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Finite-size effects in the interaction of dust particles in a plasma

Abstract. A pseudopotential model of interaction between dust particles is proposed to simultaneously take into account the finite-size and the screening effects. The consideration starts from the assumption that the dust particles are hard balls such that the interparticle distances are measured between their surfaces rather than between their centers. After that to derive the screened interaction potential between dust particles the density-response formalism is used in which the dielectric function of the plasma is taken in the form of the random-phase approximation. This procedure provides a simple analytic expression for the intergrain potential that numerically differs from the well-known Debye-Huckel (Yukawa) potential at small separations.

Key words: Dusty plasma, density-response formalism, finite-size effects, random-phase approximation.

Introduction

For the past few decades dusty plasmas have attracted a great deal of attention of plasma physics researchers since they are frequently encountered in various settings both in nature and in the laboratory. In particular, dusty plasmas appear in various astrophysical contexts [1-3], space and earth experiments [4-6], nanotechnology [7, 8], cancer therapy in medicine [9, 10], etc. Furthermore, as it happens in nuclear fusion researches [11, 12] and plasma etching in electronics [13, 14], solid microparticles readily penetrate into the plasma medium as a consequence of its contact with electrodes and chamber walls, thereby substantially modifying both surface properties of the confining material and the local plasma characteristics.

What makes dusty plasma a unique object for investigation is the presence of micron-sized dust particles, called grains, which are capable of acquiring a high, mostly negative, electric charge [15, 16] which invokes diverse manifestations of strong coupling effects [17]. For instance, first experiments with dusty plasmas clearly demonstrated that under certain conditions strong electrostatic interactions between grains took over their thermal kinetic energy resulting in the formation of the so-called plasma crystals [18-20]. The latter are quite similar, in physical properties, to ordered structures in liquids and solids, such that even phase transitions of the first and second orders are easily observed [21, 22].

In order to correctly describe the properties of non-ideal systems it is crucial to know the form of interaction energy between the constituent elements. As for dusty plasmas, the interaction potential between grains is conventionally taken in the form of the screened Coulomb (Yukawa) potential [23-25] which is only known to be valid for point-like dust particles immersed into the buffer plasma whose role is virtually reduced to shielding of the electric field. In this manuscript an effective potential of interparticle interaction in dusty plasmas is proposed to take into account finite dimensions of grains. The idea is to start counting distances between the surfaces of dust particles rather than between their centers [26] with subsequent application of the density-response formalism in which the dielectric function of the buffer plasma is taken in the form of the random phase approximation.

Dimensionless plasma parameters

Of interest in the following is the interaction of two dust particles immersed into the buffer plasma of electrons and ions. For the sake of simplicity the buffer plasma is considered to be a fully ionized hydrogen consisting of free electrons with the electric charge $-e$ and the number density n_e and of free protons with the electric charge e and the number density $n_p = n_e = n$. The microparticles are assumed to be hard balls of radius R and of the electric charge $-Z_d e$ such that their number density

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n_d is so small to satisfy the inequality $n_d Z_d << n$ which assures the total plasma neutrality.

To describe the state of the dust component of the plasma it is convenient to introduce the coupling parameter as

$$\Gamma = \frac{Z_d^2 e^2}{R k_B T}, \quad (1)$$

where k_B stands for the Boltzmann constant and T denotes the medium temperature.

Coupling parameter (1) is not conventional since it represents the ratio of the electrostatic interaction energy of two grains, separated by the distance R, to their average kinetic energy of chaotic thermal motion.

Another suitable dimensionless parameter involved is the screening parameter defined as

$$\kappa = \frac{R}{\lambda_D}, \quad (2)$$

where $\lambda_D = (k_B T / 8\pi n e^2)^{1/2}$ designates the Debye screening radius.

It has to be stressed that knowledge of dimensionless parameters (1) and (2) is perfectly enough to completely describe the interaction of two isolated hard balls placed into the buffer plasma.

Boundary condition

It is widely believed in the literature that the dust charge is determined by the normal component of the electric field strength at the particle surface. This inference is usually made from the following equation for the electric field strength $\mathbf{E}$ as applied to the cylindrical volume of the cross section S shown in Figure 1:

$$\oint \mathbf{E} \cdot \mathbf{dS} = 4\pi(\sigma S + \rho_{pl} V), \quad (3)$$

where σ stands for the surface charge density on the dust particle and ρ_{pl} refers to the plasma charge density.

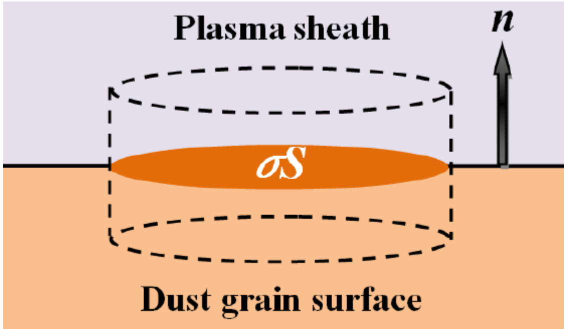

Figure 1 – To the derivation of the boundary condition.

Without any loss of generality the dust grain material can be assumed to be conductive so that the electric field strength under the grain surface turns zero and, then, applying eq. 1 to the infinitesimally thin cylinder, $V \to 0$, finally yields

$$E_n = 4\pi\sigma. \quad (4)$$

Boundary condition (4) is incorrect from the viewpoint of plasma electrodynamics because the cylinder in equation (3) cannot be taken infinitesimally thin, otherwise one has to inevitably turn to consideration of the microscopic electric field which rapidly fluctuates over time in contrast to the macroscopically averaged electric field entering equation (3).

To correctly derive the boundary condition one has to use the following explicit equation for the dielectric displacement vector $\mathbf{D}$ which stems from the plasma electrodynamics:

$$\oint \mathbf{D} \cdot \mathbf{dS} = 4\pi\sigma S, \quad (5)$$

When applied to a rather small cylinder still containing enough number of plasma particles to treat the electric field macroscopically, (5) gives rise to the correct boundary condition

$$D_n = 4\pi\sigma. \quad (6)$$

for the dielectric displacement vector in a plasma near the dust surface.

Boundary condition (6) differs significantly from (4) because the displacement vector $\mathbf{D}$ is expressed in terms of the electric field strength $\mathbf{E}$ in the static case of plasma electrodynamics via the integral relation

$$\mathbf{D}(\mathbf{r}) = \int \varepsilon(\mathbf{r}-\mathbf{r}_1)\mathbf{E}(\mathbf{r}_1)\mathbf{dr}_1 , \qquad (7)$$

where $\varepsilon(\mathbf{r})$ stands for the plasma dielectric function defined in the configurational space.

It is, thus, rather clear how to accurately work out the problem of the intergrain interaction in the buffer plasma. One has to consider a spatially finite plasma with boundary condition (6) and spatially varying plasma parameters, i.e. to construct an exact theory of the plasma sheath. This is quite a complicated problem to solve analytically and all further simplified consideration is aimed at establishing what impact expression (6) has on the interaction between plasma particles and the dust grain.

Interaction model

It is well known [27] that in spatially infinite plasmas the Fourier transform of the screened interaction potential $\tilde{\Phi}(\mathbf{k})$ between the dust particles is expressed in terms of the Fourier transform of the true microscopic interaction potential $\tilde{\varphi}(\mathbf{k})$ and the plasma static dielectric function $\varepsilon(\mathbf{k})$ as:

$$\tilde{\Phi}(\mathbf{k}) = \frac{\tilde{\varphi}(\mathbf{k})}{\varepsilon(\mathbf{k})}, \qquad (8)$$

in which the static dielectric function can be taken in the form of the random phase approximation [28] as

$$\varepsilon(k) = 1 + \frac{k_D^2}{k^2} , \qquad (9)$$

where $k_D = 1/\lambda_D$ denotes the wavenumber inversed to the Debye screening radius.

Using the convolution theorem it is convenient in the sequel to rewrite relation (8) as

$$\Phi(\mathbf{r}) = \int \varepsilon^{-1}(\mathbf{r}-\mathbf{r}_1)\varphi(\mathbf{r}_1)\mathbf{dr}_1 , \qquad (10)$$

where the kernel is found from equation (9) as

$$\varepsilon^{-1}(r) = \delta(r) - \frac{k_D^2}{2\pi r}\exp(-k_D r) . \qquad (11)$$

To practically apply formulas (10) and (11), initially worked out for an infinite plasma, to a spatially finite plasma of interest it is proposed herein to treat dust grains as point-like charges by counting all distances r from the dust grain surfaces so that the interaction micropotential acquires the form:

$$\varphi(r) = \frac{Z_d^2 e^2}{r+2R} . \qquad (12)$$

On substituting expressions (11) and (12) into (10), one ultimately gets

$$\Phi(r) = \varphi(r) - \frac{Z_d^2 e^2}{r}\left[1 - \exp(-k_D r) - k_D R\,\mathrm{B}(r)\right], \qquad (13)$$

where

$$\begin{aligned} \mathrm{B}(r) = {} & \exp(k_D(2R+r))\,\mathrm{Ei}(k_D(2R+r)) - \\ & -\exp(k_D(2R-r))\,\mathrm{Ei}(2k_D R) + \\ & +\exp(-k_D(2R+r))\left[\mathrm{Ei}(2k_D R) - \mathrm{Ei}(-k_D(2R+r)\right] \end{aligned} \qquad (14)$$

with the exponential integral function defined as

$$\mathrm{Ei}(x) = \int_x^\infty \frac{\exp(-t)}{t}dt . \qquad (15)$$

Note that it is ordinarily assumed in the literature that the screening effects start from the dust grain surface such that the Debye-like theory gives rise to the following interaction potential

$$\Phi(r) = \frac{Z_d^2 e^2}{(r+2R)(1+2k_D R)}\exp(-k_D r) . \qquad (16)$$

In Figure 2 a comparison is made between expressions (12), (13) and (16) at $\Gamma = 10$ and $\kappa = 0.5$. It is well seen that potentials (13) and (16) are effectively screened and micropotential (12) lies high above Yukawa potential (16) and the proposed potential (13) that has a gap at the origin caused by engaging of the plasma electrodynamics. It is worth noting that the onset of the gap is a straightforward consequence of boundary condition (6).

Figure 3 demonstrates the intergrain interaction potential (13) as a function of the distance r and the screening parameter κ at the fixed value of the coupling parameter $\Gamma = 100$. It is clearly observed that the increase in the screening parameter results in stronger shielding of the intergrain interaction and, at the same time, in lowering of the potential at the origin.

It is worth mentioning that the constructed potential of the intergrain interaction can be used to calculate the static correlation functions which can then be applied to find thermodynamic properties or to even evaluate dynamic characteristics of the dust component using the theory of moments [29.30].

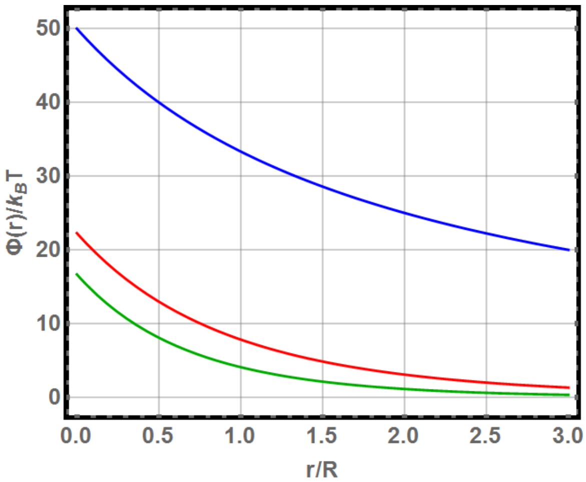

Figure 2 – The intergrain interaction potential at Γ=100 and *k*=1.0. Blue line: micropotential (12); green line: Yukawa potential (16); red line: screened potential (13).

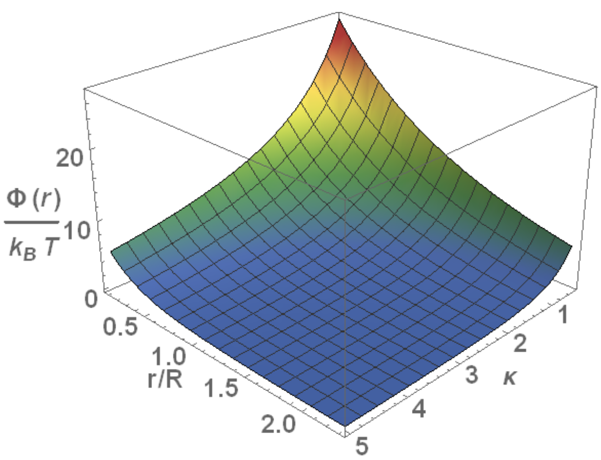

Figure 3 – The intergrain interaction potential as a function of the distance *r* and of the screening parameter *k* at Γ=100.

Conclusions

This paper has been solely concentrated on the problem of derivation of the interaction potential between two isolated dust particles immersed into a buffer plasma. The plasma electrodynamics has provided an important insight that the charge of the dust grain determines the normal component of the dielectric displacement vector near the grain surface. A simple model has been put forward to derive the screened interaction potential between dust grains. It is based on the idea of counting distances between dust particles surfaces with further application of the density-response formalism to account for the buffer plasma screening. The numerical results portray that a gap between the obtained potential and the micropotential appears at the origin and an increase in the screening parameter gives rise to stronger shielding of the interaction as well as to lowering of the potential at the origin.

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