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[1*]Zhilisbayeva K., [1]Doszhan N., [2]Saspayeva A.

[1]Faculty of Mechanics and Mathematics, al-Farabi Kazakh National University, Almaty, Kazakhstan
[2]National center space researches and technologies, Almaty, Kazakhstan
*e-mail: zhilisbaeva@mail.ru

Changing the magnetic field intensity during the motion of spacecraft

Abstract: In this paper the change of the intensity vector of geomagnetic field during the motion of the spacecraft in orbit in direct dipole model was investigated. The dependences of the intersection point coordinates of the vector intensity of the geomagnetic field with the unit radius Earth sphere were found, their graphs were built and their extremes were determined. The relations of the intersection point velocity of the vector intensity of the geomagnetic field to the unit radius Earth sphere were found, their graphs were built and their extremes determined. The graphs of each component of the geomagnetic field along the trajectory of the spacecraft and the combined graphs were built.
Key words: Magnetic field intensity, direct dipole, spacecraft, rotational motion, orientation.

Introduction

Among the spacecrafts (SC) functioning currently the majority are low-altitude (altitude less than 1000 km) spacecrafts. It is due to the broad area of use: scientific, technological and educational satellites.

To control flight and experiments conducted on the board of low-altitude spacecraft in near real timeit is needed to create a motion control system in order to solve the problem of determining the orientation quickly and autonomously. For example, information about the state vector of spacecraft with scientific and technological experiments carried out on the board is necessary for the correct interpretation of the results. Herewith, the requirements of accuracy of somestate vector elements of the spacecraft (e.g., orientation) may be low (approximately 5 ° error).

Magnetometers and sun sensors became widespread in the attitude control systems because of their high reliability and efficiency.

A lot of papers devoted to the development of methods focused on determination of orientation of the magnetometric measurements.

It is important to note that an unambiguous one-time definition of orientation only by magnetometric measurements is impossible.

The problem of removing the ambiguity of determining the orientation by magnetometric measurements is achieved either by using a model of movement relative to the center of mass or by aggregation of magnetometric data with information from the additional measuring device.

Currently existing methods of determining the orientation can be divided into two classes: the methods with usage of the movement relative to the center of mass (CM) (integral methods) and methods without usage of them (local).

The relevance of the research topic: In all satellite missions relevant issue is the problem of satellite orientations. The bulk of the logic and algorithms orientations are put into the base instrument onboard the satellite subsystems. Relevance of the topic of this research is to develop algorithms for determining the orientation of the magnetometric satellite, based on a three-axis magnetometer data aggregation.

Development and improvement of systems of orientation of magnetometric satellite. Each study in this subject is a new data, new calculations and new improved algorithms.

Purpose: The aim is to study the existing on-board subsystems designed to determine the orientation used on spacecraft for various purposes, including the development of algorithms for determining the orientation of magnetometric satellite.

Research objectives:

- To develop algorithms for determining the orientation of the magnetometric satellite based on magnetometric data aggregation;

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- To study the existing on-board subsystems to determine the orientation used in the magnetometric satellites and their analysis;
- Numerical calculations and analysis of the results.

Methods

Integral methods. The main idea is in usage of a mathematical model of the motion relative to the CM in order to combine measurements taken at a sufficient interval for the treatment of dimensional time.

Determination of orientation using the mathematical model of the motion relative to the CM may be performed on the basis of measurements of one direction. At the same time, however, significant change of direction of the vector being measured in the dimensional range is necessary; otherwise orientation of the spacecraft can be determined only up to an arbitrary rotation about the vector.

Integral methods are not applicable in the case of presence of significant underestimated disturbing moments acting on the spacecraft. Also, this method of determining the orientation is not applicable to solution of the problem of determining the orientation in the time scale near the real.

Local methods. The basis of the local methods of simultaneous determination of the orientation is a method of matching measurements of two or more vectors in the two coordinate systems. As the measured parameters in the calculation of the angular position of the spacecraft values, which characterizessome directions in connected with spacecraft coordinate system, known a priori at the base (the absolute, orbital or in another convenient) coordinate system, are used. If several directions for the spacecraftare determined at the same time, then the number of measurement functions will be sufficient to calculate the matrix of orientationat any moment of time obtaining measurements.

Local methods for determining the orientation are preferred to solve the problem of promptly determining the orientation on board of the spacecraft.

Based on the above solvable goal and objective of the thesis is being formed.

Theoretical and practical significance of the research

The practical significance of the research is the design of onboard subsystems for magnetometric satellite and study of Earth's magnetic field.

Due to the fact that magnetometric measurements are used in research work, it is necessary to consider the model of Earth's magnetic field (EMF).

In the first approximation, the magnetic field of the Earth is considered as a dipole, which it's axis is called geomagnetic axis and it's angle with the Earth rotational axis is ≈ 11,5° (Fig.1).

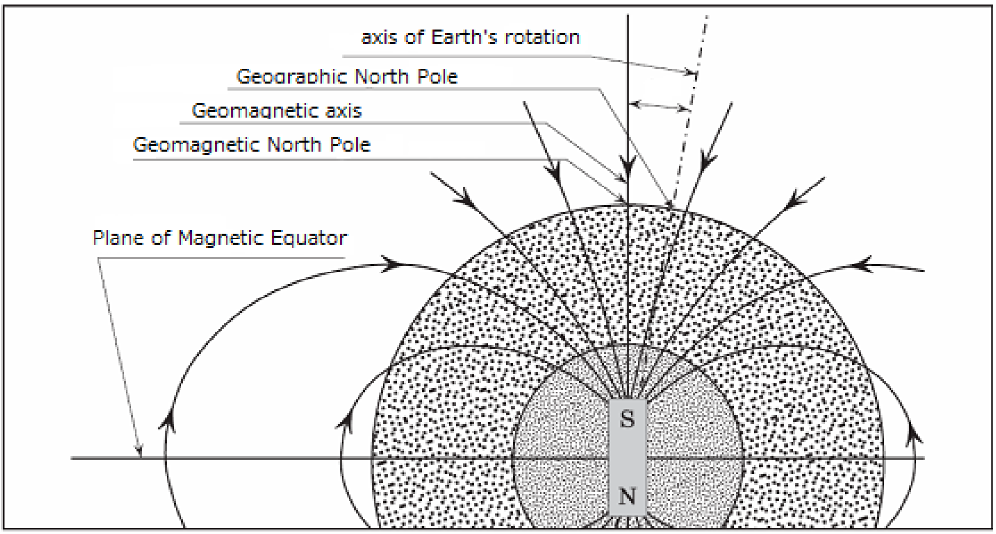

Figure 1 – Earth's magnetic field

The Earth's magnetic field represented as maps of isolines or analytically as relations ofthe intensity to geographical coordinates of the observation point. Analytical representation is more suitable for the study of the influence of Earth's magnetic field on the satellite's motion. Currently, the main part of the Earth's magnetic field is usually described by infinite series:

$$U = \overline{R}_E \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left[\left(J_n^m \cos m\lambda + i_n^m \sin m\lambda \right) \left(\frac{\overline{R}_E}{R} \right)^{n+1} + \left(E_n^m \cos m\lambda + e_n^m \sin m\lambda \right) \left(\frac{R}{\overline{R}_E} \right)^{n} \right] \widetilde{P}_n^m \cos\theta \quad (1)$$

where, $\widetilde{P}_n^m \cos\theta$ – is associated Legendre function of the first kind; λ – geographic longitude; θ – addition to the latitude of the observation point; $\overline{R}_E$ – the average radius of the Earth; R – the distance from the Earth's center to the observation point; J_n^m, i_n^m – constant coefficients corresponding to the outer part of the Earth's magnetic field; E_n^m, e_n^m – constant coefficients corresponding to the inner part of the Earth's magnetic field;

In tasks of the magnetic control and orientation in the analytical representations of EMF, the external sources are not considered.

The components of the full vector of the magnetic field of the Earth

The earth's magnetic field usually considered in a geographic coordinate system $OX_gY_gZ_g$, whose axes are directed: OX_g – along the geographic meridian to the north; OY_g – along the parallel to the east; OZ_g – along the vertical to center of the Earth (Fig.2), where, $\vec{X}$ – Northern component directed along the axis OX_g, $\vec{Y}$ – Eastern component directed along the axis OY_g, $\vec{Z}$ – Vertical component directed along the axis OZ_g, $\vec{T}$ – Full intensity vector, $\vec{H}$ – horizontal component located in a horizontal plane, D – Magnetic declination, J – magnetic inclination.

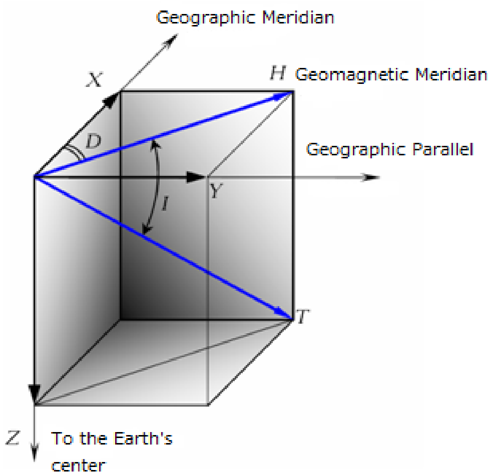

Figure 2 – The components of the full vector of the magnetic field of the Earth

In the study of motion of the spacecraft in the earth's magnetic field the geomagnetic field is represented as dipole field which located in the center of the earth and possessing a magnetic moment $M_E = 8{,}07 \cdot 10^{25}\ G \cdot cm^3$. In the dipole approximation, the different characteristics of the magnetic field are calculated using simple analytical equations:

$$U = \frac{M_E}{R^2}\cos\theta, \quad (2)$$

$$Z = -\frac{\partial U}{\partial z_g} = \frac{\partial U}{\partial R} = -\frac{2M_E}{R^3}\cos\theta, \quad (3)$$

$$H = -\frac{\partial U}{\partial x_g} = -\frac{\partial U}{R\partial\theta} = \frac{M_E}{R^3}\sin\theta, \quad (4)$$

$$T = \frac{M_E}{R^3}\sqrt{1+3\cos^2\theta}\,, \qquad (5)$$

$$tgJ = \frac{Z}{H} = -3ctg\theta = tg\Phi\,, \qquad (6)$$

where, U – dipole potential, Φ – geomagnetic latitude(the angle between the radius vector drawn from the center of the Earth, and the plane of the geomagnetic equator), $\theta = \frac{\pi}{2} - \Phi$ - addition to the magnetic latitude.

Graphs of the magnetic field along the trajectory of the spacecraft

The following characteristics of spacecraft were used as example:

Class orbit – LEO, type of orbit – sun-synchronous, inclination – 98.2°, eccentricity – 0.00190988187027, perigee – 685.0 km, apogee – 712.0 km, period – 98.74 min. According to these data by calculation program were built the graphics of components $\vec{X}$, $\vec{Y}$, $\vec{Z}$ and full intensity vector $\vec{T}$ of EMF along its trajectory (Figures 3-7).

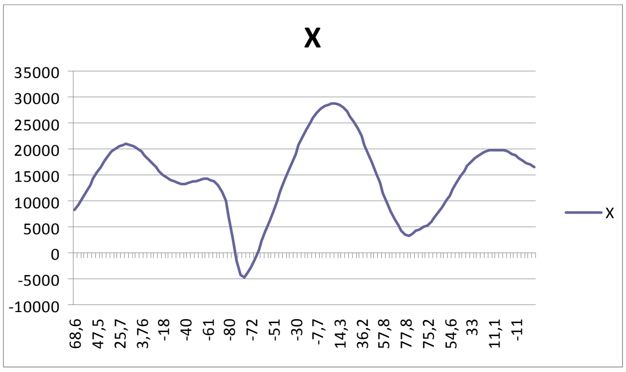

Figure 3 – Graph of X component of the Earth magnetic field along the trajectory of the spacecraft

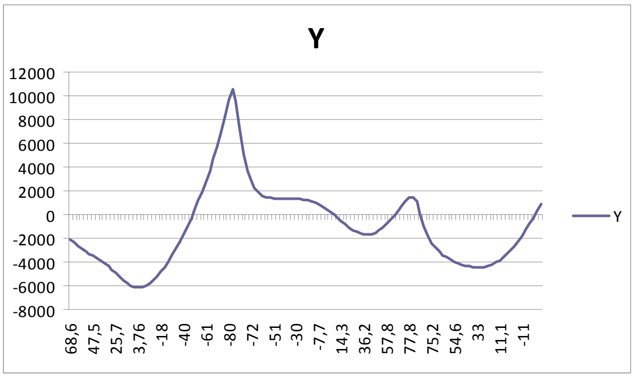

Figure 4 – Graph of Y component of the Earth magnetic field along the trajectory of the spacecraft

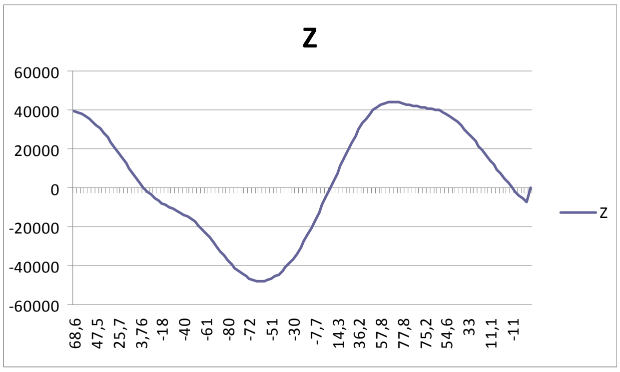

Figure 5 – Graph of Z component of the Earth magnetic field along the trajectory of the spacecraft

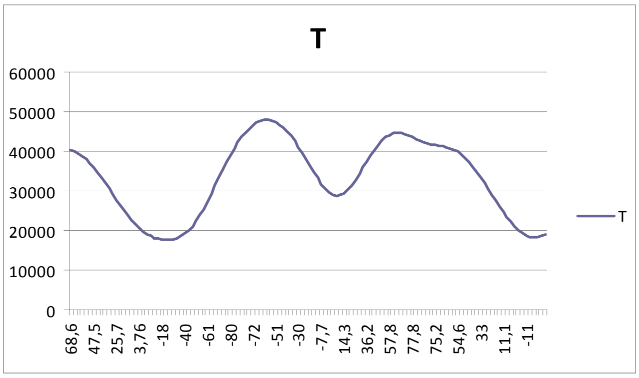

Figure 6 – Graph of intensity T of the Earth magnetic field along the trajectory of the spacecraft

For clarity and analysis, changes in the components of the magnetic field of the Earth resulting curves are combined and represented on one graph (Fig.7).

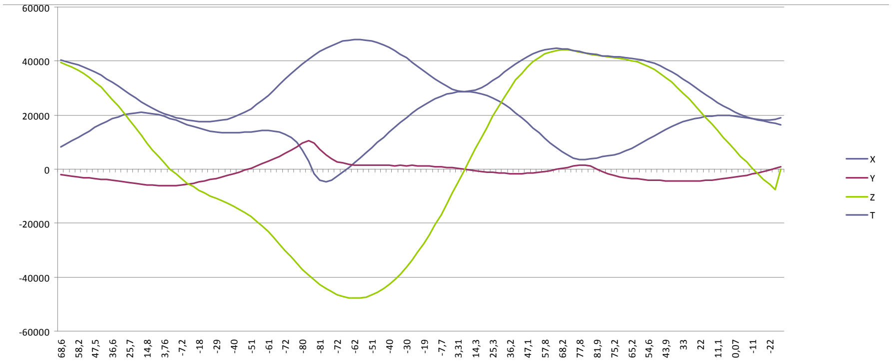

Figure 7 – Graphs of components of the Earth magnetic field along the trajectory of the spacecraft

Simulation of the geomagnetic field perceived by the spacecraft

During the motion of center of mass of a satellite along the orbit, vector of geomagnetic intensitymoves in a complicated manner in space changing at each point of the orbit.

Vector of the dipole field in a point in space with the radius vector $\vec{R}$ represented as

$$\vec{H}^E = \frac{\mu_e}{R^5}\left[R^2\vec{k}_e - 3\left(\vec{k}_e, \vec{R}\right)\vec{R}\right] \qquad (7)$$

where, $\vec{H}^E$ – the dipole geomagnetic field; μ_e – constant of Earth Magnetism (the magnetic moment of the Earth's dipole); $\vec{k}_e$ – ort of dipole axis (in a first approximation coincides with the Earth's axis of rotation), antiparallel to the magnetic moment of the Earth.

To describe the changes in vector $\vec{H}^E$ when the satellite moves in orbit in direct dipole model derive two right geocentric coordinate systems $O\overline{XYZ}$ and $OXYZ$, planes $O\overline{XY}$ and OXY lie in the Earth's equatorial plane(OX axis directed to the ascending node of the satellite orbit), and $O\overline{Z}$, OZ axes coincides with the Earth's axis of rotation. Ω_1 angle – longitude of the ascending node, ω_π – longitude in pericenter, ν – true anomaly of the satellite center of mass, u – argument of latitude. The projections of the $\vec{H}^E$ vector to the axis of the system $OXYZ$ are:

$$H_X^E = -\frac{3\mu_e}{2R^3}\sin i \sin 2u$$
$$H_Y^E = -\frac{3\mu_e}{2R^3}\sin 2i \sin^2 u \qquad (8)$$
$$H_Z^E = \frac{\mu_e}{R^3}(1 - 3\sin^2 i \sin^2 u)$$

The module of the magnetic intensity is determined by the formula:

$$\left|H^E\right| = \frac{\mu_e}{R^3}\sqrt{1 + 3\sin^2 i \sin^2 u} \qquad (9)$$

If the orbital geocentric coordinate system $O\overline{xyz}$ (fig.8) is implemented, $O\overline{z}$ axis of which coincides with the normal to the orbital plane of the satellite and $O\overline{x}$ axis passes through the ascending node of the orbit, then formulas (8) and (9) will be:

$$H_{\bar{x}}^E = -\frac{3\mu_e}{R^3}\sin i \sin u \cos u,$$
$$H_{\bar{y}}^E = \frac{\mu_e}{R^3}\sin i(1 - 3\sin^2 u), \qquad (10)$$
$$H_{\bar{z}}^E = \frac{\mu_e}{R^3}\cos i.$$
$$\left|H^E\right| = \frac{\mu_e}{R^3}\sqrt{1 + 3\sin^2 i \sin^2 u}.$$

Changing the coordinates of the magnetic field intensity during the motion of spacecraft in orbit in the direct dipole model

To investigate the changes of the $\vec{H}^E$ vector along the trajectory of the spacecraft derive $Ox_1y_1z_1$ geocentric coordinate system, which in case of $i \le 90^0$ is obtained from $OXYZ$ coordinate system by counterclockwise rotating at an angle about the OX axis (fig.8). Surface described by intensity vector in the dipole model for $\nu_1 \le 90^0$ case, is a cone (Fig.8).

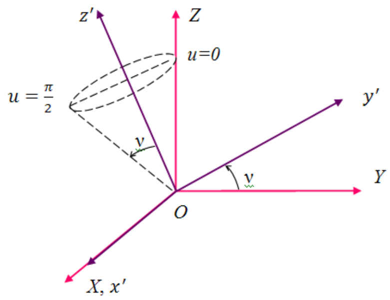

Figure 8 – Moving of an intensity vector $\vec{H}^E$

where, ν_1 is an angle determined from equations:

$$tg\nu_1 = \frac{1.5\sin 2(\pi - i)}{1-3\sin^2(\pi-i)+\sqrt{1+3\sin^2(\pi-i)}} = \frac{1.5\sin 2i}{1-3\sin^2 i+\sqrt{1+3\sin^2 i}}, \quad (11)$$

$$tg(\nu_1-(\pi-i)) = \frac{2-\sqrt{1+3\sin^2(\pi-i)}}{1+\sqrt{1+3\sin^2(\pi-i)}}tg(\pi-i) = -\frac{2-\sqrt{1+3\sin^2 i}}{1+\sqrt{1+3\sin^2 i}}tgi = tg(\nu_1+i) \quad (12)$$

To identify patterns in the distribution of $\vec{H}^E$ vector combine base of $\vec{H}^E$ with the center of the Earth's sphere of unit radius, then the points of intersection of the sphere with the $\vec{H}^E$ vector will be:

$$x_1 = -\frac{1.5\sin(\pi-i)\sin 2u}{\sqrt{1+3\sin^2(\pi-i)\sin^2 u}} = \frac{1.5\sin i\sin 2u}{\sqrt{1+3\sin^2 i\sin^2 u}},$$

$$y_1 = \frac{\sin\nu_1 - 3\sin(\pi-i)\cos(\nu_1-(\pi-i))\sin^2 u}{\sqrt{1+3\sin^2(\pi-i)\sin^2 u}} = \frac{\sin\nu_1+3\sin i\cos(\nu_1-i)\sin^2 u}{\sqrt{1+3\sin^2 i\sin^2 u}}, \quad (13)$$

$$z_1 = \frac{\cos\nu_1+3\sin(\pi-i)\sin(\nu_1-(\pi-i))\sin^2 u}{\sqrt{1+3\sin^2(\pi-i)\sin^2 u}} = \frac{\cos\nu_1-3\sin i\sin(\nu_1+i)\sin^2 u}{\sqrt{1+3\sin^2 i\sin^2 u}}.$$

All the functions in (13) are π-periodical by argument of latitude, so it is sufficient to investigate onlyvalues $u \in (0, \pi)$.

The basic laws ofmotion of intensity vector along a trajectory of the spacecraft on cone can be detected by analyzing changes of x_1 and y_1 coordinates for a half-turn of the spacecraft in orbit. To do this, derivatives of the x_1 and y_1 functions are obtained:

$$\frac{dx_1}{du} = -\frac{3\sin(\pi - i)}{\sqrt{(1+3\sin^2(\pi - i)\sin^2 u)^3}}\left(\cos 2u - 3\sin^2(\pi - i)\sin 4u\right) = \\ = -\frac{3\sin i}{\sqrt{(1+3\sin^2 i\sin^2 u)^3}}\left(\cos 2u - 3\sin^2 i\sin 4u\right), \tag{14}$$

$$\frac{dy_1}{du} = \frac{1.5\sin i\sin 2u(-2\cos(\nu_1 + i) - 3\cos(\nu_1 + i)\sin^2 i\sin^2 u + \sin\nu_1\sin i)}{\sqrt{(1+3\sin^2 i\sin^2 u)^3}}. \tag{15}$$

Substituting the value of the inclination of the orbit of spacecraft to the (11)-(15) formulas, we will have:

$$tg(\nu_1 + 98.2^0) = -\frac{2-\sqrt{1+3\sin^2 98.2^0}}{1+\sqrt{1+3\sin^2 98.2^0}}tg98.2^0, \tag{16}$$

$$tg98.2^0 = \frac{1.5\sin 196.4^0}{1-3\sin^2 98.2^0 + \sqrt{1+3\sin^2 98.2^0}}, \tag{17}$$

$$x_1 = -\frac{1.5\sin 98.2^0\sin 2u}{\sqrt{1+3\sin^2(98.2^0)\sin^2 u}},$$

$$y_1 = \frac{\sin\nu_1 + 3\sin 98.2^0\cos(\nu_1 - 98.2^0)\sin^2 u}{\sqrt{1+3\sin^2 98.2^0\sin^2 u}}, \tag{18}$$

$$z_1 = \frac{\cos\nu_1 - 3\sin 98.2^0\sin(\nu_1 + 98.2^0)\sin^2 u}{\sqrt{1+3\sin^2 98.2^0\sin^2 u}},$$

$$\frac{dx_1}{du} = -\frac{3\sin 98.2^0}{\sqrt{(1+3\sin^2 98.2^0\sin^2 u)^3}}\left(\cos 2u - 3\sin^2 98.2^0\sin 4u\right), \tag{19}$$

$$\frac{dy_1}{du} = \frac{3\sin 98.2^0\sin 2u(\sin\nu_1\sin 98.2^0 - 2\cos(\nu_1 + 98.2^0) - 3\cos(\nu_1 + 98.2^0)\sin^2 98.2^0\sin^2 u)}{2\sqrt{(1+3\sin^2 98.2^0\sin^2 u)^3}} \tag{20}$$

Using the MAPLE program the formulas (18) – (20)were implemented and the graphs of changes of x_1 and y_1 coordinates and $\dot{x}_1$, $\dot{y}_1$ velocities of intensity vectors of the geomagnetic field along the trajectory of the spacecraft were built (Figures 9-13).

The following characteristics of spacecraft were used as example:

Class orbit – LEO, type of orbit – sun-synchronous, inclination – 98.2°, eccentricity – 0.00190988187027, perigee – 685.0 km, apogee – 712.0 km, period – 98.74 min.

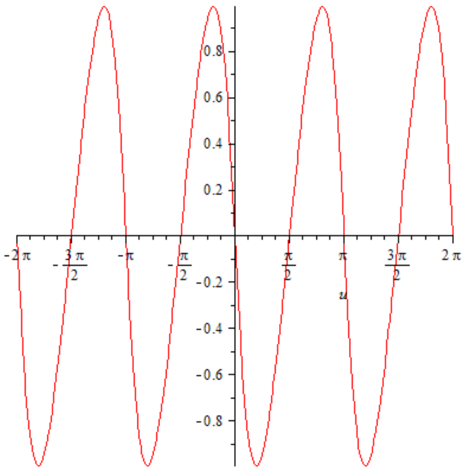

Figure 9 – Change of the x_1 coordinate of intensity vector of the geomagnetic field along the trajectory of spacecraft

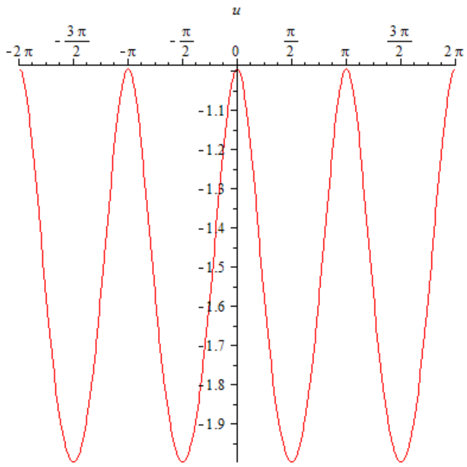

Figure 10 – Change of the y_1 coordinate of intensity vector of the geomagnetic field along the trajectory of spacecraft

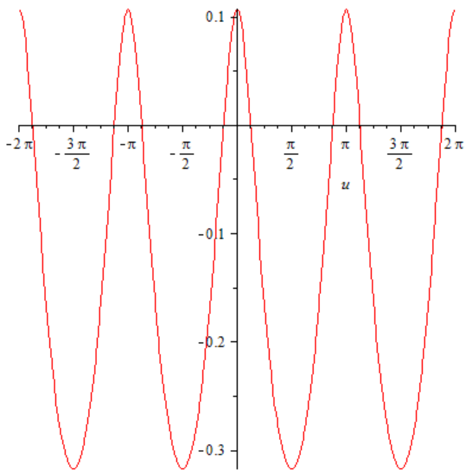

Figure 11 – Change of the z_1 coordinate of intensity vector of the geomagnetic field along the trajectory of spacecraft

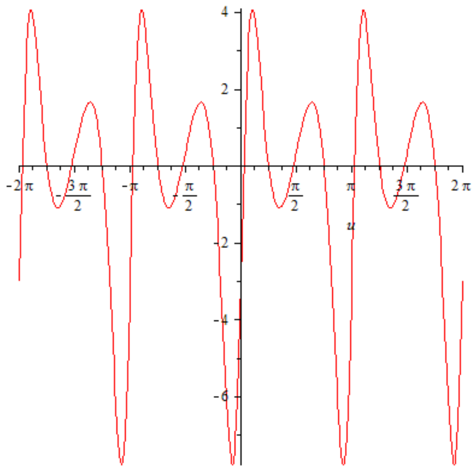

Figure 12 – Change of the $\dot{x}_1$ velocity of intensity vector of the geomagnetic field along the trajectory of spacecraft

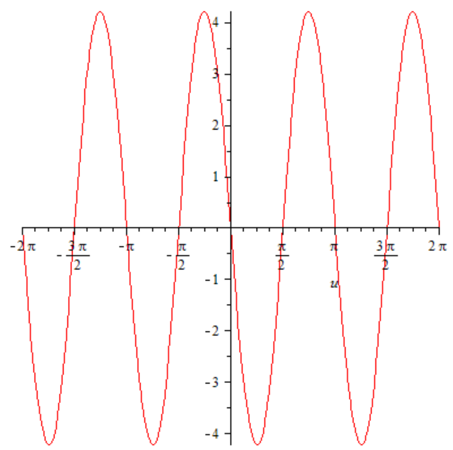

Figure 13 – Change of the $\dot{y}_1$ velocity of intensity vector of the geomagnetic field along the trajectory of spacecraft

Conclusion

The change of the intensity vector ofgeomagnetic field during the motion of the spacecraft in orbit in direct dipole model was investigated.The dependences of the intersection point coordinates of the vector intensity of the geomagnetic field with the unit radius Earth sphere were found,their graphs were built and their extremes were determined.The relations of the intersection point velocity of the vector intensity of the geomagnetic field to the unit radius Earth sphere were found, their graphs were built and their extremes determined.

The graphs of each component of the geomagnetic field along the trajectory of the spacecraft and the combined graphs were built.

The orientation of the satellite relative to the geomagnetic coordinate system was determined.

The projections of intensity of the geomagnetic field in view of components of the geomagnetic field were determined.

These results make it possible to assess the influence of each component of the geomagnetic field on the rotational motion of the satellite, to determine the optimal orientation of the satellite in the geomagnetic field, to develop necessary for this system of magnetic stabilization, etc.

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