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Difference scheme stability investigation for model of barotropic viscous gas

Abstract: In this paper, the technique of convergence of research and the stability difference schemes for the equations of gas dynamics in the presence of an electric field. The work consists in the strict mathematical definition of the conditions of stability and convergence of difference schemes, which relate to the long posed problems of computational mathematics. The studies enable significant progress in the study of the convergence and stability of difference schemes for gas dynamic tasks. The results of the thesis could be used for a wide class of problems of mathematics and mechanics. In the proof of the stability and convergence of difference schemes used by well-known theorems and inequality. All results are formulated as theorems.
Key words: difference scheme, stability investigation, gas dynamic, convergence, electric field, barotropic viscous gas.

Introduction

Many mathematical modern science and technology problems arising in practice, related to the solution of the equations of gas dynamics. Despite the numerous number of methods currently used to solve these equations, work on their further study continues to be important and relevant. Therefore, the problem encountered in the study of the mechanics of the problems are of great scientific and practical interest, as their decision is related to the further development of the theory of differential equations and difference schemes. With their help, we can solve many problems of mechanics, physics and engineering, which are, in one way or another, to the equations of gas dynamics. Such as the aerodynamics of aircraft, astrophysics, weather forecast and more.In practice, more common problem when the gas-dynamic flow affect various additional factors, such as electric, magnetic and gravitational fields, heat conduction and electrical, chemical, and others. Given these events, it follows a lot of problems and difficulties in the correct mathematical formulation and decisions. The solution of such problems analytical method is not always possible, due to the nonlinearity and complexity of the equations. The mathematical problem of determining the solutions of the equations describing the motion of a continuous medium is reduced to finding the unknown functions of three space variables and time, for example, speed, volume, pressure, temperature, density, electric and magnetic intensity. This problem is often very difficult, and it requiresto bring in additional solutions schematization associated with specific physical problems, and make valid simplify their mathematical formulation. Among the gas dynamics models important place occupied by the system of Navier-Stokes equations for viscous compressible fluid. The model takes into account both the compressibility and thermal conductivity and viscosity of the medium. This system is very complicated, it has a complex type, and the equations included in it, the non-linear. Therefore often used and other more simple model of a viscous gas. In particular, if we consider the barotropic motion, the energy equation is detached, although in this case, the system retains the main features - non-linearity and a component type.

Problem description and statement

This section describes the motion of a viscous heat-conducting gas and ions of one species in an electric field. The flow region $\Omega=(0,1)$ has impermeable walls. This chapter is devoted to the study of convergence and stability of a difference scheme for electric gas dynamic model, without taking into account the diffusion coefficient of ions, i.e., and ion mobility factor has the form $b_1 = b/\rho$,

$b = const > 0$. In this case we consider the so-called pump mode, when the relative velocity of the ions $u_{relative} = bE > 0$ everywhere in $Q_T = \Omega \times (0,T)$.

When you study the issues of sustainability and convergence of the problem is convenient to use Lagrangian coordinates. Keeping to the mass Lagrangian variable the same notation, denoting the specific volume of the system of equations can be written as follows:

$$\frac{\partial v}{\partial t} = \frac{\partial u}{\partial x}\ , \tag{1}$$

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \mu\frac{\partial}{\partial x}\left(\frac{1}{v}\frac{\partial u}{\partial x}\right) + \varepsilon E\frac{\partial E}{\partial x}\ , \tag{2}$$

$$\frac{\partial \theta}{\partial t} = -p\frac{\partial u}{\partial x} + \chi\frac{\partial}{\partial x}\left(\frac{1}{v}\frac{\partial \theta}{\partial x}\right)_x + \mu\frac{1}{v}\left(\frac{\partial u}{\partial x}\right)^2 + b\varepsilon v E^2\frac{\partial E}{\partial x}, \tag{3}$$

$$\frac{\partial E}{\partial t} = -bE\frac{\partial E}{\partial x}, \tag{4}$$

$$p = \frac{R\theta}{v}, \tag{5}$$

Conditions at the boundaries *x=0, x=1* and the initial data are

$$u(0,t) = u(1,t) = 0,\ E(0,t) = 0,$$

$$\frac{\partial\theta(0,t)}{\partial x} = \frac{\partial\theta(1,t)}{\partial x} = 0, \tag{6}$$

$$u(x,0) = u_0(x),\ v(x,0) = v_0(x),$$

$$\theta(x,0) = \theta_0(x),\ E(x,0) = E_0(x),\ x \in \overline{\Omega} \tag{7}$$

And $v_0(x)$, $\theta_0(x)$ - are strictly positive and bounded functions.

$$0 < m_0 \le (v_0(x), \theta_0(x)) \le M_0 < \infty,\ x \in \overline{\Omega}.$$

We can assume that the specific volume has the property

$$\int_0^1 v_0(x)dx = 1.$$

The difference scheme for the problem

To construct a difference scheme, approximately describing this differential problem is necessary to replace the differential region of the argument continuous variation of a discrete area of change and replace the continuous functions to grid functions and set the difference analogue for the boundary and initial conditions.

In the area $\overline{Q}_T = \overline{\Omega} \times [0,T]$ we introduce the rectangular grids $Q_{h\Delta t}$ and $Q^1_{h\Delta t}$. For grid $Q_{h\Delta t}$ points (x_i, t_{n+1}) will be assigned grid functions speed u_i^{n+1} and voltage E_i^{n+1} to the "half-integer" grid $Q^1_{h\Delta t}$ points $(x_{i-1/2}, t_{n+1})$ - grid functions specific volume $v^{n+1}_{i-1/2}$ and temperature $\theta^{n+1}_{i-1/2}$. Approximating the system of electric gas dynamics differential equations (1.5) - (1.7) with the difference analogs of derivatives we consider the following difference scheme:

$$v^{n+1}_{i-1/2\bar{t}} = u^{n+1}_{i\bar{x}},\ i = 1,...,N, \text{m} \tag{8}$$

$$u^{n+1}_{i\bar{t}} = \mu\left(\frac{u^{n+1}_{i\bar{x}}}{v^{n+1}_{i-1/2}}\right)_x - R\left(\frac{\theta^{n+1}_{i-1/2}}{v^{n+1}_{i-1/2}}\right)_x + \frac{\varepsilon}{2}\left(E^{n+1}_{i+1} + E^{n+1}_i\right)E^{n+1}_{ix}, \tag{9}$$

$$i = 1,...,N-1,$$

$$\theta^{n+1}_{i-1/2\bar{t}} = -R\frac{\theta^{n+1}_{i-1/2}}{v^{n+1}_{i-1/2}}u^{n+1}_{i\bar{x}} + \mu\frac{\left(u^{n+1}_{i\bar{x}}\right)^2}{v^{n+1}_{i-1/2}} + \frac{b\varepsilon}{2}v^n_{i-1/2}\left(E^{n+1}_i + E^n_i\right)E^{n+1}_i E^{n+1}_{i\bar{x}} + \chi\left(\frac{\theta^{n+1}_{i-1/2\bar{x}}}{\bar{v}^{n+1}_{i-1/2}}\right)_x, \tag{10}$$

$$i = 1,...,N-1,$$

$$E_{i\bar{t}}^{n+1} = -bE_i^{n+1}E_{i\bar{x}}^{n+1},\ i=1,...,N-1,$$
$$n = 0,...,M-1, \qquad (11)$$

and here is $\bar{v}_{i-1/2}^{n+1} = \dfrac{v_{i-1/2}^{n+1} + v_{i-1/2}^{n}}{2}$. Initial and boundary conditions

$$u_0^{n+1} = u_N^{n+1} = 0,\ E_0^{n+1} = 0,$$
$$\theta_{1/2\bar{x}}^{n+1} = \theta_{N+1/2\bar{x}}^{n+1} = 0,\ n = 0,...,M-1, \qquad (12)$$

$$u_i^0 = u_0(x_i),\ v_{i-1/2}^0 = v_0(x_{i-1/2}),$$
$$\theta_{i-1/2}^0 = \theta_0(x_{i-1/2}),\ E_i^0 = E_0(x_i), \qquad (13)$$
$$x_i \in \Omega_h,\ \ x_{i-1/2} \in \Omega_h^1$$

Also $v_{i-1/2}^0, \theta_{i-1/2}^0$ – strictly positive bounded functions

$$0 < m_0 \le \left(v_{i-1/2}^0, \theta_{i-1/2}^0\right) \le M_0 < \infty,$$
$$i = 1,...,N.$$

We can assume that the difference analog of the specific volume and tensions have properties

$$\sum_{i=1}^{N} v_{i-1/2}^0 h = 1,\ E_{i\bar{x}}^0 \ge 0,\ E_0^0 = 0.$$

Difference equations (1) - (4) approximate the differential equation (5), respectively, with orders $O(h^2 + \Delta t)$, $O(h + \Delta t)$, $O(h^2 + \Delta t)$, $O(h + \Delta t)$.

Upper and lower bounds for the difference analog of the specific volume

Due to the fact that the system (1.5) is non-linear, for the convergence of research and the stability of the difference scheme (2.1) - (2.6), it is necessary to obtain estimates for the difference of its decision in the rules grid analogues of Sobolev spaces. So we need to prove an auxiliary lemma.

Lemma 1. If $\left(u^0, E^0\right) \in W_{2h}^1(\Omega_h)$, $\left(v^0, \theta^0\right) \in W_{2h}^1(\Omega_h^1)$, moreover $0 < m_0 \le \left(v_{i-1/2}^0, \theta_{i-1/2}^0\right) \le M_0 < \infty,\ i = 1,...,N$. Then for the difference solution $(u,\ v,\ \theta,\ E)$ of the scheme (2.1) – (2.6) there are a priori estimates

$$\sum_{i=1}^{N}\left[\frac{\varepsilon}{2}\left(E_i^{n+1}\right)^2 v_{i-1/2}^{n+1} + \left(v_{i-1/2}^{n+1} - R\ln v_{i-1/2}^{n+1} - 1\right) + \frac{\left(u_i^{n+1}\right)^2}{2} + \left(\theta_{i-1/2}^{n+1} - \ln\theta_{i-1/2}^{n+1} - 1\right)\right]h +$$
$$+\sum_{m=0}^{n}\sum_{i=1}^{N}\left[\chi\frac{\left(\theta_{i-1/2\bar{x}}^{m+1}\right)^2}{\theta_{i-3/2}^{m+1}\theta_{i-1/2}^{m+1}\bar{v}_{i-1/2}^{m+1}} + \mu\frac{\left(u_{i\bar{x}}^{m+1}\right)^2}{\theta_{i-1/2}^{m+1}v_{i-1/2}^{m+1}} + \frac{b\varepsilon}{2}\frac{v_{i-1/2}^{m}}{\theta_{i-1/2}^{m+1}}\left(E_i^{m+1} + E_i^m\right)E_i^{m+1}E_{i\bar{x}}^{m+1}\right]h\Delta t \le C_0 < \infty, \qquad (14)$$

$$\sum_{i=1}^{N}\left[\frac{\varepsilon}{2}\left(E_i^{n+1}\right)^2 v_{i-1/2}^{n+1} + \frac{1}{2}\left(u_i^{n+1}\right)^2 + \theta_{i-1/2}^{n+1}\right]h \le C_1 < \infty, \qquad (15)$$

$$\frac{1}{2}\left\|E^{n+1}\right\|^2 + \sum_{m=0}^{n}\sum_{i=1}^{N} b\left(E_i^{m+1}\right)^2 E_{i\bar{x}}^{m+1}h\Delta t \le C_2 < \infty,\ n = 0,...,M-1. \qquad (16)$$

Proof. Let's multiply (8) to $\dfrac{\varepsilon}{2}\left(E_i^{n+1}\right)^2 + 1 - \dfrac{R}{v_{i-1/2}^{n+1}}$, **(9) to** u_i^{n+1}, **(10) to** $1 - \dfrac{1}{\theta_{i-1/2}^{n+1}}$, (11) to $\dfrac{\varepsilon}{2}\left(E_i^{n+1} + E_i^n\right)v_{i-1/2}^n$. Then, (8) (10) and (11) sum over i to N, equation (9) sum over i to N-1. All equations sum together and over m. To the sum of the right-hand side of the new expression, we apply the formula for summation by parts. Then, revealing the difference derivatives with respect to time and discarding the positive terms on the left-hand side, we obtain estimate (14)

$$\sum_{i=1}^{N}\left[\frac{\varepsilon}{2}\left(\left(E_i^{n+1}\right)^2 v_{i-1/2}^{n+1}-\left(E_i^0\right)^2 v_{i-1/2}^0\right)+\left(v_{i-1/2}^{n+1}-v_{i-1/2}^0\right)+\frac{1}{2}\left(\left(u_i^{n+1}\right)^2-\left(u_i^0\right)^2\right)-\right.$$
$$\left.-R\left(\ln v_{i-1/2}^{n+1}-\ln v_{i-1/2}^0\right)+\left(\theta_{i-1/2}^{n+1}-\theta_{i-1/2}^0\right)-\left(\ln\theta_{i-1/2}^{n+1}-\ln\theta_{i-1/2}^0\right)\right]h+$$
$$+\sum_{m=0}^{n}\sum_{i=1}^{N}\left[\mu\frac{\left(u_{\bar{ix}}^{m+1}\right)^2}{\theta_{i-1/2}^{m+1}v_{i-1/2}^{m+1}}+\frac{b\varepsilon}{2}\frac{v_{i-1/2}^{m+1}}{\theta_{i-1/2}^{m+1}}\left(E_i^{m+1}+E_i^m\right)E_i^{m+1}E_{\bar{ix}}^{m+1}+\chi\frac{\left(\theta_{i-1/2\bar{x}}^{m+1}\right)^2}{\theta_{i-1/2}^{m+1}\theta_{i-3/2}^{m+1}\bar{v}_{i-1/2}^{m+1}}\right]h\Delta t\le C_3.$$

To obtain (15) we multiply (8) to $\frac{\varepsilon}{2}\left(E_i^{n+1}\right)^2$, (9) to u_i^{n+1}, (10) remain unchanged, (11) multiply to $\frac{\varepsilon}{2}\left(E_i^{n+1}+E_i^n\right)v_{i-1/2}^n$. Then sum (8), (10) and (11) for from 1 to N, and (9) on the sum from 1 to N-1. Then we add together all the equations and sum over m from 0 to n. At the right side of the new expression the amount will be reduced after the application of the formula for summation by parts to the last three sums. We expand the derivatives with respect to time and sum over m, we have the estimate (15)

$$\sum_{i=1}^{N}\left[\frac{\varepsilon}{2}\left(E_i^{n+1}\right)^2 v_{i-1/2}^{n+1}-\frac{\varepsilon}{2}\left(E_i^0\right)^2 v_{i-1/2}^0+\theta_{i-1/2}^{n+1}-\theta_{i-1/2}^0+\frac{\left(u_i^{n+1}\right)^2}{2}-\frac{\left(u_i^0\right)^2}{2}\right]h+\sum_{m=0}^{n}\sum_{i=1}^{N}\left(u_{\bar{it}}^{m+1}\right)^2 h\left(\Delta t\right)^2=0.$$

To obtain (16) we multiply (11) to $E_i^{n+1}h\Delta t$ on and sum and m, we have required estimate 16

$$\sum_{m=0}^{n}\sum_{i=1}^{N}E_i^{m+1}E_{\bar{it}}^{m+1}h\Delta t=-\sum_{m=0}^{n}\sum_{i=1}^{N}b\left(E_i^{m+1}\right)^2E_{\bar{ix}}^{m+1}h\Delta t,$$

or

$$\sum_{i=1}^{N}\frac{1}{2}\left(\left(E_i^{n+1}\right)^2-\left(E_i^0\right)^2\right)h+\sum_{m=0}^{n}\sum_{i=1}^{N}\frac{1}{2}\left(\Delta t\right)^2\left(E_{\bar{it}}^{m+1}\right)^2h\Delta t+\sum_{m=0}^{n}\sum_{i=1}^{N}b\left(E_i^{m+1}\right)^2E_{\bar{ix}}^{m+1}h\Delta t=0.$$

Lemma 2. Let the conditions of Lemma 1 and $\Delta t=O\left(h^{\alpha+2}\right)$, $0<\alpha<1$. Then $E_{\bar{ix}}^n\ge 0$, $n=1,\dots,M$, $i=1,\dots,N$ and for the difference analogue of the specific volume the estimate is

$$0<m_1\le v_{i-1/2}^{n+1}\le M_1<\infty,\ i=1,\dots,N,\quad n=0,\dots,M-1, \qquad (17)$$

here $m_1, M_1>0$.

Let's proof that lemma, so we can get estimates. Let's write (9) as

$$u_{\bar{it}}^{n+1}=\mu\left(\frac{u_{\bar{ix}}^{n+1}}{v_{i-1/2}^{n+1}}\right)_x-R\left(\frac{\theta_{i-1/2}^{n+1}}{v_{i-1/2}^{n+1}}\right)_x+\frac{\varepsilon}{2}\left(\left(E_i^{n+1}\right)^2\right)_x, \qquad (18)$$
$$i=1,\dots,N-1,\ n=0,\dots,M-1.$$

Let's potentiate and express

$$v_{j-1/2}^{n+1}=\frac{v_{j-1/2}^{0}v_{i^*(n)-1/2}^{n+1}}{v_{i^*(n)-1/2}^{0}}\exp\left\{\frac{1}{\mu}\sum_{i=i^*(n)}^{j-1}\left(u_i^{n+1}-u_i^0\right)h-\frac{1}{\mu}\sum_{m=0}^{n}\left[R\frac{\theta_{i^*(m)-1/2}^{m+1}}{v_{i^*(m)-1/2}^{m+1}}+\mu\cdot z_{i^*(m)-1/2}^{m+1}-\right.\right.$$

$$\left.\left.-\frac{\varepsilon}{2}\left(E_{i^*(n)}^{m+1}\right)^2-R\frac{\theta_{j-1/2}^{m+1}}{v_{j-1/2}^{n+1}}-\mu\cdot z_{j-1/2}^{m+1}+\frac{\varepsilon}{2}\left(E_j^{m+1}\right)^2\right]\Delta t\right\}. \tag{19}$$

We introduce the notation

$$B(n,j)=\frac{v_{i^*(n)-1/2}^{0}}{v_{j-1/2}^{0}}\exp\left\{\frac{1}{\mu}\sum_{i=i^*(n)}^{j-1}\left(u_i^0-u_i^{n+1}\right)h+\frac{1}{\mu}\sum_{m=0}^{n}\left(\frac{\varepsilon}{2}\left(E_j^{m+1}\right)^2-\mu z_{j-1/2}^{m+1}\right)\Delta t\right\}, \tag{20}$$

$$I(n)=\exp\left\{\frac{1}{\mu}\sum_{m=0}^{n}\left(R\frac{\theta_{i^*(n)-1/2}^{m+1}}{v_{i^*(n)-1/2}^{m+1}}+\mu z_{i^*(m)-1/2}^{m+1}-\frac{\varepsilon}{2}\left(E_{i^*(m)}^{m+1}\right)^2\right)\Delta t\right\}. \tag{21}$$

From (19) using (20) and (21), we obtain the following equation

$$v_{j-1/2}^{n+1}=v_{i^*(n)-1/2}^{n+1}\left(B(n,j)\right)^{-1}\left(I(n)\right)^{-1}\exp\left\{\frac{1}{\mu}\sum_{m=0}^{n}R\frac{\theta_{j-1/2}^{m+1}}{v_{j-1/2}^{m+1}}\Delta t\right\}. \tag{22}$$

Let us show that the inequality under conditions $b>0, E_{\bar{i x}}^{0}\geq 0$, where $E_{\bar{i x}}^{n+1}\geq 0$, $i=1,\ldots,N$, $n=0,\ldots,M-1$. Let us note $E_{\bar{i x}}^{n+1}=\omega_i^{n+1}$, where $\omega_i^0\geq 0$. Let us prove that $\omega_i^{n+1}\geq 0$ for all iand n. Take the difference derivative forward from both sides of the equation (11). Then replace and multiply on equality, sum over from 0 to, using (12), we can write

$$\sum_{i=0}^{N-1}2\left(\omega_{i+1}^{n+1}\right)^-\left(\left(\omega_{i+1}^{n+1}\right)^-\right)_{\bar{t}}h\Delta t+\sum_{i=0}^{N-1}2b\left(\left(\omega_{i+1}^{n+1}\right)^-\right)^3h\Delta t=-\sum_{i=1}^{N-1}2b\left(\omega_{i+1}^{n+1}\right)^-\left(\left(\omega_{i+1}^{n+1}\right)^-\right)_{\bar{x}}h\Delta t.$$

Then, by replacing we obtain

$$2\left(\omega_{i+1}^{n+1}\right)^-\left(\left(\omega_{i+1}^{n+1}\right)^-\right)_{\bar{x}}h=\left[\left(\left(\omega_{i+1}^{n+1}\right)^-\right)^2\right]_{\bar{x}}h+h^2\left[\left(\left(\omega_{i+1}^{n+1}\right)^-\right)_{\bar{x}}\right]^2,$$

$$2\left(\omega_{i+1}^{n+1}\right)^-\left(\left(\omega_{i+1}^{n+1}\right)^-\right)_{\bar{t}}\Delta t=\left(\left(\omega_{i+1}^{n+1}\right)^-\right)^2-\left(\left(\omega_{i+1}^{n}\right)^-\right)^2+(\Delta t)^2\left[\left(\left(\omega_{i+1}^{n+1}\right)^-\right)_{\bar{t}}\right]^2.$$

By the following amounts apply the formula for summation by parts

$$-\sum_{i=1}^{N-1}bE_i^{n+1}\left[\left(\left(\omega_{i+1}^{n+1}\right)^-\right)^2\right]_{\bar{x}}h\Delta t=-bE_N^{n+1}\left(\left(\omega_N^{n+1}\right)^-\right)^2\Delta t+bE_0^{n+1}\left(\left(\omega_1^{n+1}\right)^-\right)^2\Delta t+$$

$$+\sum_{i=0}^{N-1}bE_{ix}^{n+1}\left(\left(\omega_{i+1}^{n+1}\right)^-\right)^2h\Delta t=-bE_N^{n+1}\left(\left(\omega_N^{n+1}\right)^-\right)^2\Delta t+\sum_{i=0}^{N-1}bE_{ix}^{n+1}\left(\left(\omega_{i+1}^{n+1}\right)^-\right)^2h\Delta t.$$

And we have

$$\left\|\left(\omega^{n+1}\right)^{-}\right\|^{2}-\left\|\left(\omega^{n}\right)^{-}\right\|^{2}+\sum_{i=0}^{N-1}(\Delta t)^{2}\left[\left(\left(\omega_{i+1}^{n+1}\right)^{-}\right)_{\bar{t}}\right]^{2} h+$$

$$+\sum_{i=1}^{N-1} b E_{i}^{n+1}\left(\left(\left(\omega_{i+1}^{n+1}\right)^{-}\right)_{\bar{x}}\right)^{2} h^{2} \Delta t+\sum_{i=1}^{N-1} b\left(\left(\omega_{i+1}^{n+1}\right)^{-}\right)^{3} h \Delta t=-b E_{N}^{n+1}\left(\left(\omega_{N}^{n+1}\right)^{-}\right)^{2} \Delta t .$$

Using previous estimates we obtain

$$\left(v_{j-1/2}^{n+1}\right)^{-1} \leq C_{5} N_{1}=\left(m_{1}\right)^{-1} \text{or}\, v_{j-1/2}^{n+1} \geq m_{1}>0,$$
$$j=1, \ldots, N,\, n=0, \ldots, M-1. \quad (23)$$

Using Granuoll's lemma, we conclude that

$$M_{v}^{n+1} \leq M_{1} \text{ или } v_{i-1/2}^{n+1} \leq M_{1}, \quad (24)$$
$$i=1, \ldots, N,\, n=0, \ldots, M-1.$$

Conclusion

The study of fluid dynamics problems for a wide class of their applicability in a variety of devices electric gas dynamic generator, ion-convection pump, accelerators, feeders, cages in which the conductive medium moves through the channel or pipe in the presence of an electric field, continues to be important and relevant. This thesis studied the questions of convergence and stability of implicit difference schemes for one-dimensional problems gas dynamics. The investigations are as follows:

- constructed difference scheme for model of barotropic viscous gas in the electric field;
- obtained a priori estimates of the first decision of a difference scheme for model of barotropic viscous gas in the electric field;
- a priori estimates for higher derivatives of the solution of a difference scheme for model of barotropic viscous gas in the electric field;
- the convergence and stability of a difference scheme for model of barotropic viscous gas in the electric field;

References

1. Otmar Stuetzer O. Apparent viscosity of a charged fluid // The Physics of Fluids / Published by the American Institute of Physics. – December, 1961. – Vol. 4. – No 4. – P. 1226-1237.

2. Shohet Y. Errors and Stability of the Entry Problem Equations in Laminar Magneto hydrodynamic Flow // The Physics of Fluids / Published by the American Institute of Physics. – June, 1963. – Vol. 6. – No 6. – P. 797-802.

3. Smith C.V., Melcher J.R. Electrohy-drodynamicall induced spatially periodic celluar stokes – flow // The Physics of Fluids – November, 1967. – Vol. 10. – No 11. – P. 2315-2322.

4. Itaya N. The existence and uniqueness of the solutions of the equations describing compressible viscous fluid flow. // Proc. Japan. Acad. – 1970. – Vol. 46. – No 4. – P. 379-382.

5. Tani A. On the first initial boundary value problem of compressible viscous fluid motion // Publisher Res. Institute Mathematical Science. – 1977. – Vol. 13. No 1. – P. 193-251.