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Numerical modeling of the spreading oil slick in the Caspian Sea

Abstract. This paper considers the process of filling and migration of fossil fuels - oil - in the Caspian Sea, depending on the weather conditions. As a rule, for environmentalists performing analysis of the emergency, it is necessary to know in which direction will move the oil slick, how to change the composition of the oil, how much oil will evaporate and a number of other questions that can be answered using mathematical modeling. The theoretical basis of a mathematical model for the process consists of the hydrodynamics equations of the sea, equations of transfer and changes in the concentration of the oil component, as well as the continuity equation is taken into account, expressing the an invariance of the total amount of mass of poured oil, which have formed a film on the surface of the sea. As a result, the migration trajectory, the change in concentration and temperature of the oil slick on the sea surface from a fixed source, depending on weather conditions are defined. This model allows to calculate the area and determine the migration trajectory of the oil slick on the Caspian Sea depending on the weather conditions, and carry out the numerical simulation of oil spills in the open sea at different initial masses of spilled oil.

Key words: oil spill, the Caspian Sea, oil slick, marine environment, weather conditions, mathematical modeling, oil migration, concentration of oil, numerical simulation.

Development and mastering of oil fields on the Caspian shelf represent the production of the increased risk of environmental pollution for the Caspian Sea and the environment of the surrounding areas. Any high-tech production is not insured from accidents, and in case of emergencies, the consequences can cause irreversible processes in the environment [1-4].

The real concern for the condition of the ecological environment is the multinational oil companies in relation to the protection of the environment and the sea. The program of the companies stated that in the event of a spill of a small amount of oil in the open sea, the oil will evaporate, and the case of a significant oil spill in the open sea it will be set on fire and the event of a disaster will be delivered from the UK special equipment for collecting oil, and even indicated delivery time after the accident - 20-24 hours.

According to preliminary calculations, this time is enough, with a fair wind to oil was in the coastal zone in the rushes, where it is not possible to collect [5, 6]. This carefree attitude of oil companies to environmental causes, at least, puzzling.

So currently requires new approaches to tackle the problem. This new direction is the development of methods and algorithms for the direct numerical simulation of Aero-and hydrodynamics to environmental problems when ahead in unsteady mode to follow the development of events, actually occurring in nature. Since the research is conducted on the up-to-date computer technologies and high speed of modern computers allows to stay ahead of the real physical process. At the moment time a considerable amount of work in the field of numerical simulation of pollution and oil spill, but all the research works were limited to modeling based on single-speed mathematical model.

In this paper the mathematical modeling of the spreading oil slick on the surface of the Caspian Sea with an air temperature of 200 C, at rest is considered.

For this two-dimensional mathematical model of a three-component model based on the motion equations of the multiphase medium, taking into account the velocity components of the "gas-oil-water" was examined. Figure 1 shows a multiphase environment where green is colored region of the air flow over the sea- "gas", the blue color shows the oil slick - "oil" is having physical properties of a viscous liquid, and the areadressed in red is sea water – "water".

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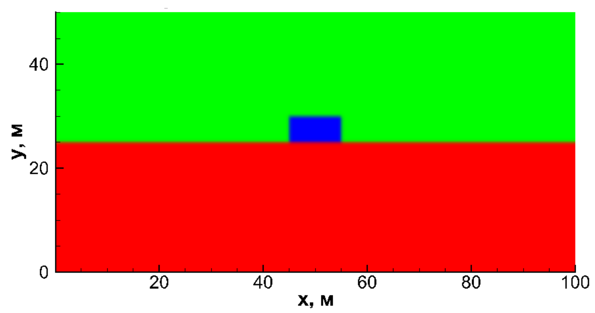

Figure 1 – Schematic illustration of the problem statement

Mathematical modeling

The theoretical basis of mathematical models for the process of the spreading of the oil slick based on the solution of the continuity equation for each component, the continuity equation for the mixture momentum equation:

$$\begin{cases} \dfrac{\partial \rho_k}{\partial t} + \operatorname{div} \rho_k \mathrm{v}_k = 0, \quad k = 1 \ldots N, & (1) \\ \operatorname{div} \mathrm{v} = 0, & (2) \\ \rho \dfrac{\partial \mathrm{v}}{\partial t} + \rho \nabla \cdot (\mathrm{v} \otimes \mathrm{v}) = -\nabla p + \nabla \tau + f, & (3) \end{cases}$$

where v_k – speed of movement components; v – speed mix; p – pressure; $f = (0, -\rho g)$ – vector of external forces; g – acceleration of free fall.

The real ρ_k^0 density of the components is defined as the weight of the k -th component in the unit volume of the component. In turn, the density (partial density) component ρ_k is the mass per unit volume of the components of the mixture. The volume fraction of the components is defined as the ratio of the components to the volume of the mixture:

$$\sum_{k=1}^{N} c_k = 1, \tag{4}$$

$$\rho_k = c_k \rho_k^0, \quad k = 1 \ldots N, \tag{5}$$

and the mixture density ρ is determined by the law

$$\rho = \sum_{k=1}^{N} \rho_k \tag{6}$$

Using (5), the first equation (1) can be obtained in terms of the continuity equation of the volume fraction

$$\frac{\partial c_k}{\partial t} + \operatorname{div} c_k \mathrm{v}_k = 0, \quad k = 1 \ldots N, \tag{7}$$

the sum of which gives equation (2). As the speed of the mixture is taken to be the volumetric average velocity

$$\mathrm{v} = \sum_{k=1}^{N} c_k \mathrm{v}_k \tag{8}$$

The viscous stress tensor τ, which is considered for non-Newtonian fluids is given by:

$$\tau = \mu_e D, \tag{9}$$

$$\mu_e = \mu_e(\dot{\gamma}), \quad \dot{\gamma} = \sqrt{\frac{1}{2} D \cdot D}, \qquad D = D_{\mathrm{i,j}} = \frac{\partial \mathrm{v_i}}{\partial \mathrm{x_j}} + \frac{\partial \mathrm{v_j}}{\partial \mathrm{x_i}} \tag{10}$$

Non-Newtonian fluids: $\mu_e = (\tau_0 + k\dot{\gamma})\dot{\gamma}^{-1}$– Bingham fluid, where τ_0 – the yield stress of visco-plastic fluid; $\dot{\gamma}$ - shear rate; n, k – parameters of rheological models.

The dynamic viscosity μ of the mixture is determined linearly

$$\mu = \sum_{k=1}^{N} c_k \mu_k \tag{11}$$

where μ_k – the dynamic viscosity of the k –thcomponents.

Components such as gas bubbles or solid particles are considered as the dispersed ($k \in K_{dispersed}$), and others – the carrying ($k \in K_{carrier}$). Phase carriers move with the same speed $v^{carrier}$:

$$\mathrm{v}_k = \mathrm{v}^{\mathrm{carrier}}, \quad k \in K_{carrier} \tag{12}$$

the velocity difference between the dispersed and carrying phases are determined by Stokes' law:

$$\mathrm{v}_k = \mathrm{v}^{\mathrm{carrier}} + u_k^{settling}, \quad k \in K_{carrier} \tag{13}$$

$$u_k^{settling} = \frac{(\rho_k^0 - \rho) \cdot d_k^2}{18\mu} g, \tag{14}$$

where d_k – the diameter of the dispersed k phase particles

Substituting (12) and (13) a definition (8) gives

$$\text{v} = \text{v}^{\text{carrier}} + \sum_{k \in K_{dispersed}} c_k u_k^{settling}. \qquad (15)$$

This yields an expression for the speed of the carrier phase through high speed mixture

$$\text{v}^{\text{carrier}} = \text{v} - \sum_{k \in K_{dispersed}} c_k u_k^{settling}. \qquad (16)$$

Speed disperse phase is now explicitly computed from (13).

The initial and boundary conditions:

At the initial moment of the bottom half of the area is sea water $(c_{water} = 1)$.

On the surface of the sea water is the oil column with dimensions 10×5 м $(c_{oil} = 1)$, in other points of the region is the air $(c_{air} = 1)$.

The boundary conditions imply an appeal to zero speed at the lower boundary, and other boundaries are turning to zero only the wall normal component of velocity.

The numerical algorithm

Numerical implementation of the model is composed by the following algorithm: at the first stage, the Navier-Stokes equation is solved without taken pressure into account, at the second stage the Poisson equation for pressure is solved, derived from the continuity equation with given velocity fields from the first stage, by using the matrix sweep method. The obtained pressure field in the next stage is used to recalculate the final velocity field [7]. At the fourth stage the concentration equation for of the components of a viscous liquid is solved according to the defined velocity field. At the last stage the equation for concentration of the components of a viscous liquid is solved according to the final recalculated velocity field.

The intermediate velocity field is defined by using the Crank-Nicholson scheme in combination with a five-point sweep method.

The horizontal component component of velocity at the grid point $\left(i + \frac{1}{2} j\right)$:

$$\frac{\partial u_1}{\partial \tau} + \frac{\partial (u_1 u_1)}{\partial x_1} + \frac{\partial (u_1 u_2)}{\partial x_2} = \frac{1}{\text{Re}}\left(\frac{\partial^2 u_1}{\partial x_1^2} + \frac{\partial^2 u_1}{\partial x_2^2}\right) \qquad (17)$$

Equation (17) at the application of the scheme Crank-Nicholson takes the following form

$$\hat{u}^{\,n+1}_{1\,i+\frac{1}{2}j} - u^{\,n}_{1\,i+\frac{1}{2}j} = -\frac{3\tau}{2}[hx]^n_{i+\frac{1}{2}j} + \frac{\tau}{2}[hxp]^{n-1}_{i+\frac{1}{2}j} + \tau[ax]^n_{i+\frac{1}{2}j} + \frac{\tau}{2}\cdot\frac{1}{\text{Re}}\cdot\left(\frac{\partial^2 \hat{u}_1}{\partial x_1^2}\right)^{n+1}_{i+\frac{1}{2}j} + \frac{\tau}{2}\cdot\frac{1}{\text{Re}}\cdot\left(\frac{\partial^2 \hat{u}_1}{\partial x_2^2}\right)^{n+1}_{i+\frac{1}{2}j} \qquad (18)$$

where

$$[hx]^n_{i+\frac{1}{2}j} = \left(\frac{\partial u_1 u_1}{\partial x_1}\right)^n_{i+\frac{1}{2}j} + \left(\frac{\partial u_1 u_2}{\partial x_2}\right)^n_{i+\frac{1}{2}j}$$

$$[hxp]^{n-1}_{i+\frac{1}{2}j} = \left(\frac{\partial u_1 u_1}{\partial x_1}\right)^{n-1}_{i+\frac{1}{2}j} + \left(\frac{\partial u_1 u_2}{\partial x_2}\right)^{n-1}_{i+\frac{1}{2}j}$$

$$[ax]^n_{i+\frac{1}{2}j} = \frac{1}{2}\cdot\frac{1}{\text{Re}}\cdot\left[\left(\frac{\partial^2 u_1}{\partial x_1^2}\right)^n_{i+\frac{1}{2}j} + \left(\frac{\partial^2 u_1}{\partial x_2^2}\right)^n_{i+\frac{1}{2}j}\right]$$

Then the left side of the equation (18) is denoted by $q_{i+\frac{1}{2}j}$

$$q_{i+\frac{1}{2}j} \equiv \hat{u}^{\,n+1}_{1\,i+\frac{1}{2}j} - u^{\,n}_{1\,i+\frac{1}{2}j} \qquad (19)$$

Component $\hat{u}^{\,n+1}_{1\,i+\frac{1}{2}j}$ is found from the equation (19)

$$\hat{u}^{\,n+1}_{1\,i+\frac{1}{2}j} = q_{i+\frac{1}{2}j} + u^{\,n}_{1\,i+\frac{1}{2}j}$$

Replacing all $\hat{u}^{\,n+1}_{1\,i+\frac{1}{2}j}$ from the equation (18) takes the following form

$$q_{i+\frac{1}{2}j} - \frac{\tau}{2}\cdot\frac{1}{\text{Re}}\cdot\left(\frac{\partial^2 q}{\partial x_1^2}\right)^{n+1}_{i+\frac{1}{2}j} - \frac{\tau}{2}\cdot\frac{1}{\text{Re}}\cdot\left(\frac{\partial^2 q}{\partial x_2^2}\right)^{n+1}_{i+\frac{1}{2}j} = -\frac{3\tau}{2}[hx]^n_{i+\frac{1}{2}j} + \frac{\tau}{2}[hxp]^{n-1}_{i+\frac{1}{2}j} + 2\cdot\tau[ax]^n_{i+\frac{1}{2}j} \qquad (20)$$

The equation (20) looks in the following way

$$\left[1 - \frac{\tau}{2}\cdot\frac{1}{\text{Re}}\cdot\frac{\partial^2}{\partial x_1^2} - \frac{\tau}{2}\cdot\frac{1}{\text{Re}}\cdot\frac{\partial^2}{\partial x_2^2}\right] q_{i+\frac{1}{2}j} = d_{i+\frac{1}{2}j} \qquad (21)$$

where

$$d_{i+\frac{1}{2}j} = -\frac{3\tau}{2}[hx]^n_{i+\frac{1}{2}j} + \frac{\tau}{2}[hxp]^{n-1}_{i+\frac{1}{2}j} + 2\cdot\tau[ax]^n_{i+\frac{1}{2}j}$$

To obtain the second order of accuracy with respect to time:

$$\left[1-\frac{\tau}{2}\cdot\frac{1}{\mathrm{Re}}\cdot\frac{\partial^2}{\partial x_1^2}\right]\left[1-\frac{\tau}{2}\cdot\frac{1}{\mathrm{Re}}\cdot\frac{\partial^2}{\partial x_2^2}\right]q_{i+\frac{1}{2}j} = d_{i+\frac{1}{2}j} \qquad (22)$$

To determine $q_{i+\frac{1}{2}j}$ the equation (22) is solved in 2 stages.

$$\left[1-\frac{\tau}{2}\cdot\frac{1}{\mathrm{Re}}\cdot\frac{\partial^2}{\partial x_1^2}\right]A_{i+\frac{1}{2}j} = d_{i+\frac{1}{2}j}$$

$$\left[1-\frac{\tau}{2}\cdot\frac{1}{\mathrm{Re}}\cdot\frac{\partial^2}{\partial x_2^2}\right]q_{i+\frac{1}{2}j} = A_{i+\frac{1}{2}j}$$

The first stage $A_{i+\frac{1}{2}j}$ is searched in the direction of x_1 coordinates:

$$\left[1-\frac{\tau}{2}\cdot\frac{1}{\mathrm{Re}}\cdot\frac{\partial^2}{\partial x_1^2}\right]A_{i+\frac{1}{2}j} = d_{i+\frac{1}{2}j}$$

$$A^{n+1}_{i+\frac{1}{2}j} - \frac{\tau}{2}\cdot\frac{1}{\mathrm{Re}}\cdot\left(\frac{\partial^2 A}{\partial x_1^2}\right)^{n+1}_{i+\frac{1}{2}j} = d_{i+\frac{1}{2}j}$$

$$A^{n+1}_{i+\frac{1}{2}j} - \frac{\tau}{2}\cdot\frac{1}{\mathrm{Re}}\cdot\frac{-A^{n+1}_{i+\frac{5}{2}j} + 16\cdot A^{n+1}_{i+\frac{3}{2}j}}{12\Delta x_1^2} +$$

$$+\frac{-30\cdot A^{n+1}_{i+\frac{1}{2}j} + 16\cdot A^{n+1}_{i-\frac{1}{2}j} - A^{n+1}_{i-\frac{3}{2}j}}{12\Delta x_1^2} = d_{i+\frac{1}{2}j}$$

$$s_1\cdot A^{n+1}_{i+\frac{5}{2}j} - 16\cdot s_1\cdot A^{n+1}_{i+\frac{3}{2}j} + (1+30\cdot s_1)\cdot A^{n+1}_{i+\frac{1}{2}j} - \qquad (23)$$
$$-16\cdot s_1\cdot A^{n+1}_{i-\frac{1}{2}j} + s_1\cdot A^{n+1}_{i-\frac{3}{2}j} = d_{i+\frac{1}{2}j}$$

where $s_1 = \dfrac{\tau}{24\cdot \mathrm{Re}\cdot \Delta x_1^2}$

This equation (23) is solved by five-point sweep method, the result of which $A^{n+1}_{i+\frac{1}{2}jk}$ is defined.

At the second stage $q_{i+\frac{1}{2}jk}$ is sought in the direction of the x_2 coordinates:

$$\left[1-\frac{\tau}{2}\cdot\frac{1}{\mathrm{Re}}\cdot\frac{\partial^2}{\partial x_2^2}\right]q_{i+\frac{1}{2}j} = A_{i+\frac{1}{2}j}$$

$$q^{n+1}_{i+\frac{1}{2}j} - \frac{\tau}{2}\cdot\frac{1}{\mathrm{Re}}\cdot\left(\frac{\partial^2 q}{\partial x_2^2}\right)^{n+1}_{i+\frac{1}{2}j} = A^{n+1}_{i+\frac{1}{2}j}$$

$$q^{n+1}_{i+\frac{1}{2}j} - \frac{\tau}{2}\cdot\frac{1}{\mathrm{Re}}\cdot\frac{-q^{n+1}_{i+\frac{1}{2}j+2} + 16\cdot q^{n+1}_{i+\frac{1}{2}j+1}}{12\Delta x_2^2} +$$

$$+\frac{-30\cdot q^{n+1}_{i+\frac{1}{2}j} + 16\cdot q^{n+1}_{i+\frac{1}{2}j-1} - q^{n+1}_{i+\frac{1}{2}j-2}}{12\Delta x_2^2} = A^{n+1}_{i+\frac{1}{2}j}$$

$$s_2\cdot q^{n+1}_{i+\frac{1}{2}j+2} - 16\cdot s_2\cdot q^{n+1}_{i+\frac{1}{2}j+1} + (1+30\cdot s_2)\cdot q^{n+1}_{i+\frac{1}{2}j} -$$

$$-16\cdot s_2\cdot q^{n+1}_{i+\frac{1}{2}j-1} + s_2\cdot q^{n+1}_{i+\frac{1}{2}j-2} = A^{n+1}_{i+\frac{1}{2}j}$$

where $s_2 = \dfrac{\tau}{24\cdot \mathrm{Re}\cdot \Delta x_2^2}$

This equation (24) is solved by a three-point sweep, as a result of which is $q^{n+1}_{i+\frac{1}{2}j}$ is obtained.

After the value determination of $q^{n+1}_{i+\frac{1}{2}j}$, the $\hat{u}^{n+1}_{1\,i+\frac{1}{2}j}$ is defined in following way

$$\hat{u}^{n+1}_{1\,i+\frac{1}{2}j} = q_{i+\frac{1}{2}j} + u^n_{1\,i+\frac{1}{2}j}$$

Components of the velocity $\hat{u}^{n+1}_{2\,ij+\frac{1}{2}}$ is obtained in the similar way.

Results of numerical simulation

The numerical simulation the following values were determined: oil film thickness of occurrence, the dynamics of oil spreading, depending on time and the diameter of the spreading oil slick.

In this problem the spreading of the pollutant is consideredat the domain L = 100 m, H = 50 m. The number of points on the x -axis is equal to 256, in y – 128. Figures 2 – 5 shows a multi-phase environment where green is colored region of the air flow over the sea-"gas", where the density and dynamic viscosity equal ρ_{gas} = 1.27 kg/m^3; μ_{gas} = 1.7 × 10^{-5} Pa·sec respectively, is shown in blue with $\rho_{pollu\ tant}$ = 900 kg/m^3 density of pollutant, having the physical properties of the viscous fluid $\mu_{pollu\ tant}$ = 0.1 × 10^{-2} Pa·sec, and the region is colored red respectively is sea water – "water", where; ρ_{water} = 1020 kg/m^3; μ_{water} = 0.9 × 10^{-3} Pa·sec.

Comparison of results of test problem and experimental task on the basis of the model

In this problem, we consider the process of collapse of the dam Figure 6, the domain size are 1.25x0.7 m. Computational domain size number of dots in the x equals 142, y-axis - 80. The sides of the rectangle are solid walls, in which is placed slip condition. Figure 7 shows the dynamics of fluid distribution.The calculation of the present work in comparison with the calculation of work Minakov [9] and experimental data by Martin [10]. The initial height of the liquid column are 0:4 m.

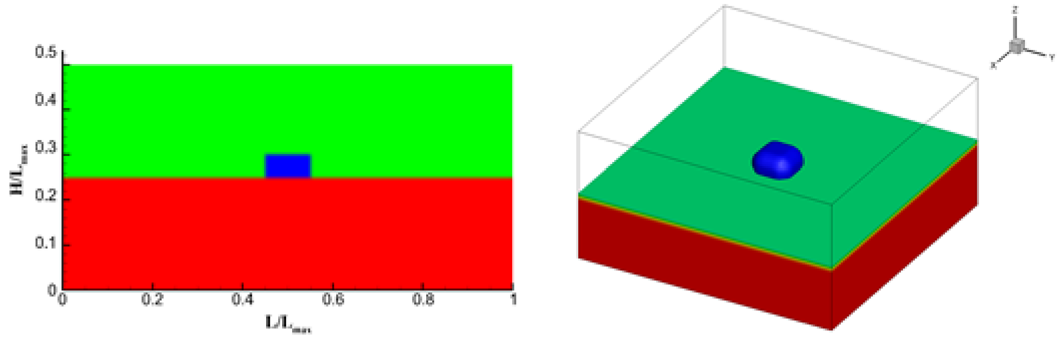

Figure 2 – The dynamics of the spreading of the pollutant on the surface of the sea at t=0 sec.

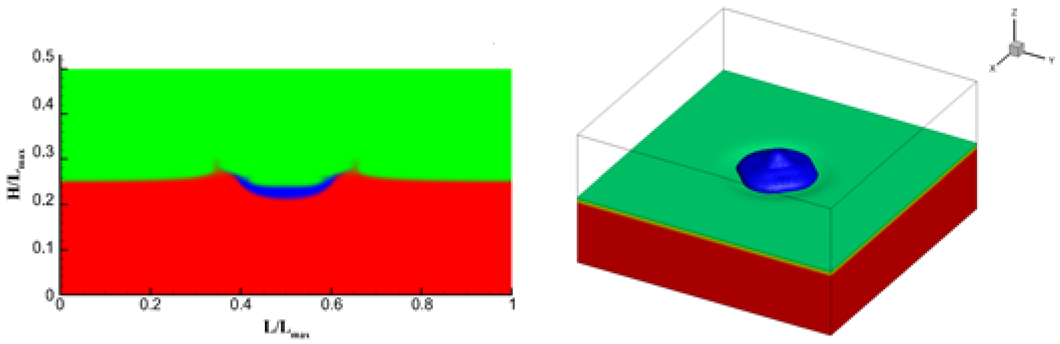

Figure 3 – The dynamics of the spreading of the pollutant on the surface of the sea at t=2 sec.

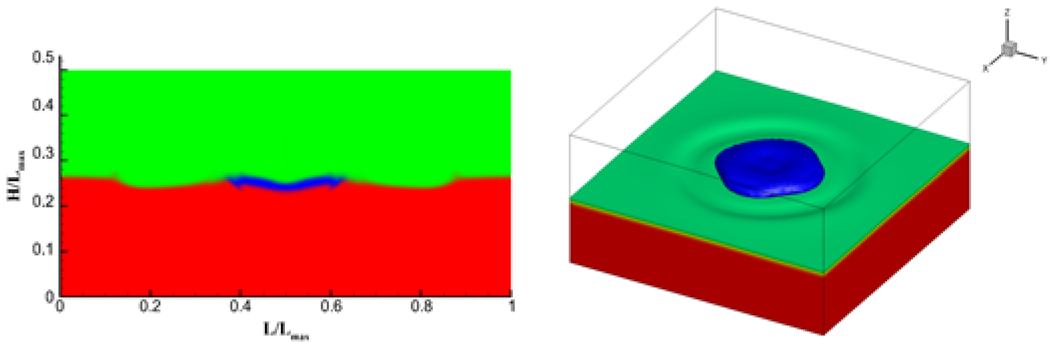

Figure 4 – The dynamics of the spreading of the pollutant on the surface of the sea at t=6 sec.

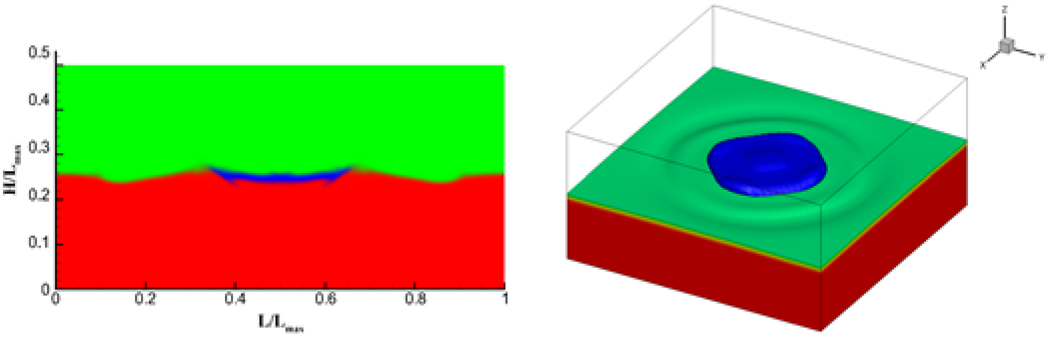

Figure 5 – The dynamics of the spreading of the pollutant on the surface of the sea at t=7sec.

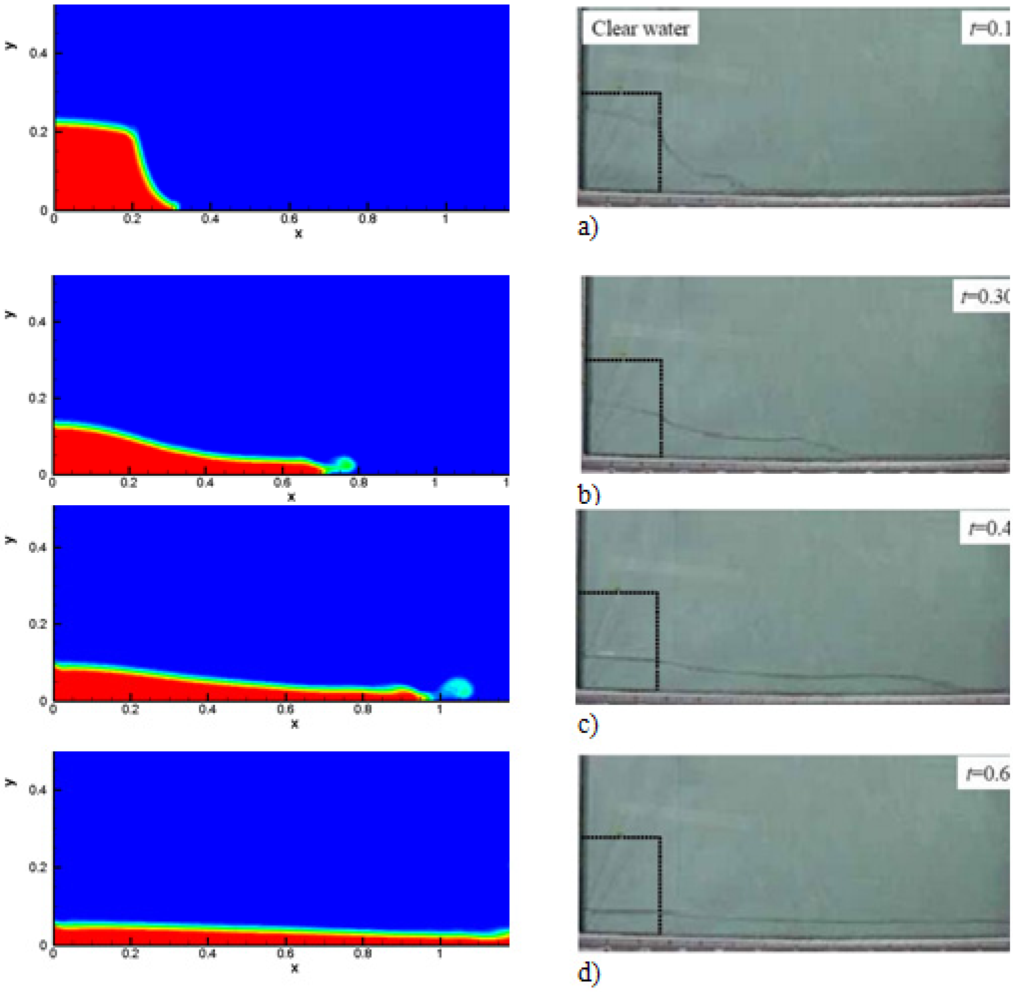

Figure 6 – Distribution of the volume fraction of water in different times
(numerical results on the left, to the right of the experimental results of Sassa and Sekiguchi [8]):
a) t=0.15 sec; b) t=0.3 sec; c) t=0.45 sec; d) t=0.6 sec.

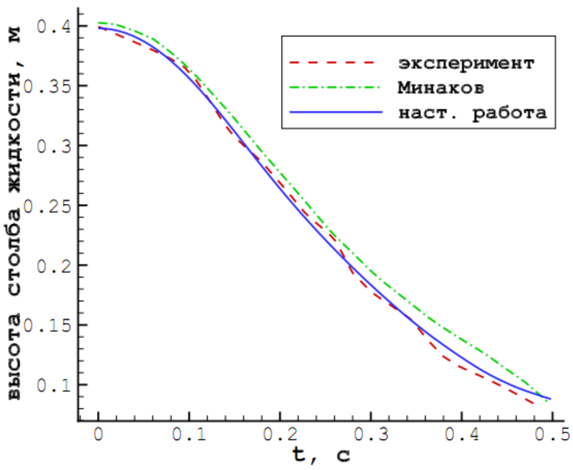

Figure 7 – the height of the liquid column near the left wall depending on time.

Conclusion

Drawing an analysis of the results it can be concluded, that the developed model of the spreading of an oil slick on the seawater surface allowing the researchers, who are involved in the evaluation of environmental damage, to determine the trajectory of the migration of an oil slick on the Caspian Sea and to obtain the most objective result of a process of spreading of oil and oil products.

The numerical simulation of oil spills in the open sea under different scenarios, including the different initial mass of spilled oil, the various types of produced and transported oil through the Caspian sea is carried out.

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