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# Angular decay distributions for cascade decay $B \to K^*(\to K\pi) + ll$

**Abstract:** This article is devoted to the study of the  $B \to K^*(\to K\pi) + \overline{ll}$  decay. The rare flavour changing neutral current decays are forbidden in the Standard Model (SM) at tree level. They proceed only via the loops in the perturbation theory. For this reason, these decays are sensitive to the possible effects of new physics beyond the Standard Model. The new heavy particles can contribute to the branching fractions and the angular decay distributions. It is generally believed that the decay mode  $B \to K^*(\to K\pi)\mu^*\mu^-$  is one of the best modes to search for new physics beyond the standard model. The angular distribution enables the independent measurement of several observables as a function of the dilepton invariant mass.

**Key words:** B-meson, rare decay, two-fold angular distributions, four-fold angle distribution, polarization observables.

### Introduction

The rare flavor changing neutral current (FCNC) decays proceed only via the loops in the perturbation theory. For this reason, these decays are sensitive to the possible effects of new physics beyond the SM. The new heavy particles can contribute to the branching fractions and the angular decay distributions. The angular distribution enables the independent measurement of several observables as a function of the dilepton invariant mass. Plenty of observables obtained in this manner enable unique tests of the standard model contributions.

In this paper we study the rare  $B \to K^*(\to K\pi) + \overline{l}l$  decay. We derive the fourfold angular decay distribution by employing the helicity formalism in a model-independent way.

The rare exclusive decays are described by the effective Hamiltonian obtained from the SM-diagrams by using the operator product expansion

$$Q_{1} = \left(\overline{s}_{a_{1}}\gamma^{\mu}P_{L}c_{a_{2}}\right), \left(\overline{c}_{a_{2}}\gamma_{\mu}P_{L}b_{a_{1}}\right),$$

$$Q_{3} = \left(\overline{s}\gamma^{\mu}P_{L}b\right)\sum_{q}\left(\overline{q}\gamma_{\mu}P_{L}q\right),$$

$$Q_{5} = \left(\overline{s}\gamma^{\mu}P_{R}b\right)\sum_{q}\left(\overline{q}\gamma_{\mu}P_{R}q\right),$$

$$Q_{7} = \frac{e}{16\pi^{2}}m_{b}\left(\overline{s}\sigma^{\mu\nu}P_{R}b\right)F_{\mu\nu},$$

$$Q_{9} = \frac{e}{16\pi^{2}}\left(\overline{s}\gamma^{\mu}P_{L}b\right)\left(\overline{l}\gamma_{\mu}l\right),$$

and renormalization group techniques. It allows one to separate the short-distance contributions and isolate them in the Wilson coefficients which can be studied systematically within perturbative QCD. The long-distance contributions are contained in the matrix elements of local operators. Contrary to the short-distance contributions the calculation of such matrix elements requires nonperturbative methods and is therefore model dependent.

The rare decay  $b \rightarrow s(d)l^+l^-$  can be described in terms of the effective Hamiltonian [1]:

$$H_{eff} = -\frac{G_F}{\sqrt{2}} \lambda_i \sum_{i=1}^{10} C_i(\mu) Q_i(\mu)$$
 (1)

where  $C_i(\mu)$  and  $Q_i(\mu)$  are the Wilson coefficients and local operators, respectively.  $\lambda_i \equiv V_{ts(d)}^{\dagger} V_{tb}$  is the product of CKM elements. The standard set [1] of local operators for  $b \rightarrow sl^+l^-$  transition is written as

$$Q_{2} = (\overline{s}\gamma^{\mu}P_{L}c), (\overline{c}\gamma_{\mu}P_{L}b),$$

$$Q_{4} = (\overline{s}_{a_{1}}\gamma^{\mu}P_{L}b_{a_{2}})\sum_{q}(\overline{q}_{a_{2}}\gamma_{\mu}P_{L}q_{a_{1}}),$$

$$Q_{6} = (\overline{s}_{a_{1}}\gamma^{\mu}P_{R}b_{a_{2}})\sum_{q}(\overline{q}_{a_{2}}\gamma_{\mu}P_{R}q_{a_{1}}),$$

$$Q_{8} = \frac{g}{16\pi^{2}}m_{b}(\overline{s}_{a_{1}}\sigma^{\mu\nu}P_{R}\mathbf{T}_{a_{1}a_{2}}b_{a_{2}})\mathbf{G}_{\mu\nu},$$

$$Q_{10} = \frac{e}{16\pi^{2}}(\overline{s}\gamma^{\mu}P_{L}b)(\overline{l}\gamma_{\mu}\gamma_{5}l),$$

$$(2)$$

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where  $\mathbf{G}_{\mu\nu}$  and  $F_{\mu\nu}$  are the gluon and photon field strengths, respectively;  $\mathbf{T}_{uv}$  are the generators of the SU(3) color group;  $a_1$  and  $a_2$  denote color indices (they are omitted in the color-singlet currents). The chirality projection operators

 $P_{L,R} = (1 \mp \gamma_5)/2$  and  $m_b$  is the running mass in the MS scheme.

By using the effective Hamiltonian defined by Eq. (1) one can write the matrix element of the exclusive transitions  $B \to K(K^*)l^+l^-$  as

$$M = \frac{G_F}{\sqrt{2}} \frac{\alpha \lambda_t}{\pi} \left\{ C_9^{eff} < K(K^*) \mid \overline{s} \gamma^{\mu} P_L b \mid B > (\overline{l} \gamma_{\mu} l) \right\}$$

$$- \frac{2m_b}{a^2} C_7^{eff} < K(K^*) \mid \overline{s} i \sigma^{\mu \nu} P_R b \mid B > (\overline{l} \gamma_{\mu} l) + C_{10} < K(K^*) \mid \overline{s} \gamma^{\mu} P_L b \mid B > (\overline{l} \gamma_{\mu} \gamma_5 l) \right\},$$

$$(3)$$

where  $C_7^{\text{eff}} = C_7 - C_5 / 3 - C_6$ . The Wilson coefficient  $C_9^{\text{eff}}$ from the four-quark operators  $Q_i$  (i = 1,...,6) and,

second, the nonperturbative effects coming from the effectively takes into account, first, the contributions cc-resonance contributions which are as usual parametrized by a Breit-Wigner ansatz [2]:

$$C_{9}^{eff} = C_{9} + C_{0} \left\{ h\left(\hat{m}_{c}, s\right) + \frac{3\pi}{\alpha^{2}} k \sum_{V_{i} = \psi(1s), \psi(2s)} \frac{\Gamma\left(V_{i} \to l^{+}l^{-}\right) m_{V_{i}}}{m_{V_{i}}^{2} - q^{2} - i m_{V_{i}} \Gamma_{V_{i}}} \right\}$$

$$- \frac{1}{2} h(1, s) \left(4C_{3} + 4C_{4} + 3C_{5} + C_{6}\right)$$

$$- \frac{1}{2} h(0, s) \left(C_{3} + 3C_{4}\right) + \frac{2}{9} \left(3C_{3} + C_{4} + 3C_{5} + C_{6}\right),$$

$$(4)$$

where  $C_0 \equiv 3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6$ . Here

$$h(\hat{m}_c, s) = -\frac{8}{9} \ln \frac{m_b}{\mu} - \frac{8}{9} \ln \hat{m}_c + \frac{8}{27} + \frac{4}{9} x$$

$$-\frac{2}{9} (2+x) |1-x|^{1/2} \begin{cases} \left( \ln \left| \frac{\sqrt{1-x}+1}{\sqrt{1-x}-1} \right| - i\pi \right) & \text{for } x \equiv \frac{4\hat{m}_c^2}{s} < 1 \\ 2 \arctan \frac{1}{\sqrt{x-1}} & \text{for } x \equiv \frac{4\hat{m}_c^2}{s} > 1, \end{cases}$$

$$h(0, s) = \frac{8}{27} - \frac{8}{9} \ln \frac{m_b}{\mu} - \frac{4}{9} \ln s + \frac{4}{9} i\pi,$$

where  $m_c = m_c / m_R$ ,  $s = q^2 / m_R^2$  u  $k = 1 / C_0$ .

### Two-fold angular distributions

Let us consider the polar angle decay distribution differential in the momentum transfer

squared  $q^2$ . The polar angle is defined by the angle between  $\vec{q} = \vec{p}_1 - \vec{p}_2$  and  $\vec{k}_1$  ( $\ell^+\ell^-$  rest frame) as shown in Figure 1. One has

$$\frac{d^{2}\Gamma}{dq^{2}d\cos\theta} = \frac{|\mathbf{p}_{2}|\upsilon}{(2\pi)^{3}} \cdot \frac{1}{8} \sum_{pol} |M|^{2} = \frac{G_{F}^{2}}{(2\pi)^{3}} \left(\frac{\alpha|\lambda_{t}|}{2\pi}\right)^{2} \frac{|\mathbf{p}_{2}|\upsilon}{8m_{1}^{2}} \\
\times \frac{1}{8} \left\{ H_{11}^{\mu\nu} \cdot tr \left[ \gamma_{\mu} \left( \mathcal{K}_{1} - m_{\ell} \right) \gamma_{\nu} \left( \mathcal{K}_{1} + m_{\ell} \right) \right] + H_{22}^{\mu\nu} \cdot tr \left[ \gamma_{\mu} \gamma_{5} \left( \mathcal{K}_{1} - m_{\ell} \right) \gamma_{\nu} \gamma_{5} \left( \mathcal{K}_{1} + m_{\ell} \right) \right] + H_{12}^{\mu\nu} \cdot tr \left[ \gamma_{\mu} \gamma_{5} \left( \mathcal{K}_{1} - m_{\ell} \right) \gamma_{\nu} \gamma_{5} \left( \mathcal{K}_{1} + m_{\ell} \right) \right] + H_{21}^{\mu\nu} \cdot tr \left[ \gamma_{\mu} \gamma_{5} \left( \mathcal{K}_{1} - m_{\ell} \right) \gamma_{\nu} \left( \mathcal{K}_{1} + m_{\ell} \right) \right] \right\} = \\
= \frac{G_{F}^{2}}{(2\pi)^{3}} \left( \frac{\alpha|\lambda_{t}|}{2\pi} \right)^{2} \frac{|\mathbf{P}_{2}|\upsilon}{8m_{1}^{2}} \cdot \frac{1}{2} \left\{ L_{\mu\nu}^{1} \cdot \left( H_{11}^{\mu\nu} + H_{22}^{\mu\nu} \right) - \frac{1}{2} L_{\mu\nu}^{2} \cdot \left( q^{2} H_{11}^{\mu\nu} + \left( q^{2} - 4m_{\ell}^{2} \right) H_{22}^{\mu\nu} \right) + L_{\mu\nu}^{3} \cdot \left( H_{12}^{\mu\nu} + H_{21}^{\mu\nu} \right) \right\}, \tag{5}$$

where  $|\mathbf{p}_2| = \lambda^{1/2} \left( m_1^2, m_2^2, q^2 \right) / 2m_1$  is the momentum of the  $K(K^*)$  - meson given in the B-rest frame and  $\beta_l = \sqrt{1 - 4m_l^2 / q^2}$ .

The Lorentz contractions in Eq. (5) can be evaluated in terms of helicity amplitudes as described in [3]. First, we define an orthonormal and

complete helicity basis  $\in^{\mu}(m)$  with the three spin 1 components orthogonal to the momentum transfer  $q_{\mu}$ , i.e.  $\in^{\mu}(m)q_{\mu}=0$  for  $m=\pm 0$ , and the spin 0 (time)-component m=t with  $\in^{\mu}(t)=q^{\mu}/\sqrt{q^2}$ . The orthonormality and completeness properties read

$$\epsilon_{\mu}^{\dagger}(m) \epsilon^{\mu}(n) = g_{mn}, \qquad \text{(orthonormality)}$$

$$\epsilon_{\mu}(m) \epsilon_{\nu}^{\dagger}(n) g_{mn} = g_{\mu\nu}, \qquad (m, n = t, \pm 0) \qquad \text{(completeness)}$$
(6)

with  $g_{mn} = diag(+,-,-,-)$ . Using the completeness property we rewrite the contraction of the lepton and hadron tensors in Eq. (5) according to

$$L^{(k)\mu\nu}H^{ij}_{\mu\nu} = L^{(k)}_{\mu'\nu'} \in^{\mu'}(m) \in^{\dagger\mu}(m')g_{mm'} \in^{\dagger\nu'}(n) \in^{\nu}(n')g_{mn'}H^{ij}_{\mu\nu} = L^{(k)}_{mn}g_{mm'}g_{nn'}H^{ij}_{m'n'}, \tag{7}$$

where we have introduced the lepton and hadron tensors in the space of the helicity components

$$L_{mn}^{(k)} = \in^{\mu} (m) \in^{\dagger \nu} (n) L_{\mu\nu}^{(k)}, \qquad H_{mn}^{ij} = \in^{\dagger \mu} (m) \in^{\nu} (n) H_{\mu\nu}^{ij}. \tag{8}$$

The lepton tensors  $L^{(k)} = (m, n)$  will be evaluated in the  $\ell\ell$ -CM system whereas the hadron tensors  $H^{ij}(m,n)$  will be evaluated in the B rest system.

The differential  $(q^2, \cos \theta)$  distribution finally reads

$$\frac{d\Gamma(H_1 \to H_2 \bar{\ell}\ell)}{dq^2 d(\cos\theta)} = \frac{3}{8} (1 + \cos^2\theta) \cdot \frac{1}{2} \left( \frac{d\Gamma_U^{11}}{dq^2} + \frac{d\Gamma_U^{22}}{dq^2} \right) + \frac{3}{4} \sin^2\theta \cdot \frac{1}{2} \left( \frac{d\Gamma_L^{11}}{dq^2} + \frac{d\Gamma_L^{22}}{dq^2} \right) - \upsilon \cdot \frac{3}{4} \cos\theta \cdot \frac{d\Gamma_P^{12}}{dq^2} + \frac{3}{4} \sin^2\theta \cdot \frac{1}{2} \frac{d\tilde{\Gamma}_L^{11}}{dq^2} - \frac{3}{4} \sin^2\theta \cdot \frac{1}{2} \frac{d\tilde{\Gamma}_L^{12}}{dq^2} - \frac{3}{4} \sin^2\theta \cdot \frac{1}{2} \frac{d\tilde{\Gamma}_L^{12}}{dq^2} + \frac{3}{4} \frac{d\tilde{\Gamma}_S^{22}}{dq^2} \right)$$
(9)

Integrating over  $cos\theta$  one obtains

$$\frac{d\Gamma\left(H_{1} \to H_{2}\overline{\ell}\ell\right)}{dq^{2}} = \frac{1}{2} \left(\frac{d\Gamma_{U}^{11}}{dq^{2}} + \frac{d\Gamma_{U}^{22}}{dq^{2}} + \frac{d\Gamma_{L}^{11}}{dq^{2}} + \frac{d\Gamma_{L}^{22}}{dq^{2}}\right) + \frac{1}{2} \frac{d\tilde{\Gamma}_{U}^{11}}{dq^{2}} - \frac{d\tilde{\Gamma}_{U}^{22}}{dq^{2}} + \frac{1}{2} \frac{d\tilde{\Gamma}_{L}^{11}}{dq^{2}} - \frac{d\tilde{\Gamma}_{L}^{22}}{dq^{2}} + \frac{3}{2} \frac{d\tilde{\Gamma}_{S}^{22}}{dq^{2}}, \quad (10)$$

where the partial helicity rates  $d\Gamma_X^{ij}/dq^2$  and  $d\tilde{\Gamma}_X^{ij}/dq^2(X=U,L,P,S;i,j=1,2)$  are defined as

$$\frac{d\Gamma_{X_{ij}}}{dq^2} = \frac{G_F^2}{(2\pi)^3} \left(\frac{\alpha \left|\lambda_t\right|}{2\pi}\right)^2 \frac{\left|\mathbf{p}_2 q^2 \upsilon\right|}{12m_1^2} H_X^{ij}, \quad \frac{d\tilde{\Gamma}_{X_{ij}}}{dq^2} = \delta_{\ell\ell} \frac{d\Gamma_X^{ij}}{dq^2}, \quad \delta_{\ell\ell} \equiv \frac{2m_\ell^2}{q^2}. \tag{11}$$

## The four-fold angle distribution

The lepton-hadron correlation function  $L_{\mu\nu}H^{\mu\nu}$  reveals even more structure when one uses the cascade decay  $B \to K^*(\to K\pi)\bar{\ell}\ell$  to analyze the polarization of the  $K^*$ . The hadron tensor now reads

$$H_{\mu\nu}^{ij} = T_{\mu\alpha}^{i} \left( T_{\nu\beta}^{i} \right)^{\dagger} \frac{3}{2 |\mathbf{p}_{3}|} \mathrm{B}r \left( K^{*} \to K\pi \right) p_{3\alpha'} p_{3\beta'} S^{\alpha\alpha'} \left( p_{2} \right) S^{\beta\beta'} \left( p_{2} \right), \tag{12}$$

where  $S^{\alpha\alpha'}(p_2) = -g^{\alpha\alpha'} + p_2^{\alpha} p_2^{\alpha'} / m_2^2$  is the standard spin 1 tensor,  $p_2 = p_3 + p_4$ ,  $p_3^2 = m_K^2$ ,  $p_4^2 = m_\pi^2$ , and  $p_3$  and  $p_4$  are the momenta of the K and the  $\pi$ , respectively. The relative configuration of the

$$(K,\pi)$$
- and  $(\overline{\ell}\ell)$ -planes is shown in Fig. 1.

Following basically the same trick as in Eq. (8) the contraction of the lepton and hadron tensors may be written through helicity components as

$$L^{(k)\mu\nu}H^{ij}_{\mu\nu} = \in^{\mu'}(m) \in^{\dagger\nu'}_{2}(n) L^{k}_{\mu'\nu'}g_{mn'}g_{nn'} \in^{\dagger\mu}(m') \in^{\nu}(n') H^{ij}_{\mu\nu}$$

$$= L^{k}_{mn}g_{mn'}g_{mn'}(\in^{\dagger\mu}(m') \in^{\dagger\alpha}(r) T^{i}_{\mu\alpha})(\in^{\dagger\nu}(n') \in^{\dagger\alpha}_{2}(s) T^{i}_{\nu\beta})^{\dagger}$$

$$\times p_{3} \in_{2}(r) \cdot p_{3} \in^{\dagger}_{2}(s) \frac{3Br(K^{*} \to K\pi)}{2|\mathbf{p}_{3}|} = \frac{3Br(K^{*} \to K\pi)}{2|\mathbf{p}_{3}|} \left(L^{k}_{u}|H^{ij}_{t}|^{2} \cdot \left(p_{3} \in^{\dagger}_{2}(0)\right)^{2}\right)$$

$$+ \sum_{m,n=\pm,0} L^{k}_{mn}H^{i}_{m}H^{\dagger j}_{n} \cdot p_{3} \in_{2}(m) \cdot p_{3} \in^{\dagger}_{2}(n)$$

$$- \sum_{n=\pm,0} L^{k}_{tn}H^{i}_{t}H^{\dagger j}_{n} \cdot p_{3} \in_{2}(0) \cdot p_{3} \in^{\dagger}_{2}(n)$$

$$- \sum_{m=\pm,0} L^{k}_{mt}H^{i}_{m}H^{\dagger j}_{t} \cdot p_{3} \in_{2}(m) \cdot p_{3} \in^{\dagger}_{2}(0).$$

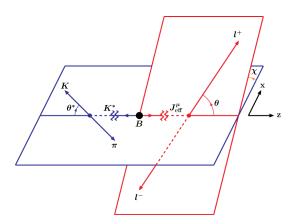
$$(13)$$

Using these results one obtains the full four-fold angular decay distribution

$$\frac{d\Gamma\left(B \to K^*\left(\to K\pi\right)\bar{l}l\right)}{dq^2d\cos\theta d\left(\chi/2\pi\right)d\cos\theta^*} = Br\left(K^* \to K\pi\right)$$

$$\times \left\{\frac{3}{8}\left(1 + \cos^2\theta\right) \cdot \frac{3}{4}\sin^2\theta^* \cdot \frac{1}{2}\left(\frac{d\Gamma_U^{11}}{dq^2} + \frac{d\Gamma_U^{22}}{dq^2}\right) + \frac{3}{4}\sin^2\theta \cdot \frac{3}{2}\cos^2\theta^* \cdot \frac{1}{2}\left(\frac{d\Gamma_L^{11}}{dq^2} + \frac{d\Gamma_U^{22}}{dq^2}\right) - \frac{3}{4}\sin^2\theta \cdot \cos2\chi \cdot \frac{3}{4}\sin^2\theta^* \cdot \frac{1}{2}\left(\frac{d\Gamma_U^{11}}{dq^2} + \frac{d\Gamma_U^{22}}{dq^2}\right) + \frac{9}{16}\sin2\theta \cdot \cos\chi \cdot \sin2\theta^* \cdot \frac{1}{2}\left(\frac{d\Gamma_I^{11}}{dq^2} + \frac{d\Gamma_I^{22}}{dq^2}\right) + \frac{9}{16}\sin\theta \cdot \sin\chi \cdot \sin2\theta^* \cdot \left(\frac{d\Gamma_H^{12}}{dq^2} + \frac{d\Gamma_H^{21}}{dq^2}\right) - \frac{9}{32}\sin2\theta \cdot \sin\chi \cdot \sin2\theta^* \cdot \left(\frac{d\Gamma_H^{11}}{dq^2} + \frac{d\Gamma_H^{22}}{dq^2}\right) + \frac{9}{32}\sin^2\theta \cdot \sin\chi \cdot \sin2\theta^* \cdot \left(\frac{d\Gamma_H^{11}}{dq^2} + \frac{d\Gamma_H^{22}}{dq^2}\right) + \frac{3}{4}\sin^2\theta \cdot \frac{3}{4}\sin^2\theta^* \cdot \frac{1}{2}\cdot\frac{d\tilde{\Gamma}_U^{11}}{dq^2} - \frac{3}{8}\left(1 + \cos^2\theta\right) \cdot \frac{3}{4}\sin^2\theta^* \cdot \frac{d\tilde{\Gamma}_U^{22}}{dq^2} + \frac{3}{4}\sin^2\theta \cdot \frac{3}{4}\sin^2\theta^* \cdot \frac{1}{2}\cdot\frac{d\tilde{\Gamma}_U^{11}}{dq^2} - \frac{3}{8}\left(1 + \cos^2\theta\right) \cdot \frac{3}{4}\sin^2\theta^* \cdot \frac{d\tilde{\Gamma}_U^{22}}{dq^2} + \frac{3}{4}\sin^2\theta \cdot \frac{3}{4}\sin^2\theta^* \cdot \frac{d\tilde{\Gamma}_U^{22}}{dq^2} + \frac{3}{4}\sin^2\theta \cdot \cos2\chi \cdot \frac{3}{4}\sin^2\theta^* \cdot \left(\frac{d\tilde{\Gamma}_H^{11}}{dq^2} + \frac{d\tilde{\Gamma}_H^{22}}{dq^2}\right) - \frac{9}{8}\sin2\theta \cdot \cos\chi \cdot \sin2\theta^* \cdot \frac{1}{2}\left(\frac{d\tilde{\Gamma}_H^{11}}{dq^2} + \frac{d\tilde{\Gamma}_H^{22}}{dq^2}\right) + \frac{3}{2}\cos^2\theta^* \cdot \frac{d\tilde{\Gamma}_U^{22}}{dq^2} + \frac{3}{4}\sin^2\theta \cdot \cos2\chi \cdot \frac{3}{4}\sin^2\theta^* \cdot \left(\frac{d\tilde{\Gamma}_H^{11}}{dq^2} + \frac{d\tilde{\Gamma}_H^{22}}{dq^2}\right) - \frac{9}{8}\sin2\theta \cdot \cos\chi \cdot \sin2\theta^* \cdot \left(\frac{d\tilde{\Gamma}_H^{11}}{dq^2} + \frac{d\tilde{\Gamma}_H^{22}}{dq^2}\right) + \frac{3}{2}\cos^2\theta^* \cdot \frac{d\tilde{\Gamma}_U^{22}}{dq^2} + \frac{9}{16}\sin2\theta \cdot \sin\chi \cdot \sin2\theta^* \cdot \left(\frac{d\tilde{\Gamma}_H^{11}}{dq^2} + \frac{d\tilde{\Gamma}_H^{22}}{dq^2}\right) - \frac{9}{8}\sin2\theta \cdot \sin2\chi \cdot \sin2\theta^* \cdot \left(\frac{d\tilde{\Gamma}_H^{11}}{dq^2} + \frac{d\tilde{\Gamma}_H^{22}}{dq^2}\right) + \frac{3}{2}\cos^2\theta^* \cdot \frac{3}{4}\frac{d\tilde{\Gamma}_H^{22}}{dq^2} + \frac{9}{16}\sin2\theta \cdot \sin\chi \cdot \sin2\theta^* \cdot \left(\frac{d\tilde{\Gamma}_H^{11}}{dq^2} + \frac{d\tilde{\Gamma}_H^{22}}{dq^2}\right) - \frac{9}{16}\sin2\theta \cdot \sin2\chi \cdot \sin2\theta^* \cdot \left(\frac{d\tilde{\Gamma}_H^{11}}{dq^2} + \frac{d\tilde{\Gamma}_H^{22}}{dq^2}\right) + \frac{3}{2}\cos^2\theta^* \cdot \frac{d\tilde{\Gamma}_H^{22}}{dq^2} + \frac{9}{16}\sin2\theta \cdot \sin\chi \cdot \sin2\theta^* \cdot \left(\frac{d\tilde{\Gamma}_H^{11}}{dq^2} + \frac{d\tilde{\Gamma}_H^{22}}{dq^2}\right) - \frac{9}{16}\sin2\theta \cdot \sin2\chi \cdot \sin2\theta^* \cdot \left(\frac{d\tilde{\Gamma}_H^{11}}{dq^2} + \frac{d\tilde{\Gamma}_H^{22}}{dq^2}\right) + \frac{9}{16}\sin2\theta \cdot \sin2\chi \cdot \sin2\theta^* \cdot \left(\frac{d\tilde{\Gamma}_H^{11}}{dq^2} + \frac{d\tilde{\Gamma}_H^{22}}{dq^2}\right) + \frac{9}{16}\sin2\theta \cdot \sin2\chi \cdot \sin2\theta^* \cdot \left(\frac{d\tilde{\Gamma}_H^{$$

Integrating Eq. (14) over  $\cos \theta^*$  and  $\chi$  one recovers the two-fold  $(q^2, \cos \theta)$  distribution of Eq. (9).



**Figure 1** – Definition of angles in the cascade decay  $B \to K^* (\to K\pi) \bar{\ell} \ell$ 

### Conclusion

Note that a similar four-fold distribution has also been obtained in Refs. [4], [5] using, however, the zero lepton mass approximation. If there are sufficient data one can attempt to fit them to the full four-fold decay distribution and thereby extract the values of the coefficient functions  $d\Gamma_X^{ij}/dq^2$  and, in the case  $\ell=\tau$  the coefficient functions  $d\tilde{\Gamma}_X^{ij}/dq^2$ . Instead of considering the full four-fold decay distribution one can analyze single angle distributions by integrating out two of the remaining angles.

We have performed a detailed analysis of the decay process  $B \rightarrow K^*(\rightarrow K\pi)\bar{\ell}\ell$  by using the helicity formalism to analyze the angular decay distribution.

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