UDK 517.538.7:517.953

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Approximation of a method of iterations of Landveber for the net equation

Abstract: In this article approach at the numerical solution of the return problem of acoustics is considered by method of iterations of Landveber. The considered approach consists in the following: for restoration of unknown coefficient in the differential equation we have statement of a direct problem and additional information on the solution of a direct problem. We write out functionality nonviscous, we receive statement of the interfaced problem. Further by means of solutions of a straight line and the interfaced problem we receive a functionality gradient nonviscous [1]. Then for the numerical solution of the return problem from statement of a direct problem we pass to a problem which we will solve on the computer in number.

Key words: the return a problem, a problem of acoustics, Landveber's iteration, discrete analog, a gradient, the interfaced problem.

Introduction

We will consider the return problem of acoustics [2]:

$$u_{tt} = u_{xx} - 2\frac{s'(x)}{s(x)}u_x, \quad t > x > 0$$

$$u_x|_{x=0} = 0, \quad t > 0,$$

$$u(x, x+0) = s(x), \quad x > 0,$$

$$u|_{x=+0} = g(t), \quad t > 0.$$

where in the given function g (t) is required to find a function f (x).

We will enter a grid $x = ih$, $t = kh$, where $i = \overline{0, N}$, $k = \overline{i, 2N - i}$, N is grid size, h = l/N is grid step. We will enter the following designations for net functions:

$$q(i,k) = (q_1[i,k], q_2[i], q_3[i]),$$
$$q_1(i,k) := q_1(ih, kh), \quad q_2(i) := q_2(ih), \quad q_3(i) := q_3(ih),$$

$$f(i,k) = (f_1[i,k], f_2[i], f_3[i]),$$
$$f_1(i,k) := f_1(ih, kh), \quad f_2(i) := f_2(ih), \quad f_3(i) := q_3(ih).$$

Objects and methods of researches

The considered approach consists in the following: for restoration of unknown coefficient in the differential equation we have statement of a direct problem and additional information on the solution of a direct problem. We write out functionality nonviscous, we receive statement of the interfaced problem. Further by means of solutions of a straight line and the interfaced problem we receive a functionality gradient nonviscous. Then for the numerical solution of the return problem from statement of a direct problem we pass to a problem which we will solve on the computer in number. Further we write out functionality nonviscous Φ[p], which approximates functionality nonviscous J[q], from statement of the interfaced problem $L_q^*\psi = 0$ we pass to a problem $\widetilde{\Lambda}_p\phi = 0$, where $\widetilde{\Lambda}_p$ is the operator of the

 Printed in Kazakhstan

numerical solution of the interfaced problem, and function ϕ is function approach ψ; we receive a ratio which approximates expression of a gradient of functionality nonviscous and further for production of minimization sequence any gradient method is used [3].

For the description of the scheme we will use method of mathematical induction.

We will set initial approach $q^0[i,k] = \left(q_1^0[i,k], q_2^0[i], q_3^0[i]\right)$.

We will assume that $q^n[i, k]$ is already known, then we calculate values

$$Aq^n[i,k]:\; A_1q^n[i,k] = q_1^n[i,k] - \frac{h}{4}\Big(q_3^n[0]\big(q_1^n[0,k+i] + q_1^n[0,k-i]\big) + 2q_3^n[i]q_1^n[i,k]\Big)$$
$$- \frac{1}{2}\sum_{j=1}^{i-1} q_3^n[j]\Big(q_1^n[j,k+i-j] + q_1^n[j,k-i+j]\Big)h,$$
$$A_2q^n[i] = q_2^n[i] + \frac{h}{4}\Big(q_3^n[0]q_2^n[0] + q_3^n[i]q_2^n[i]\Big) + \frac{1}{2}\sum_{j=1}^{i-1} q_3^n[j]q_2^n[j]h,$$
$$A_3q^n[i] = q_3^n[i] + \Big(0.5h\big(q_3^n[0]q_2^n[0] + q_3^n[i]q_2^n[i]\big) + \sum_{j=1}^{i-1} q_3^n[j]q_2^n[j]h$$
$$\times\Big(0.5h\big(q_3^n[0]q_1^n[0,2i] + q_3^n[i]q_1^n[i,i]\big) + \sum_{j=1}^{i-1} q_3^n[j]q_1^n[j,2i-j]h - 0.5\gamma f_3[i]\Big)$$
$$+ 2/\gamma\Big(0.5h\big(q_3^n[0]q_1^n[0,2i] + q_3^n[i]q_1^n[i,i]\big) + \sum_{j=1}^{i-1} q_3^n[j]q_1^n[j,2i-j]h\Big).$$

We calculate values of functionalities $J_1(q^n) = \|r_1\|_{L_2}^2 \|A_1q^n - f_1\|_{L_2}^2 = \sum_{i=0}^{N}\sum_{k=i}^{2N-i}(A_1q^n[i,k] - f_1[i,k])^2 h^2$,

$$J_2(q^n) = \|r_2\|_{L_2}^2 + \|A_2q^n - f_2\|_{L_2}^2 = \sum_{i=0}^{N}(A_2q^n[i] - f_2[i])^2 h,$$
$$J_3(q^n) = \|r_3\|_{L_2}^2 = \|A_3q^n - f_3\|_{L_2}^2 = \sum_{i=0}^{N}(A_3q^n[i] - f_3[i])^2 h,$$

and if $J_1(q^n), J_2(q^n), J_3(q^n)$ are rather small, we stop process, accepting q^n for the approximate solution of the return problem [4].

If functionalities $J_1(q^n), J_2(q^n), J_3(q^n)$ are insufficiently small, we calculate gradients of functionalities

$$J_1'(q^n)[i,k] = 2[A_1'q^n]^* r[i,k] = r_1[i,k] - 0.5q_3^n[i]\Bigg(\sum_{j=i}^{(i+k)/2} r_1[j,k+i-j]h$$
$$+ \sum_{j=1}^{N-(k-i)/2} r_1[j,k-i+j]h - 2(B_2q[(k+i)/2] + 1/\gamma)r_3[(k+i)/2]\Bigg),$$
$$J_2'(q^n)[i] = 2[A_2'q^n]^* r[i] = r_2[i]$$
$$+ 0.5q_3^n[i]\sum_{j=i}^{N}\{r_2[j] + 2r_3[j](B_4q[j] - 0.5\gamma f_3[j])\}h,$$
$$J_3'(q^n)[i] = 2[A_3'q^n]^* r[i] = r_3[i]$$
$$- 0.5\sum_{j=i}^{N}\Bigg(\sum_{p=j}^{2N-j}(q_1^n[i,p+j-i] + q_1^n[i,p-j+i])r_1[j,p]h$$
$$- q_2^n[i]r_2[j] - 2q_2^n[i]r_3[j](B_4q[j] - 0.5\gamma f_3[j])$$
$$- 4q_1^n[i,2j-i](B_2q[j] + 1/\gamma)r_3[j]\Bigg)h,$$

where

$$B_2 q[i] = \frac{1}{2}\sum_{j=0}^{i} q_3^n[j] q_2^n[j] h,$$

$$B_4 q[i] = \sum_{j=0}^{i} q_3^n[j] q_1^n[j, 2i-j] h.$$

Results and their discussion

We calculate the following approach

q^{n+1}

$$q_1^{n+1} = q_1^n - \alpha_1 J_1'(q^n),$$

$$q_2^{n+1} = q_2^n - \alpha_2 J_2'(q^n),$$

$$q_3^{n+1} = q_3^n - \alpha_3 J_3'(q^n).$$

where

$$\alpha_1, \alpha_2, \alpha_3 \in \left(0, \left\|[A'q]^*\right\|^{-2}\right).$$

Conclusions

We carry out finite and differential approximation. We have net area Ω_h, one way or another we approximate the operator L_q - differential operator. Further one way or another we approximate the operator A, differential operator A_h, and the corresponding interfaced problem $L_p^*\psi = 0$ - we replace with differential analog $\widetilde{\Lambda}^*\psi_h = 0$. From this scheme of calculations of receiving approximation of the interfaced problem, that is there is no guarantee that $\widetilde{\Lambda}^*$ coincides with $\widetilde{\Lambda}^*$, in case of their discrepancy also the discrete analog of a gradient as a result will change, that is $B \neq A_h$.

From the point of view of the theory of differential schemes, using any choice of finite approximation of the interfaced problem, it is possible to pick up exact approximation of the interfaced problem that gradients to them corresponding coincided.

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