UDC 533.9

^{1*}Arkhipov Yu.V., ¹Ashikbayeva A.B., ¹Askaruly A., ¹Dubovtsev D.Yu., ¹Syzganbayeva S.A., ²Tkachenko I.M. ²

¹Department of Plasma Physics, al-Farabi Kazakh National University, Tole Bi 96, Almaty 050012, Kazakhstan, ²Departamento de Matemática Aplicada, Universitat Politècnica de València, Camino de Vera s/n, 46022 Valencia, Spain *e-mail: yarkhipov@yahoo.ca

Optical properties of dense coulomb plasmas

Abstract. In the present work, we provide a theoretical analysis of the reflection of low-temperature dense plasma for the normal incidence of laser radiation, and the dependence of reflectivity of s - and p - polarized waves in respect to the angle of incidence. In connection with the calculations for large parameters of density, the interpolation approach based on the theory of moment's method is used for the calculation of the dielectric function. In contrast to the classical method of moments, the characteristic frequencies can be found from the simplified formulas, and the Nevanlinna parameter function is determined as in Phys.Rev.Lett. 119, 045001 (2017). The reflectivity is calculated with using of Fresnel equations. The obtained results were compared with theoretical and experimental data of other authors. It is shown that results of the present work for the reflection coefficient are close to the experimental results, in contrast to other theoretical results, in which a model with several adjustable parameters was used. The patent of RK for the present method was submitted.

Key words: dense Coulomb plasmas, reflectivity, dielectric function, method of moments.

Introduction

Nonideal plasma is the most common state of matter in the Universe. For instance, warm dense matter is formed: in giant planets cores; in the process of the electrical explosion of conductors; in the matter impacted by femtosecond laser pulse; at moving of swift heavy ion through a nuclear material etc. For further development of nonideal plasma physics, investigations of its electronic properties appear to be crucial. In particular, optical reflectance are an important diagnostic tool: the reflectivity is expected to give information on the free-charge carrier density. The reflectance of shock-compressed xenon is measured in the unique experiments of Mintsev and Zaporozhets [1-4]. The main purpose of this work is the theoretical explanation of the experimental data using the interpolation method of moments. Because this method allows you to find the dielectric function of plasma on the basis of exact relations and sum rules (frequency moments of the imaginary part of the dielectric function) directly through interpolation static characteristics of the system.

Let us specify the thermodynamic parameters characteristic of model plasmas we deal with. Consider model completely ionized hydrogen-like plasmas in thermal equilibrium. The number density of electrons, n_e is presumed to be of the order of

 10^{21} cm⁻³, the temperature, T, so that $\beta^{-1}=k_BT$. Under these conditions all characteristic lengths in the system are of the same order of magnitude and the dimensionless parameters, like $(a=(3/4\pi n)^{1/3})$ being the Wigner-Seitz radius and the density parameter, r_s=a/a_B, a_B - Bohr radius.

Self - consistent calculation method

The Fresnel formulas [5] are used for calculation of reflectivity for the normal incidence ($\theta = 0^0$) of the electromagnetic wave

$$R = \left| \frac{\left(\sqrt{\epsilon} - 1 \right)}{\left(\sqrt{\epsilon} + 1 \right)} \right|^2, \tag{1}$$

and for the s- and p-polarized reflectivity
$$R_{s} = \left(\frac{\left|cos\theta - \sqrt{\epsilon - sin^{2}\theta}\right|}{\left|cos\theta + \sqrt{\epsilon - sin^{2}\theta}\right|}\right)^{2},$$

$$R_p = \left(\frac{\left|\epsilon \cos\theta - \sqrt{\epsilon - \sin^2\theta}\right|}{\left|\epsilon \cos\theta + \sqrt{\epsilon - \sin^2\theta}\right|}\right)^2. \tag{2}$$

The dielectric function ϵ (DF), which is included in (2) and (1) is a complex function and can be expressed as

$$\epsilon^{-1}(k,\omega) = 1 + \frac{\omega_p^2(\omega + Q(k,\omega))}{\omega(\omega^2 - \omega_2^2(k)) + Q(k,\omega)(\omega^2 - \omega_1^2(k))} \ . \tag{3}$$

In general, the characteristic frequencies $\omega_1(k)$ and $\omega_2(k)$ can be calculated on the basis of the Kubo theory of linear response [6], but here, for simplicity and as a tool of preliminary but reliable estimate, we employ for them the following interpolation expressions [7,8]:

$$\omega_1^2(k) = \omega_p^2 (1 + k^2 k_D^{-2} + k^4 \kappa^4) \tag{4}$$

in a hydrogen-like plasma with

$$n_e = Zn_i$$

 n_i -density of protons,

$$\mu^{4} = \frac{16\pi e^{2}}{\hbar^{2}} a^{4} (n_{e} m_{e} + Z^{2} n_{i} m_{i}) \approx
\approx \frac{16\pi Z^{2} e^{2} n_{i} a^{4} m_{i}}{\hbar^{2}} = 12 r_{s} \frac{m_{i}}{m_{e}}.
\omega_{2}^{2}(k) = \omega_{p}^{2} (1 + H) +
+ \langle v_{e}^{2} \rangle k^{2} + (\hbar k^{2} / 2 m_{e})^{2} - v_{int}^{2} k^{2}.$$
(5)

The parameters introduced here (4,5) can be calculated immediately: $v_{int}^2 = -\frac{4}{15}\frac{\frac{3}{\Gamma^2}}{\beta m_e} \left(\frac{A_1}{\sqrt{A_2+\Gamma}} + \frac{A_3}{1+\Gamma}\right), A_1 = -0.9052, A_2 = 0.6322 \text{ M} A_3 = \frac{-\sqrt{3}}{2} - \frac{A_1}{\sqrt{A_2}}, \hbar - \text{Planck constant}, \ e - \text{charge of electron},$

 ω_p — the plasma frequency, $\langle v_e^2 \rangle$ — the average square of the electron thermal velocity, k_D^{-1} —the Debye radius, and m_e — electron mass and m_i —proton mass; Finally, the contribution H is related to the electron-ion interaction in the target plasma and we evaluate it in the modified random-phase approximation [9] as: $H = (4r_S/3)(3\Gamma + 4r_S/\Gamma + 4\sqrt{6r_S})^{-1/2}$.

In the present work, we do not reconstruct the Nevanlinna parameter function (NPF) from the very data, we use the following NPF model [10], where q = ka:

$$Q(q,\omega) = ih_0(q), \tag{6}$$

where the positive parameter $h_0(q)$ is a function of the frequencies $\omega_2^2(q)$ and $\omega_1^2(q)$.

Numerical results

The analytical results of the reflectivity coefficient R^{IMM} (1) at the plasma front, depending of the incident angle θ in comparison with experimental data at the R^{exp} [11] wavelengths of $\lambda_L=1064$ nm, $\lambda_L=694$ nm, $\lambda_L=532$ nm and the respective thermodynamic parameters are presented in tables 1, 2 and 3 (pressure P, temperature T, mass density ρ , density of free electrons n_e , density of neutral atoms n_a , ionization degree $\alpha_{ion}=\frac{n_e}{(n_a+n_e)}$ and degeneracy parameter $\theta=\frac{k_BT}{E_F}$, E_F – Fermi energy.)

Table 1 – $\lambda_L = 1064 \text{ nm}$

P, (GPa)	T, (K)	ρ, g/cm-3	$n_e \cdot 10^{21}$ cm-3	$n_a \cdot 10^{21}$ cm-3	α_{ion}	Γ	Θ	R^{exp}	R from [12]	R^{IMM}
1.6	30050	0.51	1.8	6.1	0.75	1.1	4.8	0.096	0.272	0.14
3.1	29570	0.97	3.2	1.4	0.70	1.3	3.2	0.12	0.342	0.18
5.1	30260	1.46	4.5	2.2	0.67	1.5	2.6	0.18	0.381	0.20
7.3	29810	1.98	5.7	3.5	0.62	1.6	2.2	0.26	0.404	0.23
10.5	29250	2.70	7.1	5.4	0.57	1.8	1.9	0.36	0.429	0.25

P, (GPa)	Т, (К)	ρ, g/cm-3	$n_e \cdot 10^{21}$ cm-3	$n_a \cdot 10^{21}$ cm-3	α_{ion}	Γ	0	R^{exp}	R^{IMM}
0.93	32070	0.27	1.1	2.1	0.78	0.87	7.1	0.02	0.039
1.9	32900	0.53	2.1	4.8	0.72	1.0	4.8	0.05	0.11
4.1	33100	1.1	4.0	1.3	0.69	1.3	3.2	0.11	0.15
6.1	33120	1.6	5.2	2.1	0.64	1.4	2.6	0.14	0.18
9.1	32090	2.2	6.6	3.6	0.60	1.6	2.1	0.18	0.19
12.0	32020	2.8	7.8	5.0	0.56	1.7	1.9	0.26	0.21

Table 2 – λ_L = 694 nm

Table 3 – $\lambda_L = 532 \text{ nm}$

P, (GPa)	T, (K)	ρ, g/cm-3	$n_e \cdot 10^{21}$ cm-3	$n_a \cdot 10^{21}$ cm-3	α_{ion}	Γ	0	R^{exp}	R^{IMM}
4.1	33100	1.1	4.0	1.3	0.69	1.3	3.2	0.02	0.1
6.1	33120	1.6	5.2	2.1	0.64	1.4	2.6	0.045	0.15
9.1	32090	2.2	6.6	3.6	060	1.6	2.1	0.10	0.17
12.0	32020	2.8	7.8	5.0	0.56	1.7	1.9	0.16	0.18

On the pictures (1,2) one can see theoretical values the reflectivity of electromagnetic waves, 532 Nm and 694 Nm of length with different kinds of

polarization (s-p) from the dense plasma's surface and experiment data, as seen from the graphs, there is a good agreement.

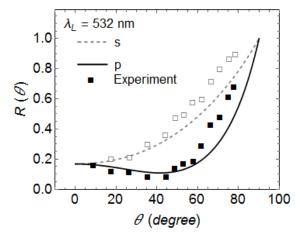


Figure 1 – Calculated reflectivity (1) in comparison to the experimental data for $\lambda_L = 532$ nm. *s*-polarisation is gray dashed line, *p*-polarisation is black line. Experimental data [13] are shown as square.

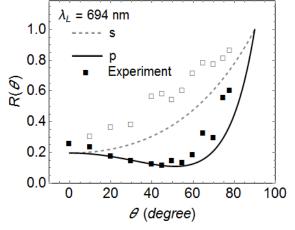


Figure 2 – Calculated reflectivity (1) in to the experimental data for $\lambda_L = 694$ nm. *s*-polarisation is gray dashed line, *p*-polarisation is black line. Experimental data [13] are shown as square.

4 Conclusions

This simplified mathematical method of moments allows describing theoretically the optical properties of dense Coulomb systems. From the results of Tab.1-3, one can see that the reflection coefficient, calculated in the present article is closest to the experimental data [11], in contrast to the approach from [12] and other theoretical results, where a model with several adjustable parameters was used.

Acknowledgements.

Work supported by the Republic of Kazakhstan Ministry of Education and Science grant No. 3119/GF4 and No. 3831/GF4.

References

[1] V.B. Mintsev., Yu.B. Zaporogets // Contrib. Plasma Phys. – 1989. – Vol. 29. – N. 4. – P. 493 – 501.

- [2] Yu. B. Zaporoghets., V. B. Mintsev., V. K. Gryaznov., V. E. Fortov. // Physics of Extreme States of Matter 2002. Chernogolovka. 2002. P. 188 189
- [3] Yu. B. Zaporoghets., V.B. Mintsev., V.K. Gryaznov., H. Reinholz., G. Röpke. // Physics of Extreme States of Matter. 2004. Chernogolovka. P. 140 141.
- [4] Yu. B. Zaporoghets., V.B. Mintsev., V.K. Gryaznov., H. Reinholz., G. Röpke., V. E. Fortov. // Physics of Extreme States of Matter. 2013. Chernogolovka. P. 194 197.
- [5] Yu.L. Klimontovich. Kinetic theory of nonideal gas and nonideal plasma // M: Nauka. 1975. P. 352
- [6] Yu. V. Arkhipov et al. // Phys. Rev. E. 2014. Vol. 90. P. 053102; *ibid* 2015. Vol. 91. P. 019903
- [7] V.M. Adamyan., I.M. Tkachenko // High Temp. 1983. Vol. 21. P. 307; J. Ortner I.M. Tkachenko // Phys. Rev E. 2001. Vol. 63. P. 0264031-11.

- [8] G. Chabrier., A. Y. Potekhin. Equation of state of fully ionized electron-ion plasmas //Phys. Rev. E. 1998. Vol. 58. P. 4941.
- [9] M.J. Corbatón., I.M. Tkachenko // International Conference on Strongly Coupled Coulomb Systems. 2008. Camerino. Italy. Book of Abstracts. P. 90.
- [10] Yu. V. Arkhipov *et al.* Direct Determination of Dynamic Properties of Coulomb and Yukawa Classical One-Component Plasmas // Phys.Rev.Lett. 2017. Vol. 119. P. 045001.
- [11] Yu. Zaporoghets., V. Mintsev V., Gryaznov V., Fortov., H. Reinholz., T. Raitza., G. Ropke. Reflectivity of nonideal plasmas // J. Phys. A: Math. Gen. 2006. Vol. 39. No 17. P. 4329-4335.
- [12] Yu. B. Zaporozhets., Y. A. Omarbakiyeva., H. Reinholz., G. Ropke., V. B. Mintsev., V. K. Gryaznov. Optical Reflectivity of Xenon Plasma Revisited // Contribution of the 15th International Conference on the Physics of Non-Ideal Plasmas. 2015. Kazakhstan. Almaty.