[1][*]Seshavatharam U.V.S., [2]Lakshminarayana S.

[1]Honorary Faculty, I-SERVE, Alakapuri, Hyderabad-35, Telangana, India
[2]Departament of Nuclear Physics , Andhra University, Visakhapatnam-03, AP, India
[*]e-mail: seshavatharam.uvs@gmail.com

Towards a workable model of final unification

Abstract: Even though 'String theory' models and "quantum gravity' models are having a strong mathematical back ground and sound physical basis, they are failing in implementing the Newtonian gravitational constant in atomic and nuclear physics and thus seem to fail in developing a 'workable' model of final unification. In this context, extending Abdus Salam's old concept of 'nuclear strong gravitational coupling' we consider two very large pseudo gravitational constants assumed to be associated with electromagnetic and strong interactions. By combining the two microscopic pseudo gravitational constants with the Newtonian gravitational constant, we make an attempt to combine the old 'strong gravity' concept with 'Newtonian gravity' and try to understand and re-interpret the constructional features of nuclei, atoms and neutron stars in a unified approach. Finally we make a heuristic attempt to estimate the Newtonian gravitational constant from the known elementary atomic and nuclear physical constants. By exploring the possibility of incorporating the proposed two pseudo microscopic gravitational constants in current unified models, in near future, complete back ground physics can be understood and observable low energy predictions can be made.
Key words: Final unification, Schwarzschild interaction, Newtonian gravitational constant, Gravitational constants associated with electromagnetic and strong interactions.

Novelty and Significance of this paper

By introducing two pseudo gravitational constants, we make an attempt to combine the old 'strong gravity' concept with 'Newtonian gravity' and try to understand and re-interpret the constructional features of nuclei, atoms, and neutron stars in a unified approach and finally making an attempt to estimate the Newtonian gravitational constant from the known elementary atomic and nuclear physical constants.

Scope of this paper

Considering the two pseudo gravitational constants assumed to be associated with strong and electromagnetic interactions,
1. Currently believed generalized physical concepts like, proton-electron mass ratio, neutron life time, weak coupling constant, strong coupling constant, nuclear charge radius, root mean square radius of proton, melting points of proton and electron, nuclear charge radii, nuclear binding energy, nuclear stability, Bohr radius of hydrogen atom, electron and proton magnetic moments, Planck's constant, atomic radii, molar mass constant and Avogadro number etc. can be reviewed in a unified approach and can be simplified.

2. Significance of the ratio of nuclear gravitational constant and Newtonian gravitational constant can be understood and thereby magnitude of the Newtonian gravitational constant can be estimated in a unified approach.

3. Proceeding further, considering the ratio of nuclear gravitational constant and Newtonian gravitational constant, neutron star mass can be understood

Table – Topics of this paper

S.No	Section heading
1	Introduction
2	Two basic assumptions of final unification
3	Important points pertaining to 'Schwarzschild interaction' and 'final unification'
4	To understand the role of Newtonian gravitational constant in nuclear physics
5	To understand the Planck's constant
6	Nuclear charge radius and root mean square radius of proton
7	To fit and understand the Fermi's weak coupling constant
8	Melting points of proton and electron
9	To fit and understand the atomic radii
10	'System of units' independent Avogadro number and Molar mass unit
11	To fit and understand the atomic radii
12	Mass and radius of a neutron star
13	Fitting and understanding the neutron life time
14	Understanding the Bohr radius, Reduced Planck's constant and magnetic moments of electron and proton
15	Understanding the nuclear charge radii
16	Discussion
17	Conclusion

1. Introduction

Even though 'String theory' models and "quantum gravity' models [1,2] are having a strong mathematical back ground and sound physical basis, they are failing in implementing the Newtonian gravitational constant [3] in atomic and nuclear physics and thus seem to fail in developing a 'workable' model of final unification.

According to Roberto Onofrio [4], weak interactions are peculiar manifestations of quantum gravity at the Fermi scale, and that the Fermi coupling constant is related to the Newtonian constant of gravitation. In his opinion, at atto-meter scale, Newtonian gravitational constant seems to reach a magnitude of $8.205 \times 10^{22} \mathrm{m}^3 \mathrm{kg}^{-1} \mathrm{sec}^{-2}$. In this context, in physics literature [5,6,7] one can see number of papers on 'strong gravity'. Based on the old and ignored scientific assumption put forward by Nobel laureate Abdus Salam, we developed and compiled many interesting relations assumed to be connected with nuclear physics, atomic physics and astrophysics. We are sure to say that, each and every relation is having its own mathematical beauty and we are working on deriving them at fundamental level. The main issue is: to understand the basics of final unification from hidden, unknown and un-identified physics! It is true that, from unification point of view, one cannot accept any relation without a derivation. It is also true that, practically,

subject of 'true unification' is beyond the scope of current human understanding. Based on the concepts of : 'workability' and 'something is better than nothing', we appeal the readers to go through the following sections in a true scientific spirit.

Clearly speaking, in this paper, by introducing two pseudo gravitational constants, we make an attempt to combine the old 'strong gravity' concept with 'Newtonian gravity' and try to understand and re-interpret the constructional features of nuclei, atoms, and neutron stars in a unified approach and finally making an attempt to estimate the Newtonian gravitational constant from the known elementary atomic and nuclear physical constants.

2. Two basic assumptions of final unification

In our recent publication [8] (Proceedings of International Intradisciplinary Conference on the Frontiers of Crystallography (IICFC-2014)), qualitatively we proposed the following two assumptions with many possible applications. It may be noted that, current main stream physics is very silent on implementing the Newtonian gravitational constant in current microscopic physics. In this context, thinking that, 'something is better than nothing', we developed this subject. We are at 'half the way' and are sure to say that the subject under development is fruitful and needs experts' hands-on experience in ripening it.

Assumption-1: Magnitude of the gravitational constant associated with the electromagnetic interaction is, $G_e \cong (2.375 \pm 0.002) \times 10^{37}$ m^{3}kg^{-1}sec^{-2} .

Assumption-2: Magnitude of the gravitational constant associated with the strong interaction is, $G_s \cong (3.328 \pm 0.002) \times 10^{28}$ m^{3}kg^{-1}sec^{-2} .

Note-1: We choose the following semi empirical relations as 'reference relations' for constructing other semi empirical relations.

$$\frac{m_p}{m_e} \cong \left(\frac{G_s m_p^2}{\hbar c}\right)\left(\frac{G_e m_e^2}{\hbar c}\right) \text{ and } \left(\frac{G_s m_p m_e}{\hbar c}\right) \cong \left(\frac{\hbar c}{G_e m_e^2}\right) \quad (1)$$

$$m_p \cong \left(\frac{G_N}{G_e}\right)^{\frac{1}{6}} \left(\frac{\hbar c m_e^2}{G_N}\right)^{\frac{1}{4}} \cong \left(\frac{G_N}{G_e}\right)^{\frac{1}{6}} \sqrt{M_{pl} m_e} \quad (2)$$

where $M_{pl} \cong \sqrt{\hbar c/G_N}$ is the Planck mass.

Note-2: It may be noted that, with reference to the operating force magnitudes, protons and electrons cannot be considered as 'black holes'. But protons and electrons can be assumed to follow the relations that black holes generally believed to follow. Clearly speaking, in the study of black holes, Newtonian gravitational constant G_N plays a major role, whereas in the study of elementary particles, G_s and G_e play the key role. For detailed information, see the following section.

Note-3: Considering the above two assumptions,

1) Currently believed generalized physical concepts like, proton-electron mass ratio, neutron life time, weak coupling constant, strong coupling constant, nuclear charge radius, root mean square radius of proton, melting points of proton and electron, nuclear charge radii, nuclear binding energy, nuclear stability, Bohr radius of hydrogen atom, electron and proton magnetic moments, Planck's constant, atomic radii, molar mass constant and Avogadro number etc. can be reviewed in a unified approach and can be simplified.

2) Significance of the ratio of nuclear gravitational constant and Newtonian gravitational constant can be understood and thereby magnitude of the Newtonian gravitational constant can be estimated in a unified approach.

3) Proceeding further, considering the ratio of nuclear gravitational constant and Newtonian gravitational constant, neutron star mass can be understood.

3. Important points pertaining to 'Schwarzschild interaction' and 'final unification'

1) If it is true that c and G_N are fundamental physical constants, then (c^4/G_N) can be considered as a fundamental compound constant related to a characteristic limiting force [9].

2) Black holes are the ultimate state of matter's geometric structure.

3) Magnitude of the operating force at the black hole surface is of the order of (c^4/G_N).

4) Gravitational interaction taking place at black holes can be called as 'Schwarzschild interaction'.

5) Strength of 'Schwarzschild interaction' can be assumed to be unity.

6) Strength of any other interaction can be defined as the ratio of operating force magnitude and the classical or astrophysical force magnitude (c^4/G_N).

7) If one is willing to represent the magnitude of the operating force as a fraction of (c^4/G_N) i.e. X times of (c^4/G_N), where $X \ll 1$, then

$$\frac{X \text{ times of } (c^4/G_N)}{(c^4/G_N)} \cong X \rightarrow \text{Effective } G \Rightarrow \frac{G_N}{X} \quad (3)$$

If X is very small, $(1/X)$ becomes very large. In this way, X can be called as the strength of interaction. Clearly speaking, strength of any interaction is $(1/X)$ times less than the 'Schwarzschild interaction' and effective G becomes (G/X).

8) With reference to Schwarzschild interaction, for electromagnetic interaction, $X \approx 2.811 \times 10^{-48}$ and for strong interaction, $X \cong 2.0 \times 10^{-39}$.

9) Characteristic operating force corresponding to electromagnetic interaction is $(c^4/G_e) \approx 3.4 \times 10^{-4}$ N and characteristic operating force corresponding to strong interaction is $(c^4/G_s) \approx 242600$ N.

10) Characteristic operating power corresponding to electromagnetic interaction is $(c^5/G_e) \approx 10990$ J/sec and characteristic operating power corresponding to strong interaction is $(c^5/G_s) \approx 7.27 \times 10^{13}$ J/sec

11) As $\left[\left(c^4/G_e\right),\left(c^4/G_s\right)\right]<<\left(c^4/G_N\right)$ and $\left[\left(c^5/G_e\right),\left(c^5/G_s\right)\right]<<\left(c^5/G_N\right)$, protons and electrons can not be considered as 'black holes', but may be assumed to follow similar relations that black holes generally believed to follow.

12) According to S.W. Hawking [10], temperature of black hole takes the following expression.

$$T_B \cong \frac{\hbar c^3}{8\pi G_N k_B M_B} \qquad (4)$$

where M_B and T_B represent the mass and temperature of a black hole respectively. It may be noted that, by combining the views of Hawking and Abhas Mithra [11] and by considering the proposed assumptions, melting points of elementary particles can be estimated and fitted.

4. To understand the role of Newtonian gravitational constant in nuclear physics

Let,

$$M_{pl} \cong \sqrt{\frac{\hbar c}{G_N}} \cong 1.220\,93(7)\times 10^{19}\ \text{GeV}/c^2 \cong \text{Planck mass} \qquad (5)$$

$$m_{npl} \cong \sqrt{\frac{\hbar c}{G_s}} \approx 546.7\ \text{MeV}/c^2 \cong \text{Nuclear Planck mass} \qquad (6)$$

After developing many relations, to a very good accuracy, it is noticed that,

$$m_p \cong \left(\frac{m_e^6 M_{pl}}{m_{npl}^2}\right)^{\frac{1}{5}} \text{ and } m_e \cong \left(\frac{m_p^5 m_{npl}^2}{M_{pl}}\right)^{\frac{1}{6}} \qquad (7)$$

In a simplified picture,

$$m_e \cong \left(\frac{G_N m_{npl}^2}{G_s m_p^2}\right)^{\frac{1}{12}} m_p \cong \left(\frac{G_N \hbar c m_p^{10}}{G_s^2}\right)^{\frac{1}{12}} \cong \left(\left(\frac{G_N}{G_s}\right)\left(\frac{\hbar c}{G_s}\right) m_p^{10}\right)^{\frac{1}{12}}$$

$$\rightarrow m_p \cong \left(\frac{G_s^2 m_e^{12}}{G_N \hbar c}\right)^{\frac{1}{10}} \cong \left(\left(\frac{G_s}{G_N}\right)\left(\frac{\hbar c}{G_s}\right)^{-1} m_e^{12}\right)^{\frac{1}{10}}$$

$$\Rightarrow \hbar \cong \left(\frac{G_s}{G_N}\right)\left(\frac{m_e}{m_p}\right)^{10}\left(\frac{G_s m_e^2}{c}\right) \qquad (8)$$

In this way, one can see the combined role of $\left(G_s, G_N\right)$ in understanding the mystery of rest masses of proton and electron. By fixing the magnitude of $\left(G_s\right)$, magnitude of $\left(G_N\right)$ can be fixed.

5. To understand the Planck's constant
Proceeding further, it is possible to show that,

$$h \cong \sqrt{\frac{m_p}{m_e}}\sqrt{\left(\frac{G_s m_p^2}{c}\right)\left(\frac{e^2}{4\pi\varepsilon_0 c}\right)} \qquad (9)$$

$$hc \cong \sqrt{\frac{m_p}{m_e}}\sqrt{\left(G_s m_p^2\right)\left(\frac{e^2}{4\pi\varepsilon_0}\right)} \qquad (10)$$

Note that, these two relations are free from arbitrary coefficients and seems to be connected with quantum theory of radiation. With further research, if one is able to derive these two relations, unification of quantum theory and gravity can be made practical and successful. Based on relation (9) and by considering the recommended values of elementary physical constants [12, 13],

$$\begin{cases} G_s \cong \dfrac{4\pi\varepsilon_0 h^2 c^2 m_e}{e^2 m_p^3} \cong 3.329560807\times 10^{28}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\[2mm] G_e \cong \dfrac{\hbar^2 c^2}{G_s m_p m_e^3} \cong \left(\dfrac{\hbar}{h}\right)^2\left(\dfrac{e^2 m_p^2}{4\pi\varepsilon_0 m_e^4}\right) \cong \left(\dfrac{e^2 m_p^2}{16\pi^3\varepsilon_0 m_e^4}\right) \cong 2.374335471\times 10^{37}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\[2mm] G_N \cong \left(\dfrac{m_e}{m_p}\right)^{12}\left(\dfrac{G_s^2 m_p^2}{\hbar c}\right) \cong \left(\dfrac{m_e}{m_p}\right)^{14}\left(\dfrac{4\pi\varepsilon_0}{e^2}\right)^2\left(\dfrac{2\pi h^3 c^3}{m_p^2}\right) \cong 6.679856051\times 10^{-11}\ \text{m}^3\text{kg}^{-1}\text{sec}^{-2} \\[2mm] \text{and } m_{npl} \cong \sqrt{\dfrac{\hbar c}{G_s}} \cong 546.6205673\ \text{MeV}/c^2 \end{cases} \qquad (11)$$

From this data it is very clear to say that, accuracy of G_N seems to depend only on the ratio of $\left(\frac{h}{\hbar}\right)$. It is a very important point to be noted here. In the forgoing sections, we use these values.

6. Nuclear charge radius and root mean square radius of proton

Nuclear charge radius [14] can be expressed with the following relation.

$$R_0 \cong \frac{2G_s m_p}{c^2} \cong 1.239290976 \times 10^{-15} \text{ m} \quad (12)$$

Considering this relation (12), magnitude of G_N can be estimated with the following relation.

$$G_N \cong \left\{\left(\frac{m_e}{m_p}\right)^{12}\left(\frac{c^3 R_0^2}{4\hbar}\right)\right\} \cong 4.349311656 \times 10^{19} R_0^2 \quad (13)$$

By measuring the nuclear charge radii of stable atomic nuclides, (R_0, G_s) both can be estimated.

Root mean square radius of proton [12,13] can be expressed with the following relation.

$$R_p \cong \frac{\sqrt{2}G_s m_p}{c^2} \cong 0.8763110532 \times 10^{-15} \text{ m} \quad (14)$$

Considering this relation (14), magnitude of G_N can be estimated with the following relation.

$$G_N \cong \left\{\left(\frac{m_e}{m_p}\right)^{12}\left(\frac{c^3 R_p^2}{2\hbar}\right)\right\} \cong 8.698623312 \times 10^{19} R_p^2 \quad (15)$$

See the following table-1.

Table 1 – RMS radius of proton Vs. Newtonian gravitational constant

RMS radius of proton (fm)	Newtonian gravitational constant $\left(10^{-11}\text{m}^3\text{kg}^{-1}\text{sec}^{-2}\right)$
0.8775	6.697994316
0.8768	6.68731232
0.8758	6.672067113
0.8751	6.661405819

7. To fit and understand the Fermi's weak coupling constant

To a great surprise, it is noticed that [12,13],

$$G_F \cong \left(\frac{m_e}{m_p}\right)^2 \hbar c R_0^2 \quad (16)$$

From above relations,

$$G_F \cong \left(\frac{m_e}{m_p}\right)^2\left(\frac{4G_s^2 m_p^2 \hbar}{c^3}\right) \cong \left(\frac{4G_s^2 m_e^2 \hbar}{c^3}\right) \quad (17)$$

$$\cong 1.440210005 \times 10^{-62} \text{ J.m}^3$$

Based on this relation (17), magnitude of G_N can be estimated with the following relation.

$$G_N \cong \left(\frac{m_e}{m_p}\right)^{10}\left(\frac{G_F c^2}{4\hbar^2}\right) \cong 6.659637481 \times 10^{-11} \text{ m}^3\text{kg}^{-1}\text{sec}^{-2} \quad (18)$$

where $G_F \cong 1.1663787 \times 10^{-5} (\hbar c)^3 \text{ GeV}^{-2} \cong 1.435850781 \times 10^{-62} \text{ Jm}^3$.

8. Melting points of proton and electron

From above concepts and relations, melting points of proton and electron can be estimated with the following relations.

Melting point of proton,

$$T_{proton} \cong \frac{\hbar c^3}{8\pi k_B G_s m_p} \cong 0.147 \text{ Trillion K} \quad (19)$$

This relation can be applied to quarks also. It may be noted that, RHIC have tentatively claimed to have created a quark–gluon plasma with an approximate temperature of 4 trillion degree Kelvin. A new record breaking temperature was set by ALICE at CERN on August, 2012 in the ranges of 5.5 trillion degree Kelvin. In June 2015, an international team of physicists have produced quark-gluon plasma at the Large Hadron Collider by colliding protons with lead nuclei at high energy inside the supercollider's Compact Muon Solenoid detector at a temperature of 4 trillion degree Kelvin [15]. These experimental temperatures are close to the predicted melting temperatures of Proton, up, down and strange quarks and seem to support the proposed pseudo gravitational constant assumed to be associated with strong interaction. Melting point of electron,

$$T_{electron} \cong \frac{\hbar c^3}{8\pi k_B G_e m_e} \cong 0.3786 \text{ Million K} \quad (20)$$

Melting point of electron is 38827 times less than proton melting point. These two estimations are for experimental verification.

9. Nuclear stability and binding energy

Proton-neutron stability [16] can be understood with the following relation

Let A_s be the stable mass number of Z.

$$A_s \cong 2Z + k(2Z)^2 \cong 2Z + 0.0016(2Z)^2 \quad (\text{Or})$$
$$Z \cong \frac{\sqrt{4kA+1}-1}{4k} \text{ where } A \text{ is any mass number} \quad (21)$$

where $k \cong \left(\frac{G_s m_p m_e}{\hbar c}\right) \cong 1.605\times 10^{-3}$. See colum-2 of table-2. With even-odd corrections, accuracy can be improved. Close to stable atomic nuclides, nuclear binding energy [17] can be understood with the following relation.

For $(Z \geq 5)$,

$$BE \cong -\left(Z - 2 + \sqrt{\frac{Z}{30}}\right)\sqrt{\left(\frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R_p}\right)\left(\frac{3}{5}\frac{G_s m_p^2}{R_p}\right)} \quad (22)$$

$$\cong -\left(Z - 2 + \sqrt{\frac{Z}{30}}\right)\times 19.8 \text{ MeV}$$

where R_p is the RMS radius of proton.

$$-\left(\frac{3}{5}\frac{e^2}{4\pi\varepsilon_0 R_p}\right) \cong -0.986 \text{ MeV and}$$

$$-\left(\frac{3}{5}\frac{G_s m_p^2}{R_p}\right) \cong -398.0 \text{ MeV represent the respective}$$

self binding energies. See the following table-2.

Table 2 – Estimated stable mass numbers and their corresponding nuclear binding energy

Proton number	Estimated Stable mass number	Estimated binding energy in MeV	Proton number	Estimated Stable mass number	Estimated binding energy in MeV
6	12. 2	88.05	60	143.0	1176.4
16	33.6	291.6	70	171.4	1376.6
26	56.3	493.4	82	207.0	1616.7
40	90.2	775.3	92	238.2	1816.7
50	116.0	976.0	100	264.0	1976.5

10. 'System of units' independent Avogadro number and Molar mass unit

If, atoms as a whole believed to exhibit electromagnetic interaction, then molar mass constant and Avogadro number, both can be understood with the following simple relation.

$$G_e (m_{atom})^2 \cong G_N (M_{mole})^2 \quad (23)$$

where m_{atom} is the unified atomic mass unit and M_{mole} is the molar mass unit or gram mole.

Thus it is very clear to say that, directly and indirectly 'gravity' plays a key role in understanding the molar mass unit.

$$\frac{M_{mole}}{m_{atom}} \cong \sqrt{\frac{G_e}{G_N}} \rightarrow M_{mole} \cong \sqrt{\frac{G_e}{G_N}} \times m_{atom} \quad (24)$$

where $\sqrt{\dfrac{G_e}{G_N}} \cong 5.96 \times 10^{23}$ and $(0.00099 > M_{mole} < 0.001)$ kg

Based on these relations, "independent of system of units" and "independent of ad-hoc selection of exactly one gram", it may be possible to explore the correct physical meaning of the famous 'Molar mass unit' and 'Avogadro number' in a unified approach [18].

11. To fit and understand the atomic radii

Considering the geometric mean of the two assumed gravitational constants associated with proton and 'atom as whole', atomic radii can be fitted in the following way. By following the periodic arrangement of atoms and their electronic arrangement, accuracy can be improved.

$$R_{atom} \cong A_s^{1/3} \sqrt{\left(\dfrac{2G_s m_n}{c^2}\right)\left(\dfrac{2G_e m_{atom}}{c^2}\right)} \tag{25}$$
$$\cong A_s^{1/3} * 33.0 \times 10^{-12} \text{ m} \cong A_s^{1/3} * 33.0 \text{ pico.meter}$$

where A_s is the stable atomic mass number of the atom, m_n is the average mass of nucleon and m_{atom} is the unified atomic mass unit. Note that, this relation resembles the famous relation for nuclear radii proposed by Rutherford [19]. See the following table-3.

Table 3 – Estimated atomic radii

Proton number	Stable Mass number	Estimated atomic radii (pico meter)	Reference data [20] (pico meter)	Proton number	Stable Mass number	Estimated atomic radii (pico meter)	Reference data [20] (pico meter)
1	1	33.0	31	40	90	147.9	175
6	12	75.6	76	47	107	156.7	145
16	32	104.8	105	60	142	172.2	201
27	57	127.0	126	70	172	183.5	187
28	62	130.6	124	81	203	193.9	145
29	63	131.3	132	89	227	201.3	215
30	66	133.4	122	92	238	204.5	196

12. Mass and radius of a neutron star
A) Mass of neutron star

According to G. Srinivasan [21]: "We began by remarking that the real stumbling block in determining the maximum mass of neutron stars is the equation of state of neutron star matter at densities above the nuclear density ~2.5x10^14 g.cm^-3. After four decades of strenuous effort by several groups there is still considerable uncertainty concerning the equation of state: is the matter in the core of the star "stiff" or "soft"! This depends on whether or not Bose-Einstein Condensates, such as pion condensate or kaon condensate, occur at supranuclear densities, and whether asymptotically free quark matter occurs at even higher densities. Till this question is resolved all one can say is that the maximum mass of neutron stars is somewhere in the range (1.5 to 6.0) solar masses. It seems to us that the best one can do at present is to appeal to observation".

Let (M_{NS}, m_n) represent masses of neutron star [21] and neutron respectively.

$$\dfrac{G_N M_{NS} m_n}{\hbar c} \cong \sqrt{\dfrac{G_s}{G_N}}$$
$$\to M_{NS} \cong \sqrt{\dfrac{G_s}{G_N}}\left(\dfrac{\hbar c}{G_N m_n}\right) \cong 3.17 \text{ Solar mass} \tag{26}$$

Alternatively, it is also noticed that,

$$\dfrac{M_{NS}}{M_{pl}} \cong \dfrac{G_s}{G_N} \text{ Or } \dfrac{G_N M_{NS}^2}{\hbar c} \cong \left(\dfrac{G_s}{G_N}\right)^2$$
$$\to G_N M_{NS} \cong G_s M_{pl} \tag{27}$$
$$\Rightarrow M_{NS} \cong \left(\dfrac{G_s}{G_N}\right) M_{pl} \cong 5.46 \text{ Solar mass}$$

Interesting point to be noted is that, ratio of neutron star mass and Planck mass is of the order of $\left(\dfrac{G_s}{G_N}\right)$.

$$\frac{\text{Mass of neutron star}}{\text{Planck mass}} \cong \frac{M_{NS}}{\left(\sqrt{G_N \hbar/c^3}\right)} \cong \frac{G_s}{G_N} \quad (28)$$

From astro-particle physics point of view, it can be given some consideration.

Note: Currently believed upper mass limit of Super massive black holes (SMBHs) [22] can be fitted with the following relation.

$$\frac{M_{SMBH}}{M_{pl}} \cong \frac{G_e}{G_N} \quad \text{Or} \quad \frac{G_N M_{SMBH}^2}{\hbar c} \cong \left(\frac{G_e}{G_N}\right)^2$$

$$\rightarrow G_N M_{SMBH} \cong G_e M_{pl} \Rightarrow M_{SMBH} \cong \left(\frac{G_e}{G_N}\right) M_{pl} \cong \quad (29)$$

$$\cong 7.74 \times 10^{39} \text{ kg} \approx 10^{10} \text{ Solar mass}$$

Point to be noted is that, ratio of upper limit of galactic black hole mass and Planck mass is of the order of $\dfrac{G_e}{G_N}$.

B) Radius of neutron star

Particle data group [13] recommended value of magnetic radius of neutron is around 0.86 fm. Qualitatively this can be compared with the following relation.

$$\frac{\sqrt{2} G_s m_n}{c^2} \cong 0.877 \text{ fm.} \quad (30)$$

Let $\left(R_{NS}, R_n\right)$ represent the radii [23,12] of neutron star and neutron respectively.

$$\frac{R_{NS}}{\left(\sqrt{2} G_s m_n/c^2\right)} \cong \frac{\text{Radius of neutron star}}{\text{Neutron magnetic radius}} \cong \sqrt{\frac{G_s}{G_N}}$$

$$\rightarrow R_{NS} \cong \sqrt{\frac{G_s}{G_N}} \left(\frac{\sqrt{2} G_s m_n}{c^2}\right) \cong 19.5 \text{ km} \quad (31)$$

It may be noted that, observed masses of neutron stars are of the order of 2 Solar masses and radii are of the order of 11 km. In this context, important point to be noted is that, ratio of neutron star radius

and neutron's characteristic radius is of the order of $\sqrt{\dfrac{G_s}{G_N}}$. It is also possible to say that, ratio of neutron star radius and Planck size is of the order of $\left(\dfrac{G_s}{G_N}\right)$.

It can be expressed in the following way.

$$\frac{\text{Radius of neutron star}}{\text{Planck length}} \cong \frac{R_{NS}}{\left(\sqrt{G_N \hbar/c^3}\right)} \cong \frac{G_s}{G_N} \quad (32)$$

$$\frac{R_{NS}}{\left(\sqrt{\hbar G_N/c^3}\right)} \cong \frac{G_s}{G_N}$$
$$\rightarrow R_{NS} \cong \left(\frac{G_s}{G_N}\right)\sqrt{\frac{\hbar G_N}{c^3}} \cong \left(\sqrt{\frac{G_s}{G_N}} \times \sqrt{\frac{\hbar G_s}{c^3}}\right) \cong 8.1 \text{ km} \quad (33)$$

where $\sqrt{\dfrac{\hbar G_s}{c^3}} \cong 3.61$ fm can be called as the nuclear Planck length. This can be compared with neutron's positively charged core of radius ~3 fm. Now the above relation (33) can be re-expressed in the following way.

$$\frac{\text{Radius of neutron star}}{\text{Nuclear Planck length}} \cong \frac{R_{NS}}{\left(\sqrt{G_s \hbar/c^3}\right)} \cong \sqrt{\frac{G_s}{G_N}} \quad (34)$$

13. Fitting and understanding the neutron life time

It may be noted that, during beta-decay, by emitting one electron and one neutrino, neutron transforms to proton.

Let, t_n be the life time of neutron, m_n be the rest mass of neutron and $\left(m_n - m_p\right)$ be the mass difference of neutron and proton. Then, quantitatively it is possible to show that,

$$\frac{\left(m_n - m_p\right)}{m_n} \cong \left(\frac{G_e}{G_N}\right)^{\frac{1}{2}} \left(\frac{G_s m_n}{c^3 t_n}\right) \quad (35)$$

Very interesting observation is that, the three gravitational constants seem to play a simultaneous role in deciding the neutron decay time and is for further analysis. Now,

$$t_n * \left(m_n - m_p\right)c^2 \cong \left(\frac{G_e}{G_N}\right)^{\frac{1}{2}} \left(\frac{G_s m_n^2}{c}\right) \quad (36)$$

$$t_n \cong \left(\frac{G_e}{G_N} \right)^{\frac{1}{2}} \left(\frac{G_s m_n^2}{(m_n - m_p) c^3} \right) \cong 896.45 \text{ sec} \quad (37)$$

With 1-2 % error, this obtained value can be compared with recommended [13] and experimental [24,25] neutron life times of $(878 \text{ to } 888) \text{ sec}$.

With reference to weak coupling constant and proposed gravitational constant associated with strong interaction,

$$t_n \cong \left(\frac{m_e}{m_p} \right)^{\frac{7}{2}} \frac{\sqrt{G_s G_F}}{2 G_N (m_n - m_p) c} \quad (38)$$

$$G_N \cong \left(\frac{m_e}{m_p} \right)^{\frac{7}{2}} \frac{\sqrt{G_s G_F}}{2 t_n (m_n - m_p) c} \quad (39)$$

Qualitatively, if one is willing to define the well believed strong coupling constant with the following relation,

$$\begin{cases} \alpha_s \cong \left(\frac{\hbar c}{G_s m_p^2} \right)^2 \cong 0.11519371 \quad \text{Or} \\ \sqrt{\alpha_s} \cong \left(\frac{\hbar c}{G_s m_p^2} \right) \cong 0.339401988, \end{cases} \quad (40)$$

error in estimation of neutron life can be minimized and can be expressed with the following relation.

$$t_n \cong \left(\frac{G_e}{G_N} \right)^{\frac{1}{2}} \left(\frac{1}{\alpha_s} \right)^{\frac{1}{2}} \left(\frac{\hbar}{(m_n - m_p) c^2} \right) \cong \frac{303.41914 \text{ sec}}{\sqrt{\alpha_s}} \quad (41)$$

With reference to recommended value [13] of $\alpha_s \cong 0.1185 \pm 0.0006$, obtained $t_n \cong 881.422 \text{ sec}$

$$\alpha_s \cong \left(\frac{G_e}{G_N} \right) \left(\frac{\hbar}{t_n (m_n - m_p) c^2} \right)^2 \cong \left(\frac{303.41914 \text{ sec}}{t_n} \right)^2 \quad (42)$$

With reference to recommended value [13,26,27] of $t_n \cong (880.3 \pm 1.1) \text{ sec}$, obtained $\alpha_s \cong 0.1188$

14. Understanding the Bohr radius, Reduced Planck's constant and magnetic moments of electron and proton

Energy conservation point of view, qualitatively and quantitatively, we noticed the following relation.

$$\frac{G_e m_e^2}{2 a_0} \cong \frac{e^2}{4 \pi \varepsilon_0 \left(2 G_s m_p / c^2 \right)} \cong \frac{e^2}{4 \pi \varepsilon_0 R_0}$$

$$\Rightarrow a_0 \cong \left(\frac{4 \pi \varepsilon_0 G_e m_e^2}{e^2} \right) \left(\frac{G_s m_p}{c^2} \right) \quad (43)$$

where $a_0 \cong 0.53 \text{ A}°$ is the Bohr radius of hydrogen atom and $\frac{G_s m_p}{c^2} \cong 0.61965 \text{ fm}$ and $R_0 \cong \frac{2 G_s m_p}{c^2} \cong 1.24 \text{ fm}$ is the nuclear charge radius. Now, potential energy of electron corresponding to Bohr radius can be expressed with the following relation.

$$\left(E_{pot} \right)_{a_0} \cong -\frac{e^2}{4 \pi \varepsilon_0 a_0} \cong -\left(\frac{e^2}{4 \pi \varepsilon_0 G_e m_e^2} \right) \frac{e^2}{4 \pi \varepsilon_0 \left(G_s m_p / c^2 \right)} \quad (44)$$

Now the basic question to be understood is : How to understand the 'discreteness'?. Important quantum mechanical result of Bohr's theory is that, maximum number of electrons that can be accommodated in any orbit is $2n^2$ where $n = 1, 2, 3, ...$. Based on this result, it can be interpreted that, in any orbit, probability of finding any one electron out of $2n^2$ electrons is $\left(\frac{1}{2n^2} \right)$. By following this interpretation and with reference to electron's total energy of 13.6 eV, 'discrete total energy' of electron in any orbit can be expressed with the following relation.

$$(E_{tot})_n \cong -\left(\frac{1}{2n^2}\right)\frac{e^2}{4\pi\varepsilon_0 a_0} \cong -\left(\frac{1}{2n^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\frac{e^2}{4\pi\varepsilon_0 \left(G_s m_p/c^2\right)}$$

$$\cong -\left(\frac{1}{n^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\frac{e^2}{4\pi\varepsilon_0 \left(2G_s m_p/c^2\right)} \cong -\left(\frac{1}{n^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0 G_e m_e^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0 R_0}\right) \qquad (45)$$

where $R_0 \cong 1.24$ fm.

Clearly speaking, in any orbit,

$$\frac{\text{Total energy of electron}}{\text{Nuclear potential}} \cong \left(\frac{1}{n^2}\right)\left(\frac{e^2}{4\pi\varepsilon_0 G_e m_e^2}\right) \quad (46)$$

Based on relation (43) and with reference to Bohr's theory of hydrogen atom,

$$\left(\frac{\hbar}{m_e}\right) \cong \frac{\sqrt{G_s m_p}\sqrt{G_e m_e}}{c} \cong \frac{\sqrt{G_s G_e}\sqrt{m_p m_e}}{c} \qquad (47)$$

Now, revolving electron's magnetic moment can be expressed as follows.

$$\mu_e \cong \frac{e\hbar}{2m_e} \cong \frac{e\sqrt{G_s G_e}\sqrt{m_p m_e}}{2c} \qquad (48)$$

Proceeding further, with reference to strong interaction and the proposed strong interaction gravitational constant, magnetic moment of proton can be expressed with the following relation.

$$\mu_p \cong \gamma * \left(\frac{eG_s m_p}{2c}\right) \cong \gamma * 1.488142 \times 10^{-26} \text{ J.Tesla}^{-1} \quad (49)$$

where γ is a coefficient of the order of unity and its approximate value is 0.952.

It may be noted that, by considering a proportionality ratio of $\left(\frac{\mu_p}{\mu_e}\right)$, planet earth's dipole magnetic moment can be expressed with the following relation.

$$\mu_{earth} \cong \left(\frac{\mu_p}{\mu_e}\right) * \left(\frac{eG_s M_{earth}}{2c}\right) \cong 8.15 \times 10^{22} \text{ J.Tesla}^{-1} \quad (50)$$

where $M_{earth} \cong 6 \times 10^{24}$ kg.

With further study and analysis, if one is willing to consider the proportionality ratio as a function of planetary physical and magnetic parameters, it may be possible to understand the weak and strong planetary magnetic moments.

15. Understanding the nuclear charge radii

For atomic number greater than 23, nuclear charge radii [30] can be fitted with the following relation.

$$R_{(Z,A)} \cong \left\{ Z^{1/3} + \left(\sqrt{Z(A-Z)}\right)^{1/3} \right\}\left(\frac{G_s m_p}{c^2}\right) \qquad (51)$$

where $Z \geq 23$ and $\dfrac{G_s m_p}{c^2} \cong 0.61965$ fm

This relation seems to be best applicable for medium, heavy and super heavy atomic nuclides. See the following table-4.

By refining the relation (51) with reference to lower atomic numbers and by knowing the nuclear charge radii of various atomic nuclides, magnitude of G_s can be estimated from nuclear experimental data.

Table 4 – To fit the nuclear charge radii

Proton number	Mass number	Neutron number A-Z	Estimated charge radii from relation (51)	Charge radii from reference [30]	%Error w.r.t relation (51)
20	40	20	3.364	3.4776	3.27
21	45	24	3.458	3.5459	2.49
22	48	26	3.522	3.5921	1.96
23	51	28	3.583	3.6002	0.47
24	52	28	3.621	3.6452	0.66
25	55	30	3.680	3.7057	0.70
26	56	30	3.716	3.7377	0.59
27	59	32	3.771	3.7875	0.43
28	60	32	3.806	3.8118	0.16
29	63	34	3.859	3.8823	0.61
30	66	36	3.910	3.9491	0.99
31	69	38	3.960	3.9973	0.93
32	72	40	4.009	4.0576	1.20
33	75	42	4.057	4.0968	0.98
34	76	42	4.087	4.1395	1.27
35	79	44	4.133	4.1629	0.73
36	86	50	4.207	4.1835	-0.57
37	87	50	4.236	4.1989	-0.88
38	88	50	4.264	4.224	-0.95
39	89	50	4.292	4.243	-1.14
40	90	50	4.319	4.2694	-1.15
41	93	52	4.360	4.324	-0.83
42	92	50	4.371	4.3151	-1.30
44	104	60	4.491	4.5098	0.41
45	103	58	4.503	4.4945	-0.20
46	108	62	4.554	4.5563	0.06
47	109	62	4.578	4.5638	-0.31
48	114	66	4.627	4.6087	-0.39
49	115	66	4.650	4.6156	-0.75
50	120	70	4.697	4.6519	-0.98
51	121	70	4.720	4.6802	-0.86
52	130	78	4.787	4.7423	-0.95
53	127	74	4.788	4.75	-0.81
54	136	82	4.853	4.7964	-1.18
55	133	78	4.854	4.8041	-1.04
56	138	82	4.897	4.8378	-1.22
57	139	82	4.918	4.855	-1.31
58	140	82	4.940	4.8771	-1.28
59	141	82	4.961	4.9174	-0.88
60	142	82	4.981	4.9123	-1.41
62	144	82	5.022	4.9524	-1.41
63	145	82	5.042	4.9663	-1.52
64	160	96	5.130	5.1734	0.83
65	159	94	5.141	5.06	-1.60
66	148	82	5.101	5.0455	-1.09

Proton number	Mass number	Neutron number A-Z	Estimated charge radii from relation (51)	Charge radii from reference [30]	%Error w.r.t relation (51)
67	165	98	5.198	5.2022	0.08
68	170	102	5.235	5.2789	0.83
69	169	100	5.245	5.2256	-0.38
70	176	106	5.290	5.3215	0.58
71	175	104	5.300	5.37	1.30
72	178	106	5.327	5.3371	0.18
73	181	108	5.354	5.3507	-0.06
74	184	110	5.381	5.3658	-0.28
75	185	110	5.399	5.3596	-0.73
76	192	116	5.441	5.4126	-0.53
77	191	114	5.451	5.3968	-1.00
78	194	116	5.476	5.4236	-0.97
79	197	118	5.502	5.4371	-1.18
80	198	118	5.519	5.4463	-1.33
81	205	124	5.559	5.4759	-1.52
82	208	126	5.584	5.5012	-1.50
83	209	126	5.601	5.5211	-1.44
84	208	124	5.610	5.5584	-0.92
86	212	126	5.650	5.5915	-1.05
87	212	125	5.662	5.5915	-1.27
88	214	126	5.682	5.6079	-1.33
90	232	142	5.773	5.7848	0.20
92	238	146	5.818	5.8571	0.66
94	239	145	5.846	5.8601	0.24
95	243	148	5.872	5.9048	0.56
96	244	148	5.887	5.8429	-0.75

16. Discussion

It may be noted that,

1) Mostly, old 'strong gravity' models seem to focus on understanding 'quark confinement,' 'basic hadron mass spectrum' and 'coupling constants'.

2) In this paper, we tried our level best in implementing the Newtonian gravitational constant along with two pseudo microscopic gravitational constants and proposed many interesting applications starting from 'electron mass' and 'neutron star mass'.

3) Relations (5) to (8) show the potential and combined role of (G_s, G_N) in nuclear and particle physics.

4) Relations (9) and (10) show the potential role of (G_s) in quantum theory of radiation.

5) Relations (12) to (23) seem to show the potential applications of (G_s, G_e) in nuclear and particle physics.

6) Relations (23), (24) and (25) clearly demonstrate the combined role of (G_s, G_e) in understanding the Avogadro number, molar mass constant and atomic radii.

7) Relations (26) to (34) seem to extend the scope of applicability of the proposed assumptions in astrophysics starting from neutron stars to galactic nuclei.

8) Relations (35) to (39) seem to play a key role in understanding the combined role of (G_s, G_e, G_N).

9) Relations (40) to (42) seem to play a key role in understanding the strong coupling constant and can be estimated from neutron life time and neutron-proton mass difference.

10) Relations (43) to (49) seem to play a key role understanding the origin of quantum mechanics and magnetic moments of electron and proton.

11) Relation (50) seems to play a key role in understanding the dipole magnetic moment of planet earth in a unified approach.

12) Relation (51) seems to play a key role in fitting and understanding the role of G_s in nuclear charge distribution.

13) Qualitatively and quantitatively in a heuristic approach we developed many characteristic relations among micro-macro physical constants with utmost possible accuracy.

14) Proceeding further, we proposed interesting and accurate analytical relations for estimating the Newtonian gravitational constant in a meaningful way and this procedure is beyond the scope of current research paradigm. One must admit this fact.

15) We admit the fact that, in this paper, we could not provide the required 'back ground physics' for understanding the proposed semi empirical relations. At the same time, one must accept the fact that, we presented all possible relations and relevant information using by which theoretically, one may be able to develop a unified and workable model of unification. We would like to inform that,

1) Based on the hierarchy of elementary physical constants,

2) Based on dimensional analysis,

3) Based on trial-error methods,

4) Based on simple mathematical functions,

5) Based on simplified computer programs,

6) Based on data fitting and

7) Based on data prediction

so far we could publish more than 20 papers on this subject. We admit that this procedure is against to the current 'scientific standards' and 'scientific procedures'. In this context, we would like to stress the fact that, even though string theory models are having strong mathematical back ground and sound physical reasoning, they are badly failing in coupling the gravitational and nuclear physical constants. Here, the problem is with 'our understanding and our perception' but not with 'scientific standards and procedures'. In the development of science and engineering, 'data fitting' and 'workability' are the two essential tools using by which physical models can be generated and validated in a progressive manner.

17. Conclusion

Considering the wide applicable range of the proposed two assumptions, we are confident to say that, with further research and analysis, 'hidden and left over physics' can easily be explored. In this context, we would also like to stress the fact that, with current understanding of String theory [28] or Quantum gravity [29], qualitatively or quantitatively, one cannot implement the Newtonian gravitational constant in microscopic physics. This 'draw back' can be considered as a characteristic 'inadequacy' of modern unification paradigm. Proceeding further, with reference to String theory models and Quantum gravity models, proposed two pseudo gravitational constants and presented semi empirical relations can be given some consideration in developing a 'workable model' of 'final unification'.

References

1. Ashoke Sen. Strong-weak coupling duality in four-dimensional string theory // International Journal of Modern Physics A. – 1994. – Vol. 9 (21). – P. 3707–3750.

2. Juan M., Maldacena Gravity. Particle Physics and Their Unification // Int.J.Mod.Phys. – 2000. – Vol. A15S1. – P. 840-852.

3. Schlamminger S. and Newman R.D. Recent measurements of the gravitational constant as a function of time // Phys. Rev. – 2015. – Vol. D 91. – P. 121101.

4. Roberto Onofrio. Proton radius puzzle and quantum gravity at the Fermi scale // EPL. – 2013. – Vol. 104. – P. 20002.

5. Abdus Salam., Strong Interactions., Gravitation and Cosmology. Publ. in: NATO Advanced Study Institute, Erice, June16-July 6, 1972.

6. Salam A., Sivaram C. Strong Gravity Approach to QCD and Confinement // Mod. Phys. Lett. – 1993. – Vol. A8(4). – P. 321- 326.

7. Sivaram C. et al. Gravity of Accelerations on Quantum Scales and its consequences. http://arxiv.org/abs/1402.5071.

8. Seshavatharam U.V.S. and Lakshminarayana S. Semi empirical procedure for estimating the Newtonian gravitational constant with elementary physical constants and Avogadro number.

Proceedings of International Intradisciplinary Conference on the Frontiers of Crystallography. – 2014. – P. 47- 60.

9. Gibbons G.W. The Maximum Tension Principle in General relativity // Found.Phys. – 2002. – Vol. 32. – P. 1891-1901.

10. Hawking S.W. Particle Creation by Black Holes // Commun.Math. Phys. – 1975. – Vol. 43. – P. 199–220.

11. Abhas Mitra. Why gravitational contraction must be accompanied by emission of radiation in both Newtonian and Einstein gravity // Phys. Rev. D. – 2006. – Vol. 74. – P. 024010.

12. Mohr P.J., Taylor B.N. and Newell D.B. CODATA Recommended Values of the Fundamental Physical Constants:2010 // by in Rev. Mod. Phys. – 2012. – Vol. 84. – P. 1527.

13. Olive K.A. et al. Review of Particle Physics // Chin. Phys. C. – 2014. – Vol. 38. – P. 090001.

14. Robert Hofstadter, Rudolf Mössbauer. The electron-scattering method and its application to the structure of nuclei and nucleons. Nobel Lecture, December 11, 1961.

15. Khachatryan V. *et al.* (CMS Collaboration). Evidence for Collective Multiparticle Correlations in p−Pb Collisions // Phys. Rev. Lett. – 2015. – Vol. 115. – P. 012301.

16. Chowdhury P.R. et al. Modified Bethe-Weizsacker mass formula with isotonic shift and new driplines // Mod. Phys. Lett. – 2005. – Vol. A20. – P.1605-1618.

17. Ghahramany N. et al. New approach to nuclear binding energy in integrated nuclear model // Journal of Theoretical and Applied Physics. – 2012. –Vol. 6. – P. 3.

18. Martin J.T. Milton A. New definition for the mole based on the Avogadro constant: a journey from physics to chemistry // Phil. Trans. R. Soc. A. – 2011. – Vol. 369. – P. 3993–4003.

19. Rutherfor E. The Scattering of α and β rays by Matter and the Structure of the Atom // Philos. Mag. – Vol. 6. – P. 21.

20. The Periodic Table of the Elements (including Atomic Radius).

www.sciencegeek.net/tables/AtomicRadius.pdf.

21. Srinivasan G. The Maximum Mass of Neutron Stars // Bulletin of Astronomic Society of India. – 2002. – Vol. 30. – P. 523-547.

22. Kohei Inayoshi. and Zoltan Haiman. Is there a maximum mass for black holes in galactic nuclei? arxiv: 1601.02611v1.

23. Sebastien Guillot. et al. Measurement of the Radius of Neutron Stars with High S/N Quiescent Low-mass X-ray Binaries in Globular Clusters. Astrophys.J. 772 (2013).

24. Yue A.T. et al. Improved Determination of the Neutron Lifetime // Phys. Rev. Lett. – 2013. – Vol. 111. – P. 222501.

25. Arimoto Y. et al. Development of time projection chamber for precise neutron lifetime measurement using pulsed cold neutron beams // Nuclear Instrument sand Methods in Physics Research A. – 2015. – Vol. 799. – P. 187.

26. Seshavatharam U.V.S. and Lakshminarayana S. Applications of gravitational model of possible final unification in both large and small scale physics // Prespacetime journal. – 2016. – Vol 7. – P. 405-421.

27. Seshavatharam U.V.S. and Lakshmi-narayana S. The Possible Role of Newtonian, Strong & Electromagnetic Gravitational Constants in Particle Physics // Prespacetime journal. – 2016. – Vol 7. – P. 857-888.

28. Ashoke Sen. Developments in Superstring theory. CERN Document server, hep-ph/9810356 (2009)
https://cds.cern.ch/record/368056/files/9810356.pdf.

29. Edward Witten. What Every Physicist Should Know About String Theory. GR Centennial Celebration, Strings 2015, Bangalore, India. (2105).
http://member.ipmu.jp/yuji.tachikawa/stringsmirrors/2015/26-06-2015-Edward-Witten.pdf.

30. Angeli I., Marinovab K.P. Table of experimental nuclear ground state charge radii: An update. Atomic Data and Nuclear Data Tables. – 2013. – Vol. 99. – P. 69–95.