UDC 532.517.4.519.6

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Mathematical modelling of air flow in the human respiratory system

Abstract: Nasal inspiration is important for maintaining the internal milieu of the lung, since ambient air is conditioned to nearly alveolar conditions (body temperature and fully saturated with water vapor) on reaching the nasopharynx. In this work conducted a two-dimensional computational study of transport phenomena in model transverse cross sections of the nasal cavity of normal human noses based on the two dimensional Navier-Stokes equation. For discretization Navier-Stokes equation used finite volume method.Projection method applied for solution of the Navier-Stokes equations. The results suggest that during breathing via the normal human nose there is ample time for heat and water exchange to enable equilibration to near intraalveolar conditions. A normal nose can maintain this equilibrium under extreme conditions. The turbinates increase the rate of local heat and moisture transport by narrowing the passageways for air and by induction of laminar swirls downstream of the turbinate wall.

Key words: Respiratory air conditioning, alveolar condition, 2D modeling, heat transfer, Navier-Stokes equations, finite volume method.

Introduction

The nasal cavity balances the inhaled air to the internal condition of the body with amazing efficiency. In Cole, [2] Inglestedta, [3] and Webb [4] papers were general agreement that the air inhaled through the nasal cavity reaches the alveolar condition (fully saturated with water vapor at temperature of the body) to the time when it reaches the throat and it is practically independent of the condition of ambient air which enters through the nares. These findings were supported by Farley and Patel [5], who collected in vivo data of air temperature from various locations along the upper airways throughout the respiratory cycle, and Hannah and Scherer [6], who measured the local mass transfer coefficients on the plaster model of the human upper respiratory tract.

Numerical studies have focused on the assessment of the dampening and regulating the temperature of the nasal cavity. In addition, as the surgical procedures are now being used at an increasing rate to restore the structure and functions of the nose [8]. For example, aromatic inhalations are used to improve airflow and to reduce congestion, and rhinoplastic procedures used to overcome injuries or aesthetic deformations. These artificial intervention cause local changes, and can affect the efficiency of transport phenomena. However, the exact characteristics of intranasal and distribution of transport phenomena is still unknown even to normal (or healthy) state [15, 16].

Experimental study of the nasal cavity is impossible, due to a complex internal structure and size, because by putting a measuring instrument or probe into the nasal cavity causes a flow disturbance. Therefore, mathematical modeling is the only approach for the study of air flow in the nasal cavity.

The purpose of this study was to use the numerical modeling to study the dynamic capacity of the nasal cavity, the process of heating and moisturizing of inhaled air.

Formulation of the problem

The structure of the nasal cavity provides a very sophisticated way to airflow. The complex structure of the nasal cavity and the full three-dimensional analysis of the flow of steam, the heat transfer in the inside of the nasal mucosa requires significant computational resources, which prevent systematic analysis of relevant factors.

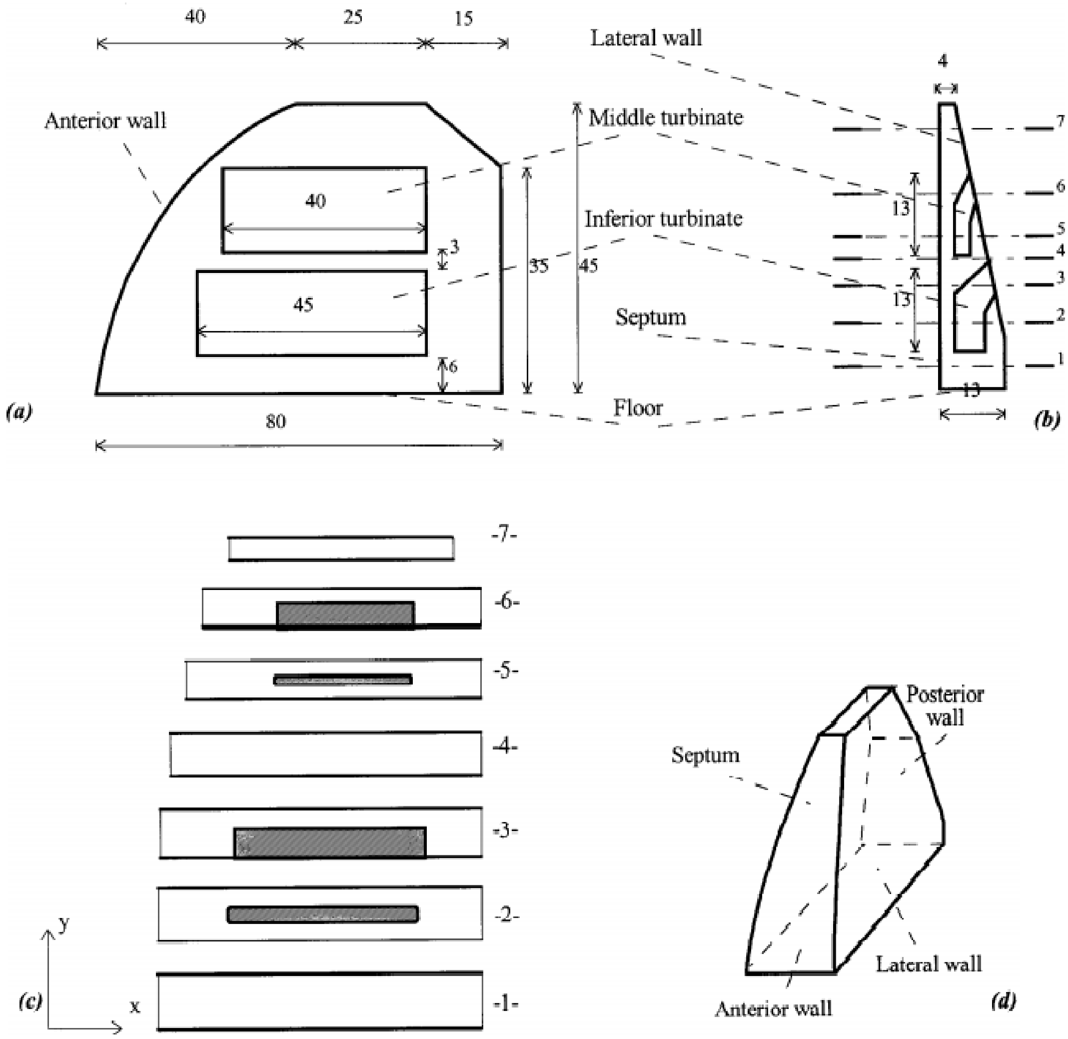

Figure 1 – Simplified nose models: a) longitudinal section, b) coronal section, c) the cross-section at height h = 3, 13, 17, 20, 26, 33, 40 mm from the bottom of the nasal cavity, d) a perspective view

Taking into account the available computing resources, a comprehensive study of transport mechanisms was conducted with two-dimensional simulation through transverse cross section of the nose.

In addition, the following assumptions were made:

- The walls of the nasal cavity and nasal turbinates are assumed to be rigid structures.
- Air flow in the nasal cavity is considered as a laminar flow, and the air as an incompressible medium.
- No-slip conditions are assumed at the interface between the air and the surface of the nasal cavity.

Subtle features of the nose do not have the exact size, as there are differences in the structure of the nasal cavities of healthy people, so we can't determine the exact model of the "normal nose".Thus, we had devised a simplified model of nose, where major essential features were identified. The sizes were taken fromaverage dimensions of human nasal cavity.

The physical area of the problem is the second cross-section (Figure 1 (c) "-2"), which is important for the study because a significant proportion of the air flow moves in this area, and it has a complex structure due to the main functions of the nasal cavity are performed.

The mathematical model is based on the Navier-Stokes equations, including the continuity equation, motion equation and the temperature transport equation [11, 12, 13]. Discretization was performed by finite volume method [9, 10, 14].

$$\nabla U = 0,$$
$$\frac{\partial U}{\partial t} + (U\cdot\nabla)U = -\frac{1}{\rho}\nabla p + v\nabla^2 U,$$
$$\frac{\partial T}{\partial t} + (U\cdot\nabla)T = \frac{k}{\rho c_p}\nabla^2 T,$$

where U is the velocity vector, t is time, p is fluid pressure , c_p is specific heat at constant pressure, T is temperature, ρ, v and k are the fluid density, kinematic viscosity and thermal conductivity, respectively, and ∇^2 is the Laplacian.

The boundary condition for the entrance of the inhaled air is given in the form of the profile, and ambient air temperature is taken as 25 °C:

$$u_{in}(t, x=0, y) = \left[2\sin^2\frac{\pi}{2} - 1\right] \times \frac{(12y - y^2)}{36}$$

$$T_{in}(t, x=0, y) = 25°C$$

On the walls of the nasal cavity and nasal turbinate:

$$u_{wall}(t,x,y) = 0,\ v_{wall}(t,x,y) = 0,$$
$$T_{wall}(t,x,y) = 37°C$$

Initial conditions:

$$u_0(t=0) = 0,$$

$$T_0(t=0) = 32°C.$$

Numerical algorithm

For the numerical solution of this system of equations is used projection method[14, 17]. At the first stage it is assumed that the transfer of momentum is carried out only by convection and diffusion. Intermediate velocity field is calculated by the fourth order Runge-Kuttamethod [13]. In the second stage, based on the found intermediate velocity field, one solves pressure field. Poisson equation for the pressure field is solved by Jacobi method. In a third step it is assumed that the transfer is carried out only by the pressure gradient. In the fourth stage, temperature transport equation is calculated by the fourth order Runge-Kutta method [13, 15, 16].

I. $$\int_\Omega \frac{\vec{u}^{\,*} - \vec{u}^{\,n}}{\tau} d\Omega = -\oint_{\partial\Omega} (\nabla \vec{u}^{\,n} \vec{u}^{\,*} - \upsilon \Delta \vec{u}^{\,*}) n_i d\Gamma$$

II. $$\oint_{\partial\Omega} (\Delta p) d\Gamma = \int_\Omega \frac{\nabla \vec{u}^{\,*}}{\tau} d\Omega$$

III. $$\frac{\vec{u}^{\,n+1} - \vec{u}^{\,*}}{\tau} = -\nabla p.$$

IV. $$\int_\Omega \frac{T^{n+1} - T^n}{\tau} d\Omega = -\oint_{\partial\Omega} (\nabla \vec{u}^{\,n} T - \frac{k}{\rho c_p} \Delta T) n_i d\Gamma$$

Computational mesh was created in Point Wise. The area was divided into 10 sub-areas to run on the software package ITFS-MKM (on the high-performance cluster T-Cluster at the Mechanics and Mathematics Faculty of al-Farabi Kazakh National University.).

The results of simulations

As a result of the numerical simulation of aerodynamics of the human nasal cavity, the following data were obtained; to verify the calculations used data from the paper [1], which describes the profiles of the longitudinal component of the velocity and temperature at three locations: at a distance x_1 = 17 mm, x_2 = 49 mm and x_3= 80 mm from the entrance (Figure 2.).

Figure 3 compares profiles between the velocitiesfrom calculation results and data from the paper Naftali S. [1]. Figure 4 shows a comparison of temperature profiles between the sections and the data from paper [1].

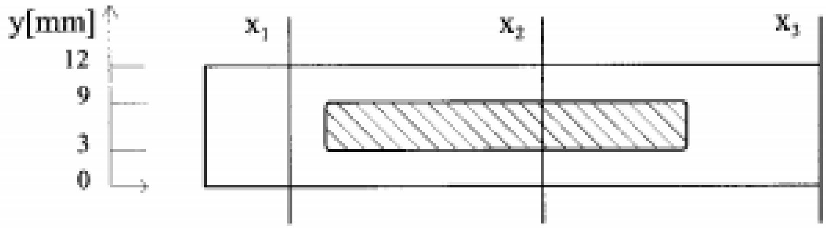

Figure 2 – Evaluation of the three zones for temperature and velocity of the cross-section

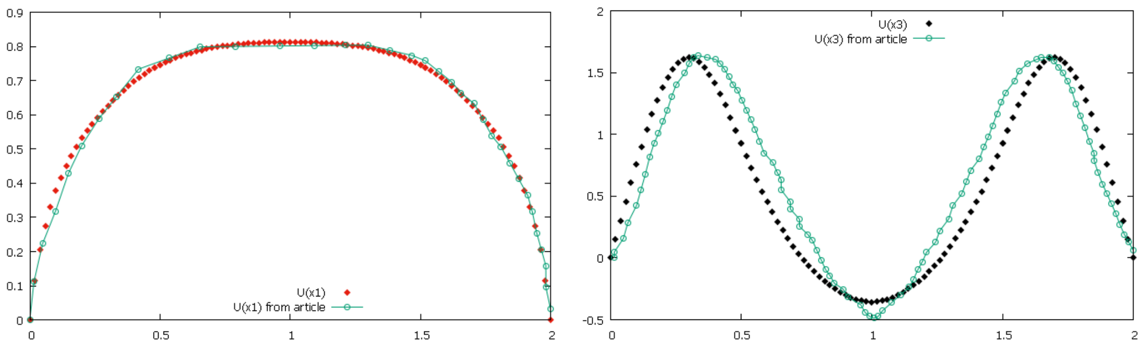

Figure 3 – Comparison of velocity component profiles with the numerical results from the paper [1] on x_1 = 17mm and x_3 = 80mm

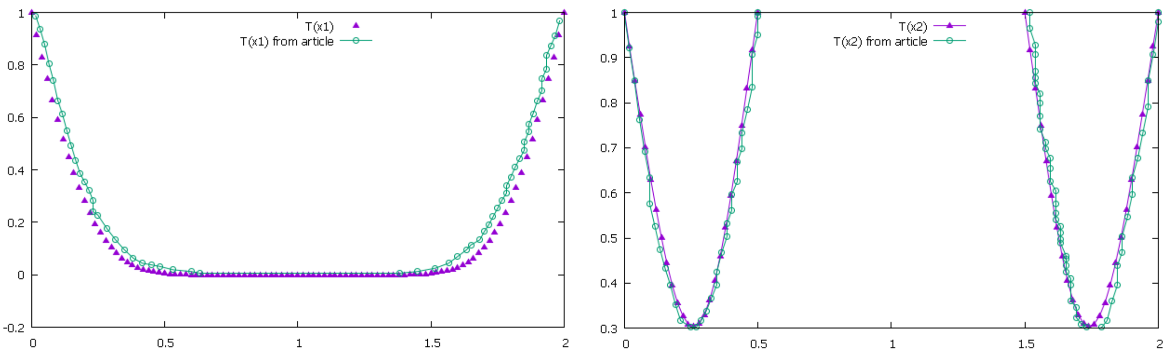

Figure 4 – Comparison of temperature profiles with the numerical results from the paper [1] on x_1 = 17mm and x_2 = 49mm

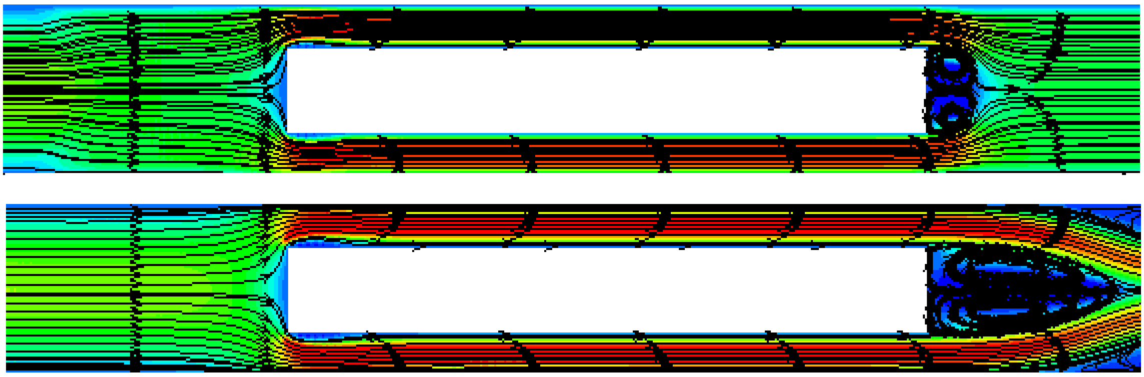

Figure 5 – The longitudinal components of the flow rate
to the cross-sectional lines with streamtracesfor different time layers

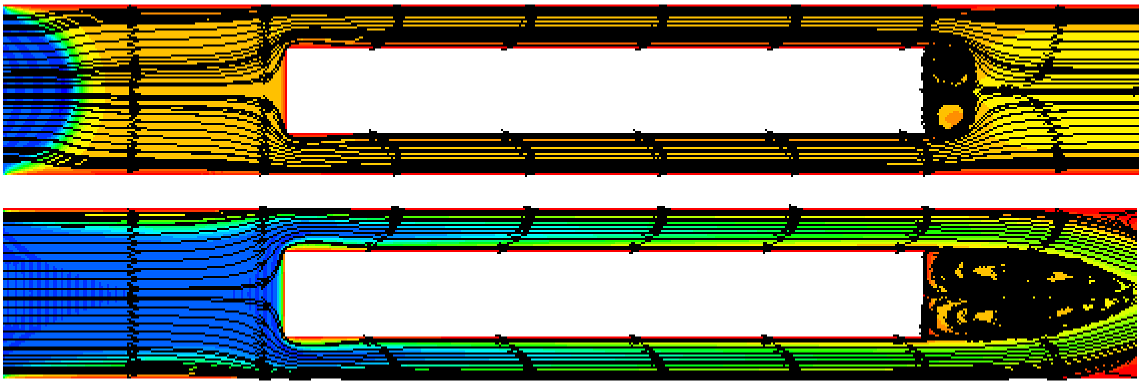

Figure 6 – Temperature for the cross-sectional lines with streamtraces
for different time layers

Figure 5 shows a longitudinal velocity component in cross section with streamtraces for different time layers. Along the streamtraces visible vortices which appear due to walls of the turbinates, which play an important role in the process of heating the air. Figure 6 shows the calculated temperature region with the flow lines for any time layers. From the figures it can be seen that the passage in the downstream narrow spaces of the nasal cavity air is heated.

Conclusion

Thus, in the research of nasal cavity we can conclude that nasal wall heats transfer air, also produces vortices which have a great importance for the transition into the alveolar air condition before reaching the nasopharynx. Studies of the nasal cavity are actually important, because nowadays thenumber of people with nasal breathing problems are increasing; this problem can be resolved by surgery, where it is important to optimally operate the structure of the nose and nasal cavity for the sake of correct functioning owing to the fact that normal breathing should be carried out with the help of the nose.

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