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# OML evaluation of the dust grain charge under quasineutrality conditions

**Abstract.** Interaction potentials of electrons and ions with dust particles are developed to consistently treat plasma electrodynamics. It is assumed for the sake for simplicity that the material, the dust particles are made of, is a perfect conductor, and, then, the linear density-response formalism in the random phase approximation is used to take into account finite dimensions of grains. Additionally, the number density of protons is kept fixed such that the negative electric charge is allocated between the free electrons and the dust particles to assure the whole quasineutrality of the system. On the ground of the developed interaction potentials the electric charge of the dust particles is then calculated within the orbital motion limited (OML) approximation, which stems from the ballistic trajectories of the plasma particles at the charging process. It is rather clear that to advocate such a technique the mean free paths of the plasma particles must be much greater than the dimension of the dust grain. It is well known that under OML assumptions the conservation laws of energy and angular momentum are sufficient to determine the absorption cross sections of electrons and ions by the dust particle. The resultant absorption cross sections are then integrated over the velocity distribution distribution function to evaluate the fluxes of plasma particles on the grain surface, and the electric charge of the dust particle is stabilized when those fluxes are finally equalized.

**Key words:** dust grain charge, interaction potential, density-response formalism, orbital motion limit approximation, velocity distribution function.

## Introduction

Over the past few decades, much attention of researchers in the field of plasma physics has been paid to the so-called dusty plasmas, which appear in a variety of situations, both in nature and in laboratory. In particular, dusty plasmas can be encountered in many kinds of astrophysical objects [1-3], space and earth experiments [4-6], nanotechnology [7.8], cancer therapy in medicine [9,10] and in many other industries. Moreover, dust plasmas are working substances in the installations designed for controlled thermonuclear fusion [11,12] and for plasma etching in the modern electronics industry [13,14] since solid particles of micron size easily penetrate into the plasma medium as a result of the destruction of electrodes and walls of plasma chambers. This results not only in immediate change of physical properties of the surface material, but also in considerable perturbation, mostly in an unpredictable way, of local plasma characteristics.

In this regard, the study of dusty plasma properties is of great scientific interest from both fundamental and applied physics points of view. An important role for the explanation of various phenomena in dusty plasmas is played by the plasma sheath theory, since, on the one hand, it is virtually impossible to completely avoid direct contact of the plasma medium with electrodes and chamber walls, and it is this contact which is responsible for dusty plasma generation. On the other hand, dust particles are themselves solid bodies, which are surrounded by the plasma sheath that ultimately governs their electric charge.

## Plasma parameters

It is widely understood that typical dusty plasma consists of at least four particle species. For simplicity, assume in the following that the buffer plasma contains free electrons with the electric charge $-e$ and the number density $n_e$, and free protons with the electric charge $e$ and the number density $n_p$. In addition, the buffer plasma is assumed to be filled with the dust particles of the same radius $R$ to constitute another species with the number density $n_d$. It is exactly this system which is conventionally called a dusty plasma. The medium is normally non-isothermal such that the electron temperature $T_e$ substantially exceeds the proton temperature $T_p$, whereas the dust particles temperature is supposed to be equal to the proton temperature. In real dusty plasmas uncharged particles are inevitably present, but in the sequel pure electrostatic interactions are in focus which somehow justifies further disregard of the neutral component.

It is rather convenient to introduce few dimensionless parameters describing the physical state of the dusty plasma. For reasons that are to become clearer later, the number density of protons is kept fixed and the coupling parameter of the buffer plasma is defined as:

$$\Gamma = \frac{e^2}{a_p k_B T_p}, \tag{1}$$

where $k_B$ denotes the Boltzmann constant, and $a_p = (3/4\pi n_p)^{1/3}$ stands for the average distance between the protons.

In addition, we introduce the ratio of the number densities of dust particles and protons as follows

$$\beta = \frac{n_d}{n_p}. \tag{2}$$

The main objective of the following is to account for the dimensions of the dust particles by incorporating the size parameter

$$D = \frac{a_d}{R}, \tag{3}$$

where $a_d = (3/4\pi n_d)^{1/3}$ refers to the average distance between the dust particles.

Electric charge of the dust particles essentially depend on mobilities of surrounding plasma particles, whose ration is described by the non-isothermality parameter of the form:

$$\tau = \frac{T_e}{T_p}. \tag{4}$$

Note that the particle mobility strongly depends on its mass, but in the following subsequently the ratio of the proton mass $m_p$ to the electron mass $m_e$ is assumed to be known and equal to $\sim 1637$

Finally, he whole system remains quasi-neutral, i.e. the following condition holds

$$n_p = n_e + Z_d n_d, \tag{5}$$

where $Z_d$ is a, still unknown, electric charge of the dust particles, which is, in the present consideration, a function of dimensionless parameters (1)-(4). Note that since the number density of of protons is fixed, condition (5) determines the balance between the charge of dust particles and the number density of electrons in the buffer plasma.

## Interaction model

When considering the interaction of dust grains immersed in a plasma it is widely believed that their electric charge is governed by the normal component of the electric field near the particle surface what is incorrect in terms of plasma electrodynamics [15]. It was also shown [16] that engaging of plasma electrodynamics results in the following interaction potential $\Phi_{ab}(r)$ between the particles of dusty plasmas:

$$\begin{aligned} \Phi_{ab}(r) = \varphi_{ab}(r) - \\ - \frac{Q_{ab}}{r}\left[1 - \exp(-k_D r) - \frac{k_D R_{ab}}{2}\mathrm{B}_{ab}(r)\right], \end{aligned} \tag{6}$$

with

$$\begin{aligned} \mathrm{B}_{ab}(r) = \exp(k_D(R_{ab}+r))\,\mathrm{Ei}(k_D(R_{ab}+r)) - \\ - \exp(k_D(R_{ab}-r))\,\mathrm{Ei}(k_D R_{ab}) + \\ + \exp(-k_D(R_{ab}+r)) \\ \left[\mathrm{Ei}(k_D R_{ab}) - \mathrm{Ei}(-k_D(R_{ab}+r))\right] \end{aligned} \tag{7}$$

and the exponential integral function

$$\mathrm{Ei}(x) = \int_x^{\infty} \frac{\exp(-t)}{t}\,dt. \tag{8}$$

Here the micropotential is defined as $\varphi_{ab}(r) = Q_{ab}/(R_{ab}+r)$, $r$ is counted from the dust surface, $Q_{ed} = -Q_{pd} = Z_d e^2$, $R_{ed} = R_{pd} = R$ and $Q_{dd} = Z_d^2 e^2$, $R_{dd} = 2R$. Note that the screening in (6) is only due to electrons and ions of the buffer plasma such that the Debye wave number reads as:

$$k_D = \sqrt{\frac{4\pi n_e e^2}{k_B T_e} + \frac{4\pi n_p e^2}{k_B T_p}}. \tag{9}$$

It is known from the literature that the shielding effects start to appear from the surface of the dust particles and the Debye theory together with the assumption that the electric charge of the dust particle is determined by the normal component of the electric field strength at the surface leads to the following interaction potential:

$$\Phi_{ab}(r) = \frac{Q_{ab}}{(r+R_{ab})(1+2k_D R)}\exp(-k_D r). \quad (10)$$

Figure 1 shows comparative curves of the micropotential, potentials (6) and (10) as functions of the dimensionless distance. It is clearly observed that screened potential (6) and the micropotential lie higher than Yukawa-type potential (10). The resulting difference between the micropotential and the screened potential (6) at small interparticle separations is associated with the consistent use of plasma electrodynamics [16].

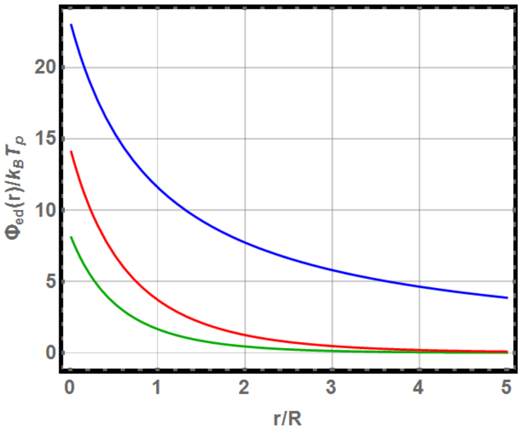

**Figure 1** – Interaction potential $\Phi_{ed}(r)$ between the electron and the dust particle in a plasma at $\tau = 5$, $\Gamma = 0.05$, $\beta = 10^{-4}$, $D = 10$ and $Z_d = 1000$. Blue line: micropotential; green line: Yukawa potential (10); red line: screened potential (6)

## OML for dust grain charge evaluation

To determine the charge of the dust particle of great significance is the orbital motion limited approximation, which is widely used in the theory of plasma probes and allows one to calculate the absorption cross sections of electrons and protons by a dust particle, based on single knowledge of the conservation laws of energy and angular momentum. It is assumed within that electrons and protons freely move on ballistic trajectories in the collective field, created by the dust particle and the plasma sheath, so that the following condition holds:

$$r_D \ll \ell_{e(p)}, \quad (11)$$

where $r_D = k_D^{-1}$ denotes the Debye screening length, and $\ell_{e(p)}$ designates the mean free paths of electrons and protons, respectively.

Consider the process of the dust particle charging in the orbital motion limited approximation [17]. The absorption cross section of protons is found from the conservation laws of energy and angular momentum as:

$$\sigma_p = \pi R^2\left(1 - \frac{2\Phi_{pd}(0)}{m_p v_p^2}\right), \quad (12)$$

where $v_p$ is the incident proton velocity, and $\Phi_{pd}(0)$ stands for the interaction potential energy of the dust particle with the proton on its surface.

Figure 2 reveals the dependence of the absorption cross section of protons as a function of their kinetic energy. As the proton kinetic energy grows, the corresponding absorption cross section decreases and it increases when the size parameter goes up, which is due to mutual attraction of those particles. Figure 3 shows the absorption cross section of protons as a function of the dust grain charge. Those dependences are practically linear and the cross section rapidly increases with the growth of the dust particle charge which is prescribed to strengthening of attraction of incident protons.

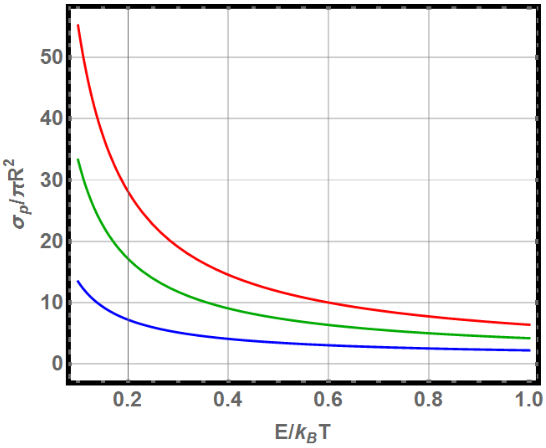

**Figure 2** – Absorption cross section of protons (12) by the dust particle as a function of the proton kinetic energy at $\tau = 1$, $\Gamma = 0.1$, $\beta = 10^{-3}$ and $Z_d = 500$. Blue line: $D = 5$; green line: $D = 10$; red line: $D = 15$

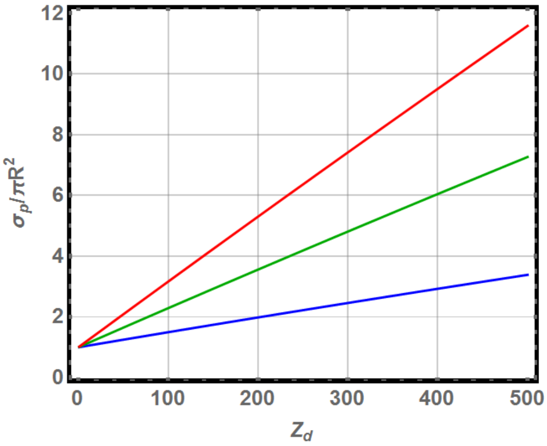

**Figure 3** – Absorption cross section of protons (12) by the dust particle as a function of the dust grain charge at $\tau = 1$, $\Gamma = 0.1$, $\beta = 10^{-3}$, and $E/k_B\,T = 5$. Blue line: $D = 5$; green line: $D = 10$; red line: $D = 15$

Similarly, the absorption cross section of electrons is obtained as

$$\sigma_e = \begin{cases} 0, & m_e v_e^2 < 2\Phi_{ed}(0), \\ \pi R^2 \left(1 - \dfrac{2\Phi_{ed}(0)}{m_e v_e^2}\right), & m_e v_e^2 \geq 2\Phi_{ed}(0), \end{cases} \quad (13)$$

where $v_e$ is the incident electron velocity, and $\Phi_{ed}(0)$ stands for the interaction potential energy of the dust particle with the electron on its surface. Note that since the dust particle is negatively charged, an electron must possess a certain threshold kinetic energy to be absorbed by the dust particle.

Figure 4 displays the dependence of the absorption cross section of electrons as a function of their kinetic energy. Since electrons are repelled by the negatively charged dust particle, there exists a threshold kinetic energy at which the absorption turns possible and the corresponding cross section becomes non-zero. As the kinetic energy of electrons grows the absorption cross section increases, and, at the same time, it decreases when the size parameter goes up. Figure 5 demonstrates the dependence of the absorption section of electrons as a function of the dust grain charge. Those dependences are practically linear and the cross section is greatly reduced with the growth of the dust particle charge due its repulsion of bombarding electrons.

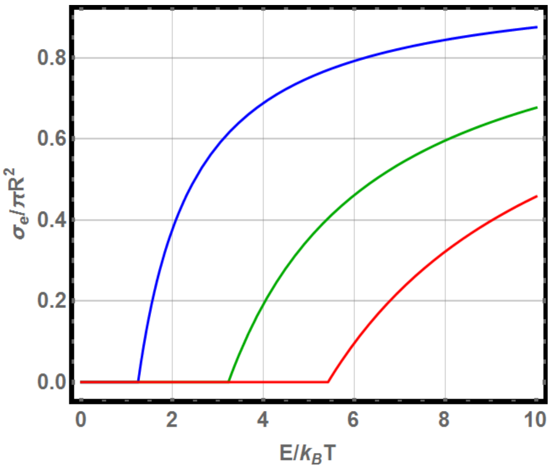

**Figure 4** – Absorption cross section of electrons (13) by the dust particle as a function of the electron kinetic energy at $\tau = 1$, $\Gamma = 0.1$, $\beta = 10^{-3}$ and $Z_d = 50$. Blue line: $D = 5$; green line: $D = 10$; red line: $D = 15$

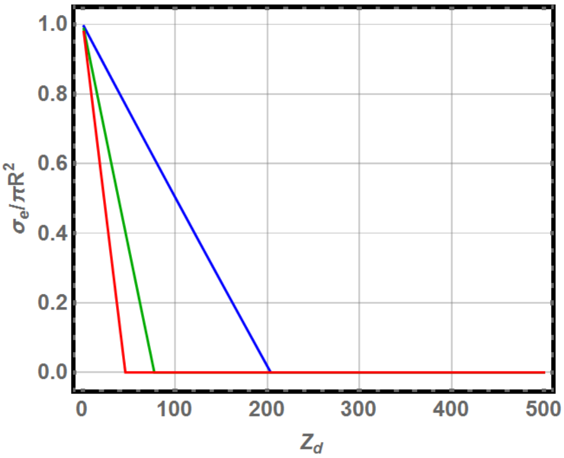

**Figure 5** – Absorption cross section of electrons (13) by the dust particle as a function of the dust grain charge at $\tau = 1$, $\Gamma = 0.1$, $\beta = 10^{-3}$, and $E/k_B\,T = 5$. Blue line: $D = 5$; green line: $D = 10$; red line: $D = 15$

It is widely known that the flux $I_a$ of particle species $a$ on the surface of the dust grain is derived from the corresponding absorption cross section by integrating over the velocity distribution as:

$$I_a = n_a \int v \sigma_a f_a(v) d^3\mathbf{v} \quad (14)$$

where $f_a(v) = (2\pi v_{T,a}^2)^{-3/2} \exp(-v^2/2v_{T,a}^2)$ simply denotes the Maxwell distribution function with the thermal velocity $v_{T,a} = \sqrt{k_B T_a / m_a}$ .

In expression (14) the corresponding absorption cross sections for electrons (13) and protons (12) are substituted, and the following expressions for the fluxes of electrons and protons on the dust grain surface are obtained:

$$I_e = 2\sqrt{2\pi} n_e R^2 v_{T,e} \exp\left( -\frac{\Phi_{ed}(0)}{k_B T_e} \right), \qquad (15)$$

$$I_p = 2\sqrt{2\pi} n_p R^2 v_{T,p} \left( 1 - \frac{\Phi_{pd}(0)}{k_B T_p} \right). \qquad (16)$$

The stationary electric charge of the dust particle can be considered established if it absorbs the same number of protons and electrons in a unit of time, i.e. the equality $I_e = I_p$ is to be satisfied and with the help of expressions (15) and (16) the following generalized equation is found

$$\frac{n_p}{n_e}\sqrt{\frac{m_e T_p}{m_p T_e}} \left( 1 - \frac{\Phi_{pd}(0)}{k_B T_p} \right) = \exp\left( -\frac{\Phi_{ed}(0)}{k_B T_e} \right), \qquad (17)$$

and it retains its validity for any kind of interaction potential between the dusty plasma particles.

In case of the Coulomb interaction between the dusty plasma particles, expression (17) is reduced to the following classical form:

$$\frac{n_p}{n_e}\sqrt{\frac{m_e T_p}{m_p T_e}} \left( 1 + \frac{Z_d e^2}{k_B T_p R} \right) = \exp\left( -\frac{Z_d e^2}{k_B T_e R} \right). \qquad (18)$$

In case of the Yukawa potential (10), equation (17) reads as:

$$\frac{n_p}{n_e}\sqrt{\frac{m_e T_p}{m_p T_e}} \left( 1 + \frac{Z_d e^2}{k_B T_p (1 + k_D R) R} \right) = \\ = \exp\left( -\frac{Z_d e^2}{k_B T_e (1 + k_D R) R} \right). \qquad (19)$$

It should be emphasized that equations (18) and (19) implicitly imply that the charge of dust particle is determined by the normal component of the electric field strength at its surface, which is incorrect from the viewpoint of plasma electrodynamics.

In case of effective interaction potential (6) that takes into account the influence of the plasma sheath on the interaction with the dust particle of finite size, the following equation holds:

$$\frac{n_p}{n_e}\sqrt{\frac{m_e T_p}{m_p T_e}} \left( 1 + \frac{Z_d e^2}{k_B T_p R} \left[ 1 - k_D R (1 + k_D R \exp(k_D R) \,\mathrm{Ei}(-k_D R)) \right] \right) = \\ = \exp\left( -\frac{Z_d e^2}{k_B T_e R} \left[ 1 - k_D R (1 + k_D R \exp(k_D R) \,\mathrm{Ei}(-k_D R)) \right] \right). \qquad (20)$$

Equation (20) correctly treats the boundary condition, which is imposed by the plasma electrodynamics and states that the charge of the dust particle is determined by the normal component of the electric displacement vector at its surface.

Figure 6 indicates the dependence of the dust particle charge, calculated according to formulas (18), (19) and (20), on the non-isothermality parameter. For all three cases, the charge of the dust particles increases with the growth of the non-isothermality parameter, since the mobility of electrons in the plasma considerably increases. It is also seen that the use of the Coulomb potential greatly underestimates the charge of the dust particle, because the screening phenomena significantly weaken the repulsion of electrons that must inevitably lead to an increase in the corresponding flux, and, hence, to a growth of the dust particle charge. On the other hand the use of the correct boundary condition within the plasma electrodynamics, as it is done at the derivation of interaction potential (6), results in a reduction of the dust grain charge compared with the case of straightforward application of the Yukawa potential (10). Figure 7 shows the dependence of the dust particle charge on the size parameter. Again in all three cases, the charge of the dust particle increases with its size since a floating plasma potential remains virtually unchanged. Note that the Coulomb interaction leads to lower values of the dust particle charge because the repulsion of electrons is dramatically weakened.

Analysis of the numerical results reveal that the discrepancy, due to the use of different potentials, manifests itself for the values of the size parameter $D \leq 20$. It is vastly understood in terms of the difference between the proposed screened potential and the micropotential at short interparticle separations, and strengthening of the screening reduces the developed potential at small distances thereby increasing the electron and decreasing the proton absorption cross sections.

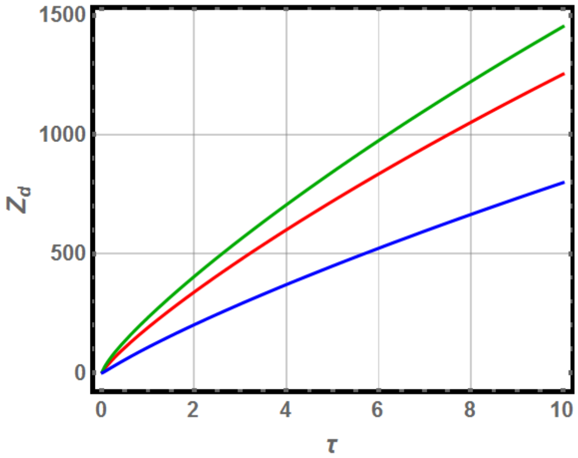

**Figure 6** – Dust grain charge $Z_d$ as a function of the non-isothermality parameter $\tau$ at $D = 10$, $\Gamma = 0.05$ and $\beta = 10^{-4}$. Red line: the formula (20); green line: formula (19); blue line: formula (18)

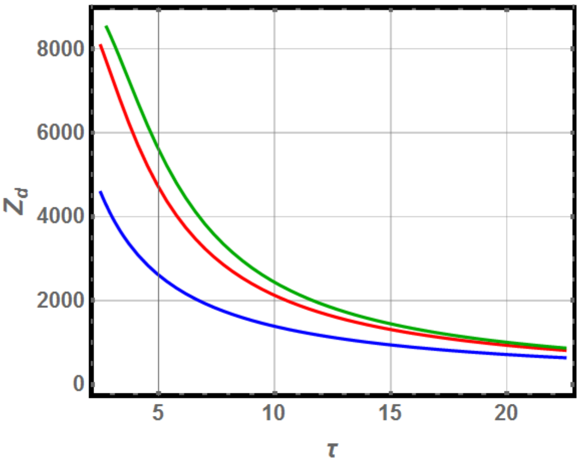

**Figure 7** – Dust grain charge $Z_d$ as a function of the size parameter $D$ at $\tau = 20$, $\Gamma = 0.05$ and $\beta = 10^{-4}$. Red line: formula (20); green line: formula (19); blue line: formula (18)

## Conclusions

On the basis of the investigations conducted it is possible to make the following conclusions:

1) With increasing the proton kinetic energy, the corresponding absorption cross section decreases, and it increases with the growth of the size parameter, which is explained by mutual attraction of protons and dust particles. It is also found that the absorption cross section of protons increases almost linearly with the increase of the dust particles charge.

2) Since electrons are repelled by the negatively charged dust particles, there exists a threshold kinetic energy at which the absorption becomes possible and the corresponding cross section turns non-zero. While the kinetic energy of electrons increases the absorption cross section drops, whereas it decreases with the growth of the size parameter. All these inferences are easily understood because dust particles repel electrons.

3) A general equation is derived to determine the charge of the dust particles within the OML approximation, which is valid for any kinf of the potential distribution around a dust grain.

4) The charge of the dust particles increases with the non-isothermality parameter, since the mobility of electrons in the plasma increases considerably. Using the Coulomb potential greatly underestimates the charge of the dust particles, because the screening is responsible for significant reduction in the repulsion of electrons that must inevitably lead to an increase of the corresponding flux, and, hence, to a decline of the dust particle charge. On the other hand the use of the strict boundary condition stemming from the plasma electrodynamics leads to a reduction of the dust graim charge compared with the case of direct use of the Yukawa potential. The charge of the dust particles increases with its size because a floating plasma potential remains virtually unchanged and numerical investigations demonstrate that the discrepancy between different potential model is observed for the size parameter $D \leq 20$. The latter is mostly due to the fact that the difference between the proposed screened potential and the micropotential manifests itself at short distances, and strengthening of the screening phenomena reduces the value of the screened potential at the origin, and, hence, increases the electron and decreases the proton absorption cross sections.

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