM). This model is based on two hypotheses. First, quark confinement is accomplished through averaging over some vacuum gluon fields which are assumed to provide the confinement of any colour states. Second, hadrons are treated as collective colourless excitations of quark-gluon interactions [1].

On the basis of nonlocal three-quark current of Δ-isobar and by using covariant confined quark model, we calculate mass operator (self-energy diagram), coupling constant and matrix element of $\Delta \to N\pi$ decay.

The covariant confined quark has been applied to a large number of elementary particle processes [4, 5]. This model can be viewed as an effective quantum field approach to hadronic interactions, which based on an interaction Lagrangian of hadrons interacting with their constituent quarks. The coupling strength is determined by the compositeness condition $Z_H = 0$ where Z_H is the renormalization constant of the hadron wave function. The hadron field renormalization constant Z_H characterizes the overlap between the bare hadron field and the bound state formed from the constituents. Once this constant is set to zero, the dynamics of hadron interactions is fully described by constituent quarks in quark loop diagrams with local constituent quark propagators. Matrix elements are generated by a set of quark loop diagrams according to the $1/N_c$ expansion. The ultraviolet divergences of the quark loops are regularized by including vertex functions for the hadron-quark vertices which, in addition, describe finite size effects due to the non-pointlike structure of hadrons. Quark confinement was implemented into the model [6] by introducing an infrared cutoff on the upper limit of the scale integration to avoid the appearance of singularities in any matrix element. The infrared cutoff parameter λ is taken to have a common value for all processes. The covariant confined quark model contains only a few model parameters: the light and heavy constituent quark masses, the size parameters that describe the size of the distribution of the constituent quarks inside the hadrons and the infrared cutoff parameter λ. They are determined by a fit to available experimental data.

Effective Lagrangian

The coupling of a Δ-isobar to its constituent quarks q_1, q_2 and q_3 is described by the Lagrangian

$$L_\Delta(x) = ig_\Delta \bar{\Delta}^{k_1k_2k_3}_{\mu\alpha}(x)(J^{\mu\alpha})^{k_1k_2k_3}(x) + h.c. \tag{1}$$

where

$$(J^{\mu\alpha}_{3q})^{k_1k_2k_3}(x) = \iiint dx_1dx_2dx_3 F_\Delta(x;x_1,x_2,x_3)(J^{\mu\alpha}_{3q})^{k_1k_2k_3}(x_1,x_2,x_3) \tag{2}$$

$$(J^{\mu\alpha}_{3q})^{k_1k_2k_3}(x_1,x_2,x_3) = \varepsilon^{a_1a_2a_3}\Gamma^\mu_{\alpha;\alpha_1,\alpha_2,\alpha_3} q^{k_1}_{a_1\alpha_1}(x_1) q^{k_2}_{a_2\alpha_2}(x_2) q^{k_3}_{a_3\alpha_3}(x_3),$$

here,

$$\Gamma^\mu_{\alpha;\alpha_1,\alpha_2,\alpha_3} = (\Gamma_1)_{\alpha\alpha_1} \otimes (\Gamma^\mu_2)_{\alpha_2\alpha_3} = (I)_{\alpha\alpha_1} \otimes (C\gamma^\mu)_{\alpha_2\alpha_3} - \frac{i}{2}(\gamma_\nu)_{\alpha\alpha_1} \otimes (C\sigma^{\mu\nu})_{\alpha_2\alpha_3}$$

is a Dirac matrix which projects onto the spin quantum number of the isobar field. The function F_Δ is characterizes the finite size of the isobar. To satisfy translational invariance the function F_Δ has to fulfill the identity $F_\Delta(x + a; x_1 + a, x_2 + a, x_3 + a) = F_\Delta(x; x_1, x_2, x_3)$ for any four-vector ***a***. In the following we use a specific form for the scalar vertex function

$$F_\Delta(x;x_1,x_2,x_3) = \delta\left(x - \sum_{i=1}^{3} x_i\omega_i\right)\Phi_\Delta\left(\sum_{i<j}\left(x_i - x_j\right)^2\right), \tag{3}$$

where Φ_M is the correlation function of the constituent quarks with masses m_{q_1}, m_{q_2}, m_{q_3} and the mass ratios $\omega_i = m_{q_i}/(m_{q_1} + m_{q_2} + m_{q_3})$.

We choose a simple Gaussian form of the vertex function $\tilde{\Phi}_\Delta(-k^2)$. The minus sign in the argument of this function is chosen to emphasize that we are working in the Minkowski space. One has

$$\tilde{\Phi}_\Delta(-k^2) = exp(k^2/\Lambda^2_\Delta) \tag{4}$$

where the parameter Λ_Δ characterizes the size of the meson. Since k^2 turns into $-k^2_E$ in the Euclidean space, the form (4) has the appropriate fall-off behavior in the Euclidean region. We emphasize that any choice for Φ_M is appropriate as long as it falls off sufficiently fast in the ultraviolet region of the Euclidean space to render the corresponding Feynman diagrams ultraviolet finite. We choose a Gaussian form for Φ_M for calculation convenience.

Strong form factors of Δ(1232) in the covariant quark model

We will study strong decay of Δ^{++} to proton and positive pion to investigate the strong form factors of Δ(1232).

The matrix element corresponding to that Feynman diagram represented as

$$\begin{aligned} M(\Delta \to p\pi) &= g_\Delta g_p g_\pi \int dx \int dy \int dz \cdot e^{ip'x - ipy + iqz} \times \\ &\times \bar{u}_p(p')\left\langle 0 \middle| T\left\{J_p(x)\bar{J}^\mu_\Delta(y)J_\pi(z)\right\} \middle| 0 \right\rangle u^\mu_\Delta(p), \end{aligned} \tag{5}$$

The Feynman diagram which describes this process is given in Figure 1.

Three-quarks current of proton and pion given in similar way with Δ-isobar,

$$\begin{aligned} J_p(x) &= \int dx_1dx_2dx_3 F_N(x;x_1,x_2,x_3) J_{3q}(x_1,x_2,x_3), \\ J_{3q}(x_1,x_2,x_3) &= \\ &= \Gamma^A\gamma^5 d^{a_1}(x_1)\left[\varepsilon^{a_1a_2a_3} u^{a_2}(x_2) C\Gamma_A u^{a_3}(x_3)\right], \end{aligned} \tag{6}$$

$$J_\pi(z) = \int dz_1dz_2 F_\pi(z;z_1,z_2) J_{3q}(z_1,z_2),$$
$$J_{3q}(z_1,z_2) = \bar{d}(z_1) i\gamma^5 u(z_2).$$

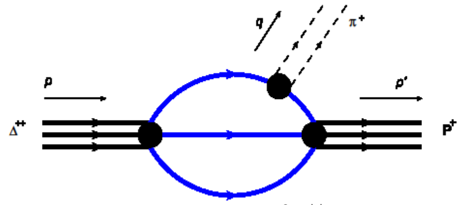

Figure 1 – Decay of $\Delta^{++} \to p\pi$

The matrix $C=\gamma^0\gamma^2$ is the usual charge conjugation matrix and the a_i (i = 1, 2, 3) are color indices. There are two possible kinds of nonderivative three-quark currents: $\Gamma^A \otimes \Gamma_A = \gamma^\alpha\otimes\gamma_\alpha$ (vector current) and $\Gamma^A\otimes\Gamma_A = 1/2\ \sigma^{\alpha\beta}\otimes\sigma_{\alpha\beta}$ (tensor current) with $\sigma^{\alpha\beta} = i/2\ (\gamma^\alpha\gamma^\beta - \gamma^\beta\gamma^\alpha)$.

Let us write the T-production for the matrix element in eq.(5)

$$\left\langle 0\left|T\left\{J_p(x)\bar{J}^\mu_\Delta(y)J_\pi(z)\right\}\right|0\right\rangle = 12\Gamma^A\gamma^5 S_d\left(x_1-z_1\right)i\gamma_5 S_u\left(z_2-y_1\right)\Gamma_1\cdot tr\left[S_u\left(y_2-x_2\right)\Gamma_A S_u\left(x_3-y_3\right)\Gamma_2^\mu\right]+ \\ +24\Gamma^A\gamma^5 S_d\left(x_1-z_1\right)i\gamma_5 S_u\left(z_2-y_1\right)\Gamma_2^\mu S_u\left(y_2-x_2\right)\Gamma_A S_u\left(x_3-y_3\right)\Gamma_1$$

Then we rewrite the M-martix as

$$M\left(\Delta\to p\pi\right)=12ig_\Delta g_p g_\pi\int dx\int dy\int dz e^{ip'x-ipy+iqz}\int dx_1dx_2dx_3\delta\left(x-\sum_{i=1}^{3}x_iv_i\right)\Phi_p\left[\sum_{i<j}\left(x_i-x_j\right)^2\right] \\ \int dy_1dy_2dy_3\delta\left(y-\sum_{i=1}^{3}y_iw_i\right)\Phi_p\left[\sum_{i<j}\left(y_i-y_j\right)^2\right]\int dz_1dz_2\delta\left(z-\sum_{i=1}^{2}z_i\eta_i\right)\Phi_\pi\left[\sum_{i<j}\left(z_1-z_2\right)^2\right] \\ \bar{u}_p\left(p'\right)\begin{Bmatrix}\Gamma^A\gamma^5 S_d\left(x_1-z_1\right)\gamma_5 S_u\left(z_2-y_1\right)\Gamma_1\cdot tr\left[S_u\left(y_2-x_2\right)\Gamma_A S_u\left(x_3-y_3\right)\Gamma_2^\mu\right] \\ +2\Gamma^A\gamma^5 S_d\left(x_1-z_1\right)\gamma_5 S_u\left(z_2-y_1\right)\Gamma_2^\mu S_u\left(y_2-x_2\right)\Gamma_A S_u\left(x_3-y_3\right)\Gamma\end{Bmatrix}u^\mu_\Delta\left(p\right) \\ \int dx\int dy\int d\rho_1^x\int d\rho_2^x\int d\rho_1^y\int d\rho_2^y\int dz_1\int dz_2 \\ \times\exp\begin{bmatrix}ip'x-ipy+iq\left(\eta_1z_1+\eta_2z_2\right)-i\omega_1^x\rho_1^x-i\omega_2^x\rho_2^x-i\omega_1^y\rho_1^y-i\omega_2^y\rho_2^y-il\left(z_1-z_2\right) \\ -ik_1\left(x_1-z_1\right)-ik_1\left(z_2-y_1\right)-ik_3\left(y_2-z_2\right)-ik_4\left(z_3-y_3\right)\end{bmatrix}$$

After integrating by space coordinates we get a set of δ-functions, which help us to vanish some momentum integrations. As a result we have

$$M\left(\Delta^{++}\to p\pi\right)=\left(2\pi\right)^4 i\delta\left(p-p'-q\right)T\left(\Delta^{++}\to p\pi^+\right)$$

where

$$T\left(\Delta^{++}\to p\pi^+\right)=-12g_\Delta g_p g_\pi\int\frac{d^4k_1}{\left(2\pi\right)^4 i}\int\frac{d^4k_2}{\left(2\pi\right)^4 i}\tilde{\Phi}_p\left[-\omega_x^2\right]\tilde{\Phi}_\Delta\left[-\omega_y^2\right]\tilde{\Phi}_p\left[-\left(k_1-\eta_2q\right)^2\right] \\ \bar{u}_p\left(p'\right)\begin{Bmatrix}\Gamma^A\gamma^5 S_d\left(k_1-q\right)\gamma_5 S_u\left(k_1\right)\Gamma_1\cdot tr\left[S_u\left(k_2\right)\Gamma_A S_u\left(k_2-k_1+p\right)\Gamma_2^\mu\right] \\ +2\Gamma^A\gamma^5 S_d\left(k_1-q\right)\gamma_5 S_u\left(k_1\right)\Gamma_2^\mu S_u\left(k_2\right)\Gamma_A S_u\left(k_2-k_1+p\right)\Gamma_1\end{Bmatrix}u^\mu_\Delta\left(p\right)= \\ =G_{\Delta p\pi}\cdot p'_\mu\cdot\bar{u}_p\left(p',\lambda'\right)u^\mu_\Delta\left(p,\lambda\right)$$

We use the next definitions

$$\begin{cases}\omega_1^x=\dfrac{1}{\sqrt{2}}\left[-k_1+k_3+p-v_3p'\right] \\ \omega_2^x=\dfrac{1}{\sqrt{6}}\left[k_1+k_3-p+(2v_2+v_3)p'\right]\end{cases},\begin{cases}\omega_1^y=\dfrac{1}{\sqrt{2}}\left[k_1-k_3-\left(\omega_1+\omega_2\right)p\right] \\ \omega_2^y=\dfrac{1}{\sqrt{6}}\left[-k_1-k_3+(\omega_1-\omega_2)p\right]\end{cases}$$

After loop integration we get 4-integral, which are numerically calculated. It is convenient to approximate the result of numerical calculations by dipole function

$$G\left(Q^2\right)=\frac{1}{\left[1+Q^2/\Lambda_D^2\right]^2}$$

where, parameter $\Lambda_D=0.96$ GeV. Function behavior in the region $0\le Q^2=-q^2\le 2.5$ GeV^2 represented at Figure 2.

For comparison we use graphics from others theoretical approaches such as [1]-[3] and results of lattice QCD calculations [4]. For graphic 3 we use parameterization given in work [2]

$$G\left(\vec{q}^{\,2}\right)=\frac{1}{\left[1+\vec{q}^{\,2}/\lambda_1^2+\vec{q}^{\,4}/\lambda_2^4\right]}$$

where $\lambda_1=0.594\,\text{GeV}$, $\lambda_2=0.998\,\text{GeV}$. $\vec{q}^{\,2}=q_0^2+Q^2$.

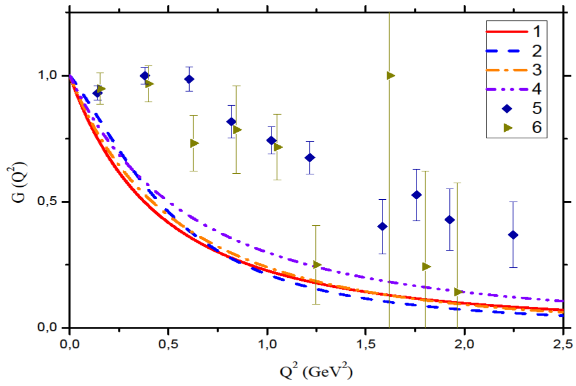

Figure 2 – Strong form factor of Δ-isobar

Also we represent Table 1 with numerical values of strong coupling constants $G_{\Delta p\pi}$.

Table 1 – Strong constant of decay

	Exp	Our work	[1]	[2]	[3]	[5]	[6]	[7]	[8]
$G_{\Delta p\pi}[GeV^{-1}]$	15.4±2.9	15.2	17.0	11.14	14.98	14.85	17.76	15.2	13.4±5.4

Conclusion

We calculate coupling constant of delta-isobar. The calculated value of coupling constant is in good agreement with the experimental data.

We built the graphic of G(Q^2) in Euclidian region of squared momentum transfer Q^2=-q^2. We compare our results with [1]-[3], [5]-[8] works. Our results are close to work [2].

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