UDC 533.9

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Stopping power of non-ideal plasmas: the moment approach

Abstract. The moment approach to the description of dynamic properties of non-ideal plasmas takes into account convergent sum rules automatically and depends on the model of the Nevanlinna parameter function (NPF). A new model of the two-componet plasma NPF is suggested to satisfy the Perel'-Eliashberg high-frequency asymptote, to reproduce the static conductivity value, and the stopping power slow projectile asymptote. The coefficient of the latter is calculated in the extended random-phase approximation. The solution is reduced to that of a transcendental equation.
Key words: Stopping power, method of moments, non-ideal plasma, nevanlinna parameter

Introduction: hypothesis

The moment approach to the reconstruction of a plasma inverse dielectric function (IDF) $\varepsilon^{-1}(k,\omega)$ which satisfies three non-zero sum rules expresses it in terms of the Nevanlinna parameter function (NPF), $Q(k,\omega)$, as [1, 2]

$$\varepsilon^{-1}(k,\omega) = = 1+\frac{\omega_p^2\left(\omega+Q(k,\omega)\right)}{\omega\left(\omega^2-\omega_2^2(k)\right)+Q(k,\omega)\left(\omega^2-\omega_1^2(k)\right)}, \qquad (1)$$

where the characteristic frequencies, $\omega_2(k)$ and $\omega_1(k)$ are determined by three successive convergent non-zero power moments of the loss function, $\mathcal{L}(k,\omega) = -Im\varepsilon^{-1}(k,\omega)/\omega$ or the sum rules:

$$\omega_1(k)=\sqrt{\frac{C_2}{C_0(k)}}, \quad \omega_1(k)=\sqrt{\frac{C_4(k)}{C_2}},$$

$$C_\nu(k)=\frac{1}{\pi}\int_{-\infty}^{\infty}\omega^\nu \mathcal{L}(k,\omega)\,d\omega, \quad \nu=0,2,4.$$

Notice that odd-order moments vanish due to the parity of the integrand, higher even-order moments diverge [3].

The aim of the present research note is to specify the NPF for two-component plasmas (TCPs) so that we could simultaneously satisfy three limiting conditions:

1. the high-frequency Perel'-Eliashberg form for the plasma dielectric function [3];
2. the static conductivity value;
3. and the correct behavior of the plasma (polarizational) stopping power for slow projectiles together with the possibility to calculate the stopping power straggling [2].

Our hypothesis for the dimensionless NPF is the following:

$$\frac{Q(k,z)}{\omega_p}=\frac{A\omega_p\sqrt{\omega_p z}\,(1+i)}{\omega_2^2(k)-\omega_1^2(k)}+ +i\left(Ba^2k^2+\frac{\omega_p H}{\nu(z)+(1-i)\sqrt{\omega_p z}}\right). \qquad (2)$$

Here: H is the electron-ion correlation contribution to the fourth moment of the loss function:

$$A=3^{-5/4}Z^3 r_s^{3/4}\sqrt{2}\,; \qquad (3)$$

$$\nu(z)=\frac{\omega_p^2 H\nu}{\omega_p^2 H-iz\nu}, \qquad (4)$$

where is the static collision frequency such that the static conductivity

$$\sigma_0 = \sigma(0,0) = \frac{\omega_p^2}{4\pi\nu}.$$

The static collision frequency was calculated by [4]. As always, let us introduce the electronic Wigner-Seitz radius a, the Bohr radius, a_B, the system temperature in energy units, β^{-1}; then $\Gamma = \beta e^2 / a$ and $r_s = a / a_B$ are the coupling and degeneracy parameters, and $B > 0$ is the dimensionless parameter dependent on the plasma thermodynamic conditions selected so that the dimensionless polarizational stopping power of slow projectiles [2]

$$-\left(\frac{dE}{dx}\right)\beta a\Bigg|_{v\to 0} = \frac{2\left(Z_p e\right)^2 \beta a}{\pi v^2}\int_0^\infty \frac{dk}{k}\int_0^{kv}\omega^2 \mathcal{L}(k,\omega)\,d\omega\Bigg|_{v\to 0} \simeq C\frac{v}{v_F} \tag{5}$$

and the dimensionless straggling,

$$\Omega^2(v)\beta^2 a = \frac{2\left(Z_p \beta e\right)^2 a\hbar}{\pi v^2}\int_0^\infty \frac{dk}{k}\int_0^{kv}\omega^3 \mathcal{L}(k,\omega)\coth\frac{\beta\hbar\omega}{2}\,d\omega, \tag{6}$$

has a finite value. Here, v_F is the Fermi velocity and $Z_p e$ is the charge of the projectiles which traverse the target plasma with the velocity v.

The framework

The idea is, for a given set of thermodynamic parameters, to calculate the constant C in (5) in the extended random-phase approximation for a TCP and then solve the transcendental equation

$$\int_0^\infty \frac{a^3\omega_p^2 k^2\left[\omega_2^2(k)-\omega_1^2(k)\right]}{\left(Ba^2k^2+\dfrac{\omega_p H}{\nu}\right)\omega_1^4(k)}dk = \frac{3\pi\sqrt{3}}{2\sqrt[3]{\dfrac{9\pi}{4}}}\frac{C\sqrt{r_s}}{Z_p^2\Gamma}, \tag{7}$$

or

$$\int_0^\infty \frac{q^2\left[x_2^2(q)-x_1^2(q)\right]dq}{\left(Bq^2+\dfrac{\omega_p H}{\nu}\right)x_1^4(q)} = \frac{3\pi\sqrt{3}}{2\sqrt[3]{\dfrac{9\pi}{4}}}\frac{C\sqrt{r_s}}{Z_p^2\Gamma}, \tag{8}$$

stemming from (5) for small values of the projectile velocity since the zero-frequency value of the loss function is

$$\mathcal{L}(k,0) = \frac{\omega_p\left[\omega_2^2(k)-\omega_1^2(k)\right]}{\left(Ba^2k^2+\dfrac{\omega_p H}{\nu}\right)\omega_1^4(k)}. \tag{9}$$

In addition, in (8) we have introduced dimensionless variables:

$$x = \frac{\omega}{\omega_p}, \quad x_1 = \frac{\omega_1}{\omega_p}, \quad x_2 = \frac{\omega_2}{\omega_p}, \quad q = ak.$$

The convergence of the integral on the l.h.s. of (7) and (8), can be guaranteed if we take into account the quantum-mechanical nature of the process and write the characteristic frequency $\omega_1(k)$ as in [5]:

$$\omega_1(q) = \omega_p\frac{\sqrt{q^4\kappa^2+q^2\varkappa^4+\kappa^2\varkappa^4}}{\kappa\varkappa^2} \underset{q\to\infty}{\simeq} q^2, \tag{10}$$

where, in a hydrogen-like plasma with

$$n_e = Zn_i,$$

$$\kappa^2 = 4\pi e^2 \beta a^2 \left(n_e + Z n_i\right),$$

$$\varkappa^4 = \frac{16\pi e^2}{\hbar^2} a^4 \left(n_e m_e + Z^2 n_i m_i\right) \approx$$

$$\approx \frac{16\pi Z e^2 n_e a^4 m_i}{\hbar^2} = 12 r_s \frac{m_i}{m_e}.$$

Then, in the short-wavelength approximation,

$$\frac{q^2}{Bq^2 + \frac{\omega_p H}{\nu}} \left(\frac{x_2^2(q)}{x_1^4(q)} - \frac{1}{x_1^2(q)} \right) \underset{q\to\infty}{\simeq} \frac{\alpha}{q^4}$$

and the integrals with B converge without any additional efforts applied.

The stopping of slow projectiles

As it was already mentioned, we precalculate the dimensionless constant C in the extended RPA and for hydrogen-like plasmas. Then the system dielectric function can be written as

$$\varepsilon\left(k,\omega\right) = 1 + \frac{3\Gamma}{q^2} \left(\frac{\Pi_e(k,\omega)}{\beta n_e} + \frac{Z \Pi_i(k,\omega)}{\beta n_i} \right), \quad (11)$$

where $\Pi_e(k,\omega)$ and $\Pi_i(k,\omega)$ are the subsystems' partial polarization operators. We presume that it suffices to introduce the local field correction $G(k)$ in the electronic component only:

$$\frac{\Pi_e(q,\omega)}{\beta n_e} = \frac{\left(\frac{\Pi_{e0}(q,\omega)}{\beta n_e}\right)}{1 - \frac{3\Gamma}{q^2} G(q) \left(\frac{\Pi_{e0}(q,\omega)}{\beta n_e}\right)}.$$

Expressions for $\Pi_{e0}(k,\omega)$ and $G(k)$ were provided in [2], while here we can simply put

$$\frac{\Pi_i(q,\omega)}{\beta n_i} = \frac{\Pi_{i0}(q,\omega)}{\beta n_i} = \frac{\Pi_{i0}(q,0)}{\beta n_i} = 1,$$

since for the present problem we need only the static value of the polarization operators and their imaginary parts, which vanish at $\omega = 0$.

Let

$$\frac{\Pi_{e0}\left(k,\omega\right)}{\beta n_e} = X\left(q,x\right) + iY\left(q,x\right),$$

where, then

$$X\left(q,0\right) = \frac{3 r_s}{q\Gamma} \left(\frac{9\pi}{4}\right)^{-\frac{1}{3}} \int_0^\infty \frac{y\,dy}{\exp\left(D y^2 - \eta\right) + 1} \ln \left| \frac{q + 2\left(\sqrt[3]{\frac{9\pi}{4}}\right) y}{q - 2\left(\sqrt[3]{\frac{9\pi}{4}}\right) y} \right|,$$

$$Y\left(q, x \to 0\right) = \frac{2x}{\sqrt{3}} \frac{r_s^{\frac{3}{2}}}{\Gamma q} \frac{e^{\eta - \frac{\Gamma q^2}{8 r_s}}}{e^{\eta - \frac{\Gamma q^2}{8 r_s}} + 1}.$$

We have now that

$$C_{\text{xRPA}} = \frac{4 Z_p^2}{3\pi} \left(\Gamma r_s\right) \left(\sqrt[3]{\frac{9\pi}{4}}\right) \int_0^\infty \frac{\left(1 + \exp\left(\frac{\Gamma q^2}{8 r_s} - \eta\right)\right)^{-1} \frac{dq}{q}}{\left(1 + \frac{3\Gamma X}{q^2}\left(1 + Z - G\right) - \left(\frac{3\Gamma X}{q^2}\right)^2 Z G\right)^2}. \quad (12)$$

Finally, we arrive to the following equation for the parameter B:

$$\int_0^\infty \frac{q^2\left[x_2^2(q)-x_1^2(q)\right]dq}{\left(Bq^2+\frac{\omega_p H}{\nu}\right)x_1^4(q)} = \int_0^\infty \frac{2\sqrt{3}r_s^{\frac{3}{2}}\left[1+\exp\left(\frac{\Gamma q^2}{8r_s}-\eta\right)\right]^{-1}\frac{dq}{q}}{\left[1+\frac{3\Gamma X}{q^2}(1+Z-G)-\left(\frac{3\Gamma X}{q^2}\right)^2 ZG\right]^2}, \quad (13)$$

where

$$X = X(q,0), \quad G = G(q), \quad Z = 1$$

in hydrogen, and, as usually, the electronic dimensionless chemical potential $\eta = \beta\mu$ is defined by the normalization equation:

$$F_{1/2}(\eta) = \frac{2}{3}D^{3/2}$$

with

$$F_\nu = \int_0^\infty \frac{x^\nu dx}{\exp(x-\eta)+1},$$

$$D = \beta E_F = \beta\hbar^2 k_F^2 / 2m = \beta\hbar^2\left(3\pi^2 n\right)^{2/3} / 2m,$$

where $F_\nu(\eta)$, E_F, and k_F are the ν-th order Fermi integral, Fermi energy, and wavenumber, respectively.

Numerical results

We have calculated the coefficient C, the constant B (from (8) or (13) with (10)), computed the corresponding stopping and straggling with the NPF from (2), studied the asymptotic (at low velocities, $v/v_{th} \lesssim 0.1$, $v_{th} = \sqrt{\beta^{-1}m}$) behavior of the stopping power. The results are presented on the following figures. The number density of the electronic subsystem of the target plasma has been chosen to be equal to $n_e = 10^{23}\ cm^{-3}$ so that $r_s = 2.5256$. The high-velocity asymptotic form of the dimensionless stopping power taking into account the target plasma electron-ion interaction [6],

$$-\left(\frac{dE}{dx}\right)\beta a\Big|_{v\to\infty} = \left(\frac{Z_p e\omega_p}{v^2}\right)^2 \beta a \ln\frac{2mv^2}{\hbar\omega_p\sqrt{1+H}},$$

is represented by continuous lines. Protons were chosen as projectiles.

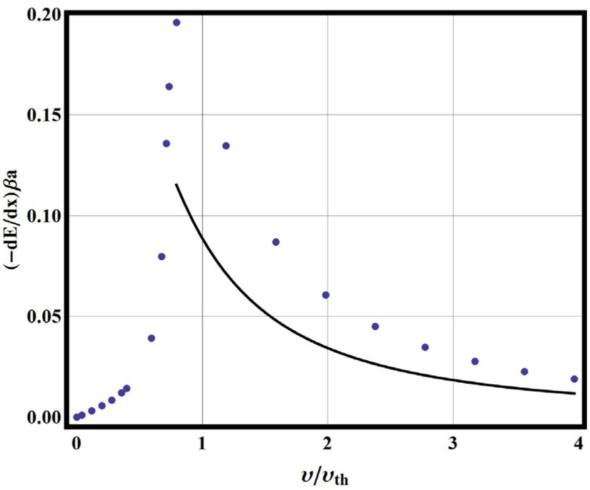

Figure 1a – The TCP stopping power for Γ = 0.1077

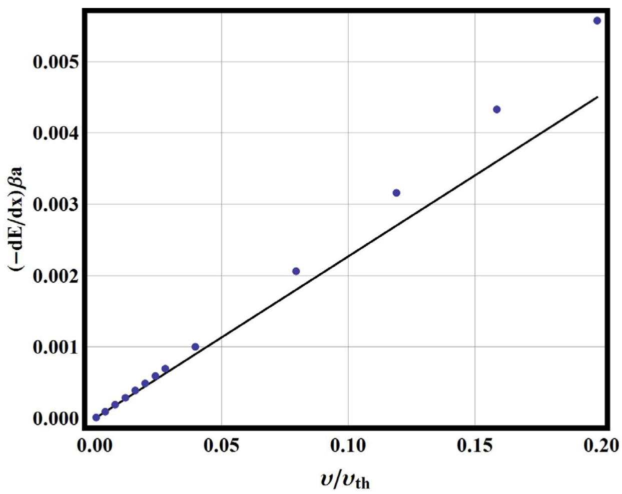

Figure 1b – The TCP stopping power for Γ = 0.1077, slow projectiles

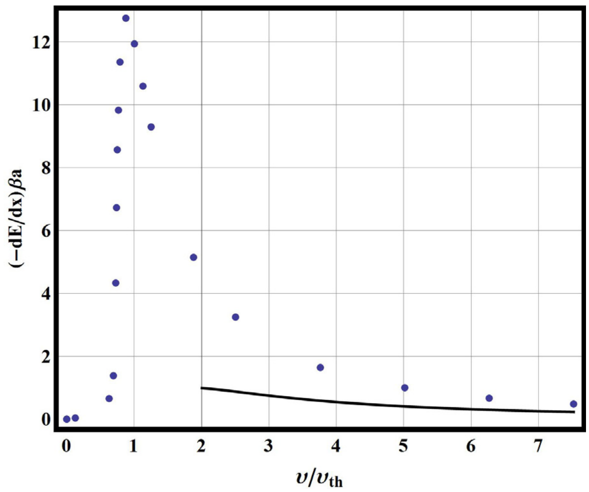

Figure 2a – As in Fig. 1a, but for Γ = 1.077

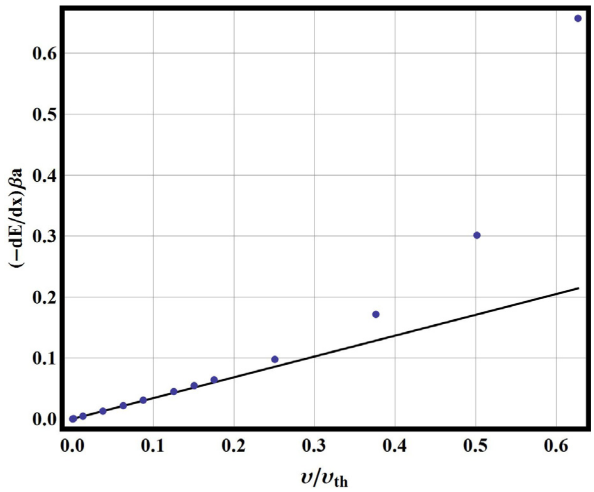

Figure 2b – As in Fig. 1b, but for Γ = 1. 077

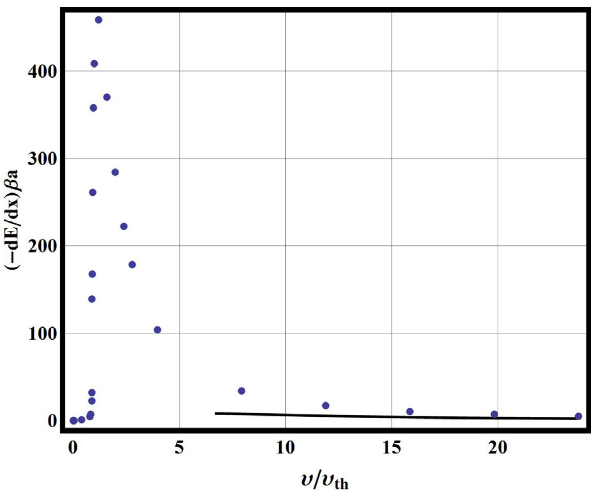

Figure 3a – As in Fig. 1a, but for Γ = 10.77

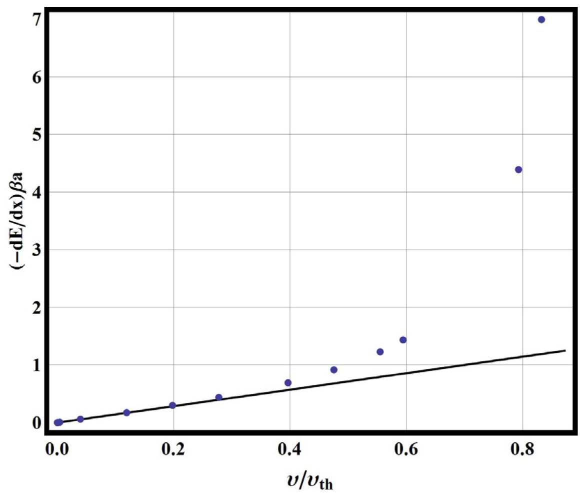

Figure 3b – As in Fig. 1b, but for Γ = 10.77

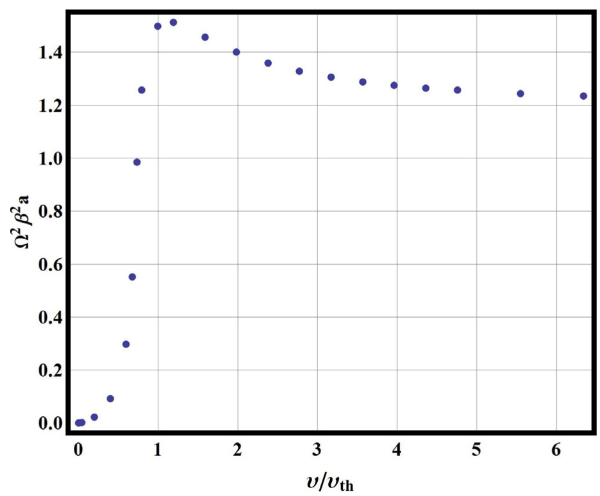

Figure 4a – The TCP straggling for Γ = 0.1077

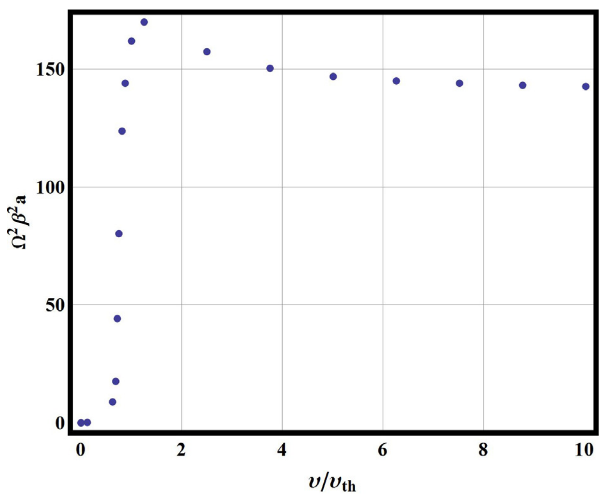

Figure 4b – As in Fig. 4a but for Γ = 0.1077

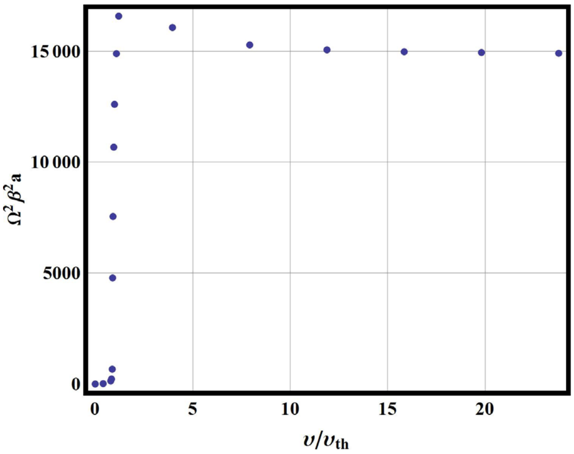

Figure 4c – As in Fig. 4a but for Γ = 10.77

The values of the parameters C and B are provided in the Table below for the reference.

Γ = 0.1077	Γ = 1.077	Γ = 10.77
C = 0.00774	C = 0.42848	C = 5.6685
B = 9.56152	B = 39.511	B = 491.671

Conclusions

We observe a significant enhancement of both stopping power and straggling of non-ideal or medium-coupled TCPs. This effect is to be confirmed experimentally. On the other hand, the form of the NPF is to be specified further in an attempt to describe simultaneously other dynamic characteristics, like the dynamic structure factor and mode dispersion [7].

Acknowledgements

Work supported by the Republic of Kazakhstan Ministry of Education and Science grant No. 3119/6GF4, 3831/GF4 and by the Spanish Ministerio de Economía, Ciencia e Innovación Project ESP2013-41078-R. A. A. expresses gratitude for the financial support provided by the grant "The Best Lecturer".

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