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On the study of Rigid Body Inertia

Abstract. In the classical finite element literature beams and plates are not considered as isoparametric elements since infinitesimal rotations are used as nodal coordinates. As a consequence, exact modeling of an arbitrary rigid body displacement cannot be obtained, and rigid body motion does not lead to zero strain. In order to circumvent this problem in flexible multibody simulations, an intermediate element coordinate system, which has an origin rigidly attached to the origin of the deformable body coordinate system and has axes which are parallel to the axes of the element coordinate system in the undeformed configuration was introduced. The correct equations of motion, however, can be obtained if the coordinates are defined in terms of global slopes. Using this new definition of the element coordinates, an absolute nodal coordinate formulation that leads to a constant mass matrix for the element can be developed. Using this formulation, in which no infinitesimal or finite rotations are used as nodal coordinates, beam and plate elements can be treated as isoparametric elements.

Key words: rigid body, inertia, finite, element method, multibody systems, dynamics.

Introduction

With regard to the dynamics of constrained deformable bodies that undergo large rigid body rotations, there are three well known solution procedures which are briefly summarized below:

(1) Floating Frame of Reference. In this approach, a coordinate system is assigned to each deformable body in the multibody system. The configuration of the deformable body is identified using two sets of coordinates that define the location and orientation of the deformable body coordinate system, as well as the deformation of the body with respect to its coordinate system. Cartesian Coordinate formulations as well as recursive methods are often used with the floating frame of reference approach.

(2) Incremental Finite Element Approach. In this approach, the nodal coordinates of the finite elements are used to describe incrementaly the large rigid body rotations of the elements. The equations of motion are formulated in terms of the nodal coordinates only and a convected element coordinate system is used to define the current element configuration.

(3) Large Rotation Vector. This approach was introduced recently in order to avoid the linearization that results from the use of the incremental finite element approach. In this approach, the element rotations are described using a finite rotation vector defined at the element nodal.

In the large deformation analysis, rate constitutive equations must correctly represent the relationship between the stress rate and the arbitrary rigid body translation and rotation. The displacement increment over the step was defined and the gradient of this displacement was used to define the strain and rotation tensors, which are, in turn, used to define the algorithm for integrating the constitutive equations. Flanagan and Taylor presented a numerical algorithm for the integration of constitutive equations under both large deformations and/or large rotations. The examples showed the relative efficiency of the unrotated configuration, and the computational efficiency and accuracy of the numerical algorithms in dealing with large rotation problems and its insensitivity to orientation.

Several numerical integration algorithms were proposed in order to improve the computational efficiency and accuracy of the solution for dynamic structural systems. In some of these numerical algorithms, criteria were introduced in order to preserve some basic rigid body quantities such as the linear and angular moment and the kinetic energy. These criteria ensure that these basic quantities are not compromised during the process of the numerical integration provided that the exact rigid body equations of motion including the exact mass moments and products of inertia are used. Using this hypothesis, it is assumed that the error in the solution is mainly the result of the numerical integration. As demonstrated in this paper and in previous publications, some of the commonly used shape functions and the associated nodal coordinates can not be used to define the exact rigid body equations of motion, and therefore, it becomes necessary to quantify the errors in the basic dynamic equations prior to investigating the accuracy of the numerical integration methods.

In this formulation, no infinitesimal or finite rotations are used as nodal coordinates, instead, absolute displacements and slopes are used to define the element configuration in the global coordinate system. The absolute slopes can be determined in the undeformed reference configuration using spatial rigid body kinematic equations.

Finite Element and Rigid Body Inertia

In the classical finite element literature, beams and plates are not considered as isoparametric elements. The use of the infinitesimal rotations as nodal coordinates leads to linearized kinematic equations which do not describe exact rigid body motion. In order to utilize existing finite element methodologies and computer programs, the concept of the intermediate element coordinate system was introduced in order to obtain an exact modeling of the rigid body inertia using the conventional finite element shape functions.

Since the conventional element shape functions contain rigid body modes that describe arbitrary translations, the exact location of an arbitrary point on the beam, in an intermediate coordinate system which differs from the element coordinate system by a translation in the undeformed configuration, can be defined using this element shape function. As a consequence, the exact rigid body mass moments and products of inertia as well as the moments of mass can be evaluated using the element shape function and the vector of element nodal coordinates.

Rigid Body Inertia. In the three dimensional analysis, the inertia forces of the rigid body are defined in terms of the inertia tensor and the moments of mass. The rigid body inertia tensor for a spatial system is defined as

$$\overline{I}_{\theta\theta} = \int_V \rho \tilde{\overline{u}}^T \tilde{\overline{u}} dV \tag{1}$$

where the superscript T indicates a transpose of a vector or a matrix, is the inertia tensor defined in the body coordinate system, $\boldsymbol{\rho}$ is the mass density, $\boldsymbol{V}$ is the volume and $\tilde{\overline{u}}$ is the skew symmetric matrix associated with the vector $\overline{u}$ that defines the local position of an arbitrary point on the body. The skew symmetric matrix $\tilde{\overline{u}}$ can be written as

$$\tilde{\overline{u}} = \begin{bmatrix} 0 & -\overline{u}_3 & \overline{u}_2 \\ \overline{u}_3 & 0 & -\overline{u}_1 \\ -\overline{u}_2 & \overline{u}_1 & 0 \end{bmatrix} \tag{2}$$

where $\overline{u}_1$, $\overline{u}_2$ and $\overline{u}_3$ are the components of the vector $\overline{u}$, that is

$$\overline{u} = [\overline{u}_1 \quad \overline{u}_2 \quad \overline{u}_3]^T$$

The inertia tensor for the rigid body can be written more explicitly as

$$\overline{I}_{\theta\theta} = \begin{bmatrix} i_{11} & i_{12} & i_{13} \\ & i_{22} & i_{23} \\ symmetric & & i_{33} \end{bmatrix} = \int_V \rho \begin{bmatrix} \overline{u}_2^{\,2} + \overline{u}_3^{\,2} & -\overline{u}_2\overline{u}_1 & -\overline{u}_3\overline{u}_1 \\ & \overline{u}_1^{\,2} + \overline{u}_3^{\,2} & -\overline{u}_3\overline{u}_2 \\ symmetric & & \overline{u}_1^{\,2} + \overline{u}_2^{\,2} \end{bmatrix} dV \tag{3}$$

In the case of a slender beam element, the vector u can be written as

$$\overline{u} = [x \quad 0 \quad 0]^T$$

where $\boldsymbol{x}$ is the position of the arbitrary point from the endpoint which defines the origin of the beam coordinate system .

Using the vector $\bar{u}$ the inertia tensor of the slender beam element can be obtained as

$$\bar{I}_{\theta\theta} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{ml^2}{3} & 0 \\ 0 & 0 & \frac{ml^2}{3} \end{bmatrix}$$

where m is the mass of the beam element and l is its length.

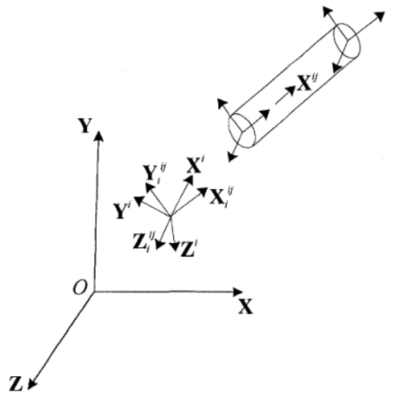

Figure – Three dimensional beam element

The moment of mass of the beam is defined as

$$M_0 = \int_V \rho \bar{u}\, dV$$

Using the vector u of the slender beam, one can show that the moment of mass is defined as

$$M_0 = \begin{bmatrix} \frac{ml}{2} \\ 0 \\ 0 \end{bmatrix}$$

Shape Function. If the shape function can be used to describe an arbitrary rigid body translation, the element nodal coordinates and the shape function can be used to define the location of an arbitrary point on the element with respect to the element coordinate system. In this case, the vector $\bar{u}$ can be written as

$$\bar{u} = Se$$

where $\boldsymbol{S}$ is the shape function matrix of the element and $\boldsymbol{e}$ is the vector of the element nodal coordinates. If the effect of rotary inertia is neglected, the shape function $\boldsymbol{S}$ can be defined in the case of three dimensional beam element as

$$S^T = \begin{bmatrix} 1-\xi & 0 & 0 \\ 0 & 1-3\xi^2+2\xi^3 & 0 \\ 0 & 0 & 1-3\xi^2+2\xi^3 \\ 0 & 0 & 0 \\ 0 & 0 & l\left(-\xi+2\xi^2-\xi^3\right) \\ 0 & l\left(\xi-2\xi^2+\xi^3\right) & 0 \\ \xi & 0 & 0 \\ 0 & 3\xi^2-2\xi^3 & 0 \\ 0 & 0 & 3\xi^2-2\xi^3 \\ 0 & 0 & 0 \\ 0 & 0 & l\left(\xi^2-\xi^3\right) \\ 0 & l\left(\xi^3-\xi^2\right) & 0 \end{bmatrix} \quad (4)$$

where ξ = x/l, and l is the length of the beam element. If the beam element is considered as a rigid body, then the vector of nodal coordinates defined in the element coordinate system is given by

$$e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & l & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

In this case one has

$$\bar{u} = Se = \begin{bmatrix} x \\ 0 \\ 0 \end{bmatrix}$$

and as a consequence, the mass moments of inertia of the rigid beam are defined in terms of the element shape function as

$$\left.\begin{aligned} i_{11} &= e^T\left(S_{22}+S_{33}\right)e = 0 \\ i_{22} &= e^T\left(S_{11}+S_{33}\right)e = \frac{ml^2}{3} \\ i_{33} &= e^T\left(S_{11}+S_{22}\right)e = \frac{ml^2}{3} \end{aligned}\right\} \quad (5)$$

and the products of inertia are

$$i_{jk} = -e^T S_{jk} e = 0$$

$$j,k = 1,2,3 \qquad j \neq k$$

where

$$S_{jk} = \int_V \rho S_j^T S_k dV$$

and S_k is the kth row in the element shape function. Further-more, the moment of mass is defined as

$$M_0 = \int_V \rho SedV = \bar{S}e = \begin{bmatrix} \frac{ml}{2} \\ 0 \\ 0 \end{bmatrix}$$

where

$$\bar{S} = \int_V \rho SdV$$

In the case of the beam element shape function, the matrix S is given by

$$\bar{S} = \begin{bmatrix} \frac{m}{2} & 0 & 0 & 0 & 0 & 0 & \frac{m}{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{m}{2} & 0 & 0 & 0 & \frac{ml}{12} & 0 & \frac{m}{2} & 0 & 0 & 0 & -\frac{ml}{12} \\ 0 & 0 & \frac{m}{2} & 0 & -\frac{ml}{12} & 0 & 0 & 0 & \frac{m}{2} & 0 & \frac{ml}{12} & 0 \end{bmatrix}^T \tag{6}$$

It is clear from the analysis presented in this section that all the rigid body inertia quantities can be evaluated using the element shape integrals $\bar{S}$, and S_{jk}.

Intermediate Element Coordinate System

The intermediate element coordinate system shown in Fig. 1 is introduced in flexible multibody simulation in order to obtain exact modeling of the rigid body inertia when the structures undergo arbitrary large rotations. In the initial undeformed configuration, this intermediate coordinate system differs from the element coordinate system by a rigid body translation. The results of the parallel axis theorem, often used in rigid body dynamics, can be obtained, using the element shape function and the intermediate element coordinate system, by utilizing the fact that the position coordinates of an arbitrary point on the finite element can be defined in the intermediate coordinate system using the element nodal coordinates. Before we demonstrate this fact in the general case of three dimensional displacement, we consider the case in which the. origin of the intermediate element coordinate system is located at the center of mass C of the element. In this case, the vector of coordinates defined in the intermediate element coordinate system is

$$e_C = \begin{bmatrix} -\frac{l}{2} & 0 & 0 & 0 & 0 & 0 & \frac{l}{2} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

Using this vector of nodal coordinates, the mass moments of inertia can be calculated as

$$\left.\begin{aligned} i_{11} &= e_C^T (S_{22} + S_{33}) e_C = 0 \\ i_{22} &= e_C^T (S_{11} + S_{33}) e_C = \frac{ml^2}{12} \\ i_{33} &= e_C^T (S_{11} + S_{22}) e_C = \frac{ml^2}{12} \end{aligned}\right\} \tag{7}$$

and the products of inertia are

$$i_{jk} = -e_C^T S_{jk} e = 0 \quad j,k = 1,2,3 \quad j \neq k$$

where the matrices S_{jk} are defined in the previous section. The use of equations shows that the moment of mass about the center of mass is

$$M_C = \bar{S} e_C = 0$$

where the matrix $\bar{S}$ in the case of the three dimensional beam element is defined.

General Translations. If the element coordinate system differs from the intermediate element coordinate system by a general three dimensional displacement defined by the vector

$$D = \begin{bmatrix} d_x & d_y & d_z \end{bmatrix}^T \tag{8}$$

the vector of nodal coordinates defined in the intermediate element coordinate system is given by

$$e_I = \begin{bmatrix} d_x & d_y & d_z & 0 & 0 & 0 & d_x + l & d_y & d_z & 0 & 0 & 0 \end{bmatrix}^T$$

In this case, the mass moments and products of inertia can be evaluated using the element shape function **S** and the vector of nodal coordinates as

$$\left.\begin{aligned} i_{11} &= e_I^T \left(S_{22} + S_{33}\right) e_I = m\left(d_y^2 + d_z^2\right) \\ i_{22} &= e_I^T \left(S_{11} + S_{33}\right) e_I = \frac{ml^2}{3} + m\left(d_x(d_x + l) + d_z^2\right) \\ i_{33} &= e_I^T \left(S_{11} + S_{22}\right) e_I = \frac{ml^2}{3} + m\left(d_x(d_x + l) + d_y^2\right) \\ i_{12} &= -e_I^T S_{12} e_I = -d_y\left(md_x + \frac{1}{2}ml\right) \\ i_{13} &= -e_I^T S_{13} e_I = -d_z\left(md_x + \frac{1}{2}ml\right) \\ i_{23} &= -e_I^T S_{23} e_I = -md_y d_z \end{aligned}\right\} \tag{9}$$

which are the exact expressions for the mass moments and products of inertia that can be obtained using the parallel axis theorem used in rigid body dynamics.

The moment of mass is defined in the intermediate element coordinate system as

$$M_I = \bar{S} e_I = \begin{bmatrix} m\left(\dfrac{l}{2} + d_x\right) \\ md_y \\ md_z \end{bmatrix}$$

The analysis presented in this section demonstrates that exact modeling of the basic inertia quantities can be obtained by using the intermediate element coordinate system, and as a consequence, the use of this coordinate system in the nonlinear flexible multibody formulation does lead to the exact equations of motion of the spatial rigid body.

Examples

In this section, we present three different forms of the equations of motion of the beam obtained using the three different approaches discussed in the preceding sections. The first form represents the exact equations of motion of the beam, the second form is obtained using the convected system approach, while the third form is obtained using the early linearization scheme. In the three cases, for simplicity, we consider a beam which has an arbitrary rigid body translation, and it is allowed to rotate about its **Z** axis.

Exact Equations. The exact equations of motion of the system are

$$\begin{bmatrix} m & 0 & 0 & -\frac{1}{2}ml\sin\theta \\ 0 & m & 0 & \frac{1}{2}ml\cos\theta \\ 0 & 0 & m & 0 \\ -\frac{1}{2}ml\sin\theta & \frac{1}{2}ml\cos\theta & 0 & \frac{1}{3}ml^2 \end{bmatrix} \cdot \begin{bmatrix} \ddot{R}_x \\ \ddot{R}_y \\ \ddot{R}_z \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} Q_{ex} \\ Q_{ey} \\ Q_{ez} \\ Q_{e\theta} \end{bmatrix} + \begin{bmatrix} Q_{vx} \\ Q_{vy} \\ Q_{vz} \\ Q_{v\theta} \end{bmatrix} \tag{10}$$

where R_x, R_y, R_z, are the translational coordinates of the reference point of the beam, в is the angle that defines the beam orientation, Q_{ex}, Q_{ey}, Q_{ez}, and $Q_{e\theta}$ are the components of the vector of generalized external forces and Q_{vx}, Q_{vy}, Q_{vz} and $Q_{v\theta}$ are the components of the vector of centrifugal forces. The components of the vector of centrifugal forces are

$$Q_v = \begin{bmatrix} \frac{1}{2} ml\dot{\theta}^2 \cos\theta \\ \frac{1}{2} ml\dot{\theta}^2 \sin\theta \\ 0 \\ 0 \end{bmatrix} \quad (11)$$

Convective System. For the beam model used in this section, we define the system generalized coordinate vector as

$$q = \begin{bmatrix} R_x & R_y & R_z & \theta \end{bmatrix}^T \quad (12)$$

In terms of the components of this vector, the vector of nodal coordinates, in the case of the convective system, can be written as

$$e = \begin{bmatrix} R_x & R_y & R_z & 0 & 0 & 0 & R_x + l\cos\theta & R_x + l\cos\theta & R_z & 0 & 0 & 0 \end{bmatrix}^T$$

and the velocity vector is

$$\dot{e} = B\dot{q} \quad (13)$$

where B is the velocity influence coefficient matrix defined as

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -l\sin\theta & l\cos\theta & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad (14)$$

The kinetic energy of the beam is

$$T = \frac{1}{2}\int_V \rho \dot{\bar{u}}^T \dot{\bar{u}} dV = \frac{1}{2}\dot{q}^T \mathrm{M}\dot{q} \quad (15)$$

where

$$\overline{u} = Se, \quad \mathrm{M} = \mathrm{B}^T \mathrm{M}_{ff} \mathrm{B},$$

$$\mathrm{M}_{ff} = \int_V \rho S^T S dV \quad (16)$$

Using Lagrange's equation, the equations of motion of the beam are

$$\begin{bmatrix} m & 0 & 0 & -\frac{1}{2} ml\sin\theta \\ 0 & m & 0 & \frac{1}{2} ml\cos\theta \\ 0 & 0 & m & 0 \\ -\frac{1}{2} ml\sin\theta & \frac{1}{2} ml\cos\theta & 0 & \frac{1}{3} ml^2 \left(1 + \frac{4}{35}\cos^2\theta\right) \end{bmatrix} \cdot \begin{bmatrix} \ddot{R}_x \\ \ddot{R}_y \\ \ddot{R}_z \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} Q_{ex} \\ Q_{ey} \\ Q_{ez} \\ Q_{e\theta} \end{bmatrix} + \begin{bmatrix} Q_{vx} \\ Q_{vy} \\ Q_{vz} \\ Q_{v\theta} \end{bmatrix} \quad (17)$$

and in the case of connected coordinates, the vector of centrifugal forces is written as

$$Q_v = \begin{bmatrix} \frac{1}{2} ml\dot{\theta}^2 \cos\theta \\ \frac{1}{2} ml\dot{\theta}^2 \sin\theta \\ 0 \\ -\frac{1}{2} m\dot{\theta}^2 \left(\frac{4l^2}{105} \sin 2\theta \right) \end{bmatrix} \tag{18}$$

The errors in these equations are readily seen by comparing with the exact equations previously presented in this section.

Early Linearization. In the case of a simple rotation в about the Z axis, the global position vector of an arbitrary point on the beam element can be written as

$$u = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{bmatrix} x\cos\theta \\ x\sin\theta \\ 0 \end{bmatrix} \tag{19}$$

The slope in this case is defined as

$$\frac{\partial u_2}{\partial x} = \sin\theta$$

Using this as definition of the slope in the vector of nodal coordinates, one has

$$e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \sin\theta & l\cos\theta & l\sin\theta & 0 & 0 & 0 & \sin\theta \end{bmatrix}^T$$

It can be demonstrated that the use of this vector and the element shape function leads to the exact rigid body inertia quantities and the exact equations of motion presented previously in this section. If the vector e, on the other hand, is linearized, one has

$$e = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \theta & l & l\theta & 0 & 0 & 0 & \theta \end{bmatrix}^T$$

In order to see the error that might result in the equations of motion from the use of such an early linearization, we consider the case of a more general displacement as defined by the vector of generalized coordinates $\boldsymbol{q}$. In this case, the vector e is given by

$$e = \begin{bmatrix} R_x & R_y & R_z & 0 & 0 & \theta & R_x + l & R_y + l\theta & R_z & 0 & 0 & \theta \end{bmatrix}^T$$

and the time derivative of this vector is

$$\dot{e} = \mathrm{B}\dot{q}$$

where the vector $\boldsymbol{q}$ is defined , and the matrix B in this case of early linearization is

$$\mathrm{B} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & l & 0 & 0 & 0 & 1 \end{bmatrix}^T \tag{20}$$

The kinetic energy of the beam is

$$T = \frac{1}{2}\int_V \rho \dot{\bar{u}}^T \dot{\bar{u}} dV = \frac{1}{2}\dot{q}^T \mathrm{M}\dot{q} \tag{21}$$

where M is the mass matrix of the rigid beam defined as

$$\mathrm{M} = \mathrm{B}^T \mathrm{M}_{ff} \mathrm{B} \tag{22}$$

and M_{ff} is as defined in equation. The use of Lagrange's Equation leads to the following matrix equation of motion in the case of early linearization.

$$\begin{bmatrix} m & 0 & 0 & 0 \\ 0 & m & 0 & \frac{1}{2}ml \\ 0 & 0 & m & 0 \\ 0 & \frac{1}{2}ml & 0 & \frac{1}{3}ml^2 \end{bmatrix} \cdot \begin{bmatrix} \ddot{R}_x \\ \ddot{R}_y \\ \ddot{R}_z \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} Q_{ex} \\ Q_{ey} \\ Q_{ez} \\ Q_{e\theta} \end{bmatrix} + \begin{bmatrix} Q_{vx} \\ Q_{vy} \\ Q_{vz} \\ Q_{v\theta} \end{bmatrix} \tag{23}$$

where the vector Q_v is defined as

$$Q_v = \begin{bmatrix} Q_{vx} \\ Q_{vy} \\ Q_{vz} \\ Q_{v\theta} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{24}$$

Note that in the case of early linearization, the vector Q of centrifugal forces is identically equal to zero. Observe also the error in the definition of the moments of mass in the mass matrix.

Conclusion

In this paper, two conceptually different finite element methods that lead to exact modeling of the spatial rigid body inertia of beams are discussed. The first method is used when infinitesimal rotations are used as nodal coordinates for the finite element. This method allows the use of the classical finite element formulations in the flexible multibody simulations. In the second method, absolute displacements and slopes are used as nodal coordinates, instead of using infinitesimal rotations. This method has a potential in solving large deformation problems in varieties of flexible multibody applications.

In the flexible multibody formulations of elements that have infinitesimal rotations as coordinates, an intermediate element coordinate system is introduced. This coordinate system does not follow the element deformation, and is used only to define the locations of the nodes in the undeformed state, thus preserving the exactness of the rigid body inertia. This coordinate system is rigidly attached to the structure (not the element) coordinate system. The position of an arbitrary point on the element can be defined in the element coordinate system as $u = S(e_0 + e_f)$, where S is the element shape function, e_0 is the vector of nodal locations in the undeformed state (This is not the vector of rigid body displacements), and e_f the vector of nodal displacements defined in the intermediate element coordinate system. The preceding equation can be used to develop a nonlinear formulation that leads to an exact model for the spatial rigid body inertia.

The concept of the intermediate element coordinate system has been successfully used in the analysis of small deformations in many flexible multibody applications. The limitations of this approach in the analysis of large deformations stem from the fact that infinitesimal rotations are used as nodal coordinates. In this case beam elements are not considered as isoparametric elements. It is demonstrated in this paper that beam elements can be considered as isoparametric elements if absolute slopes instead of infinitesimal rotations, are used as nodal coordinates.

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