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Conclusion of discrete analog for the inverse problem of acoustics

Abstract: In this article approach at the numerical solution of the return problem of acoustics is considered by method of iterations of Landveber. The offered [1, 2] second approach, consists in the following: it is necessary for search of unknown coefficient from statement of a direct problem to pass to a problem which we will realize on the computer. For this purpose we write out functionality nonviscous, we will receive statement of the interfaced problem. Further by means of solutions of the direct interfaced problem we will receive a functionality gradient nonviscous after that it is possible to solve a minimization problem. This approach at the numerical solution of the return problem is more preferable as, first, equality optional has to be carried out $\widetilde{\Lambda}_p v = \Lambda_p^* v$, secondly, expression $B(v,\phi)$ cannot be a functionality gradient nonviscous *of* $\Phi[p]$. On the other hand at realization of this approach bulky calculations that pays off the best convergence are received. Thus, this approach has "pluses" and "minuses".

Key words: the inverse problem, a problem of acoustics, Dalamber's formula, Landveber's iteration, discrete analog, a gradient, the interfaced problem.

Introduction

In the article the one-dimensional return problem of acoustics in finite-difference statement by definition of acoustic rigidity is considered. The comparative analysis of discrete analogue of a method of iterations of Landveber is carried out.

We will consider the return problem of acoustics in a finite and differential look [3]. Assuming that all considered functions rather smooth, it is possible to write down the return problem [4] acoustics in a finite and differential form

$$u_{i+1}^k = u_i^{k+1} + u_i^{k-1} - u_{i-1}^k + h^2 Q_i^k, \tag{1}$$

$$u_1^k = \frac{g^{k+1} + g^{k-1}}{2}, \tag{2}$$

$$u_i^i = s_i, \tag{3}$$

$$u_0^k = g^k. \tag{4}$$

Objects and methods of researches

Using a known technique [5], we will receive discrete analog of a formula of Dalamber. Shifting indexes we will receive a chain of equalities

$$u_i^{k+1} = u_{i-1}^{k+2} + u_{i-1}^k - u_{i-2}^{k+1} + h^2 Q_{i-1}^{k+1},$$

$$u_{i-1}^{k+2} = u_{i-2}^{k+3} + u_{i-2}^{k+1} - u_{i-3}^{k+1} + h^2 Q_{i-2}^{k+2},$$

$$\vdots$$

$$u_{i-j+1}^{k+j} = u_{i-j}^{k+j+1} + u_{i-j}^{k+j-1} - u_{i-j-1}^{k+j} + h^2 Q_{i-j}^{k+j},$$

$$\vdots \quad j = i-2$$

$$u_3^{k+i-2} = u_2^{k+i-1} + u_2^{k+i-3} - u_1^{k+i-2} + h^2 Q_2^{k+i-2},$$

$$u_2^{k+i-1} = u_1^{k+i} + u_1^{k+i-2} - u_0^{k+i-1} + h^2 Q_1^{k+i-1},$$

From where we will receive

$$u_{i+1}^k = u_i^{k-1} + u_1^{k+i} - u_0^{k+i-1} + h^2 \sum_{j=1}^{i} Q_j^{k+i-j}. \tag{5}$$

chain of equalities

$$u_{i+1}^k = u_i^{k-1} + \frac{g^{k+i+1} - g^{k+i-1}}{2} + h^2 \sum_{j=1}^{i} Q_j^{k+i-j},$$

$$u_i^{k-1} = u_{i-1}^{k-2} + \frac{g^{k+i-1} - g^{k+i-3}}{2} + h^2 \sum_{j=1}^{i-1} Q_j^{k+i-j-2},$$

$$\vdots$$

$$u_{i-m+1}^{k-m} = u_{i-m}^{k-m-1} + \frac{g^{k+i-2m+1} - g^{k+i-2m-1}}{2} +$$

$$+h^2 \sum_{j=1}^{i-m} Q_j^{k+i-j-2m},$$

$$\vdots \quad m = i-1$$

$$u_2^{k-i+1} = u_1^{k-i} + \frac{g^{k-i+3} - g^{k-i+1}}{2} + h^2 \sum_{j=1}^{1} Q_j^{k-i-j+2},$$

$$u_1^{k-i} = u_0^{k-i-1} + \frac{g^{k-i+1} - g^{k-i-1}}{2}.$$

Dalamber's formula in a discrete form

$$u_{i+1}^{k} = \frac{g^{k+i+1} - g^{k-i-1}}{2} + h^2 \sum_{m=0}^{i-1} \sum_{j=1}^{i-m} Q_j^{k+i-j-2m}, \quad (6)$$

or, having made replacement of s = i - m, we will receive

$$u_{i+1}^{k} = \frac{g^{k+i+1} - g^{k-i-1}}{2} + h^2 \sum_{s=1}^{i} \sum_{j=1}^{s} Q_j^{k-i-j+2s}. \quad (7)$$

The problem registers in a discrete operator form:

$$Aq := q + Bq = f, \quad q^{i,k} = (r_i^k, p_i, l_i),$$

$$f_1^{i,k} = \frac{g^{k+i+1} - g^{k+i-1}}{4h} - \frac{g^{k-i+1} - g^{k-i-1}}{4h}, \quad (8)$$

$$f_2^i = \begin{cases} p_{0,} & i - even, \\ p_{1,} & i - odd, \end{cases}$$

$$f_3^i = \begin{cases} p_0 \dfrac{g^{2i+2} - g^{2i-2}}{2h}, & i - even \\ p_1 \dfrac{g^{2i+2} - g^{2i-2}}{2h}, & i - odd \end{cases} \quad (9)$$

$$B_1 q^{i,k} = -\frac{h}{2} l_i r_i^k - \frac{h}{2} \sum_{j=1}^{i-1} l_j (r_j^{k+i-j} + r_j^{k-i+j}),$$

$$i = \overline{2, N}, k = \overline{i, 2N-i} \quad (10)$$

$$B_1 q^i = \begin{cases} \dfrac{h}{2} \sum\limits_{j=1}^{i/2} l_{2j-1} (p_{2j-2} + p_{2j}), & i - \text{even} \\ \dfrac{h}{2} \sum\limits_{j=1}^{(i-1)/2} l_{2j} (p_{2j-1} + p_{2j+1}), & i - \text{odd} \end{cases} \quad i = \overline{2, N}. \quad (11)$$

$$B_3 q^i = \begin{cases} -[p_0 - \frac{h}{2} \sum\limits_{j=1}^{i/2} l_{2j-1} (p_{2j-2} + p_{2j})] \cdot [2h \sum\limits_{j=1}^{i-1} l_j r_j^{2i-j} + \frac{h}{2} l_i (r_i^i + r_i^{i+1})] \\ + \dfrac{g^{2i+2} - g^{2i-2}}{4} \sum\limits_{j=1}^{i/2} l_{2j-1} (p_{2j-2} + p_{2j}), i - \text{even}, \\ -[p_1 - \frac{h}{2} \sum\limits_{j=1}^{(i-1)/2} l_{2j} (p_{2j-1} + p_{2j+1})] \cdot [2h \sum\limits_{j=1}^{i-1} l_j r_j^{2i-j} + \frac{h}{2} l_i (r_i^i + r_i^{i+1})] \\ + \dfrac{g^{2i+2} - g^{2i-2}}{4} \sum\limits_{j=1}^{(i-1)/2} l_{2j} (p_{2j-1} + p_{2j+1}), \quad i - \text{odd}, \end{cases} \quad i = \overline{2, N-1}, \quad (12)$$

The problem is solved by method of iterations of Landveber [6]:

$$q_{n+1} = q_n - 2\alpha [A' q_n]^* (A q_n - f), \quad \alpha \in (0, \|A' q\| h^{-2}).$$

$$B'(q)\delta q^{i,k} \approx B_1 (q + \delta q)^{i,k} - B_1 q^{i,k} = -\frac{h}{2} (l_i + \delta l_i)(r_i^k + \delta r_i^k)$$

$$-\frac{h}{2} \sum_{j=1}^{i-1} \left[(l_j + \delta l_j)(r_j^{k+i-j} + \delta r_j^{k+i-j}) + (l_j + \delta l_j)(r_j^{k-i+j} + \delta r_j^{k-i+j}) \right] + \frac{h}{2} l_j r^k i + \frac{h}{2} \sum_{j=1}^{i-1} (l_j r_j^{k+i-j} + l_j r_j^{k-i+j})$$

$$-\frac{h}{2} (l_j \delta r_i^k + \delta l_i r_i^k) - \frac{h}{2} \sum_{j=1}^{i-1} \left[l_j \delta r_j^{k+i-j} + \delta l_i r_i^{k+i-j} + l_j \delta r_j^{k-i+j} + \delta l_j r_j^{k-i+j} \right] + O(\delta q^2).$$

$$B'(q)\delta q^{i,k} \approx -\frac{h}{2}(l_j \delta r_i^k + \delta l_i r_i^k) - \frac{h}{2}\sum_{j=1}^{i-1}\left[(l_j(\delta r_j^{k+i-j} + \delta r_j^{k-i+j}) + \delta l_j (r_j^{k+i-j} + r_j^{k-i+j})\right] + O(\delta q^2),$$

$$i = \overline{2,N} \quad k = \overline{\overline{i, 2N-i}}$$

$$B'(q)\delta q^{0,k} = 0, \qquad B_1'(q)\delta q^{1,k} = -\frac{h}{2}(\delta l_1 r_1^k + l_1 \delta r_1^k), \quad B_2'(q)\delta q^i \approx B_2(q+\delta q)^i - B_2 q^i$$

$$B_2'(q)\delta q^i = \begin{cases} \frac{h}{2}\sum_{j=1}^{i/2}\left[\delta l_{2j-1}(p_{2j-2} + p_{2j}) + l_{2j-1}(\delta p_{2j-2} + \delta p_{2j})\right], \\ i - \text{even}, \\ \frac{h}{2}\sum_{j=1}^{(i-1)/2}\left[\delta l_{2j}(p_{2j-1} + p_{2j+1}) + l_{2j}(\delta p_{2j-1} + \delta p_{2j+1})\right], \\ i - \text{odd}, \end{cases} \qquad i = \overline{2,N} \tag{13}$$

$$B_2'(q)\delta q^i = 0, \qquad i = \{0,1\},$$

$$B_3'(q)\delta q^i = \begin{cases} \frac{h}{2}\sum_{j=1}^{i/2}\left[\delta l_{2j-1}(p_{2j-2} + p_{2j}) + l_{2j-1}(\delta p_{2j-2} + \delta p_{2j})\right] \\ \times [2h\sum_{j=1}^{i-1} l_j r_j^{2i-j} + \frac{h}{2} l_i (r_i^i + r_i^{i+2})] \\ -[p_0 - \frac{h}{2}\sum_{j=1}^{i/2} l_{2j-1}(p_{2j-2} + p_{2j})] \cdot [2h\sum_{j=1}^{i-1}\left(\delta l_j r_j^{2i-j} + l_j \delta r_j^{2i-j}\right) \\ + \frac{h}{2}\delta l_j (r_i^i + r_i^{i+2}) + l_j(\delta r_i^i + \delta r_i^{i+2})\Big] \\ + \frac{g^{2i+2} - g^{2i-2}}{4}\sum_{j=1}^{i/2}\left[\delta l_{2j-1}(p_{2j-2} + p_{2j}) + l_{2j-1}(\delta p_{2j-2} + \delta p_{2j})\right], \\ i - \text{even}, \qquad i = \overline{2,N}, \\ \frac{h}{2}\sum_{j=1}^{(i-1)/2}\left[\delta l_{2j}(p_{2j-1} + p_{2j+1}) + l_{2j}(\delta p_{2j-1} + \delta p_{2j+1})\right] \\ \times [2h\sum_{j=1}^{i-1} l_j r_j^{2i-j} + \frac{h}{2} l_i (r_i^i + r_i^{i+2})] \\ -\left[p_1 - \frac{h}{2}\sum_{j=1}^{(i-1)/2} l_{2j}(p_{2j-1} + p_{2j+1})\right] \cdot \Big[2h\sum_{j=1}^{i-1}\left(\delta l_j r_j^{2i-j} + l_j \delta r_j^{2i-j}\right) \\ + \frac{h}{2}\delta l_j (r_i^i + r_i^{i+2}) + \frac{h}{2} l_j(\delta r_i^i + \delta r_i^{i+2})\Big] \\ + \frac{g^{2i+2} - g^{2i-2}}{4}\sum_{j=1}^{(i-1)/2}\left[\delta l_{2j}(p_{2j-1} + p_{2j+1}) + l_{2j}(\delta p_{2j-1} + \delta p_{2j+1})\right], i - \text{even}. \\ i - \text{odd}, \end{cases}$$

Results and their discussion

The formula of the discrete interfaced operator is received

$$[A'q]^{*}\overline{q} := \overline{q} + [B'q]^{*}\overline{q}.$$

$$[B_1'q]^{*}\overline{q}^{i,k} = -\frac{h}{2} l_1 \sum_{j=i}^{\frac{k+i}{2}} \overline{r}_j^{k-j+i} - \frac{h}{2} l_i \sum_{j=i+1}^{N-\frac{k-i}{2}} \overline{r}_j^{k+j-i} - 2 l_i \overline{l}_{\frac{k+i}{2}} F_{\frac{k+i}{2}}, \; i=\overline{2,N-2},\; k=\overline{i+4,2N-i-2},$$

$$[B_2'q]^{*}\overline{q}^{i} = \frac{h}{2} l_i(\overline{p}_i + D_i\overline{l}_i) + (l_{i+1}+l_{i-1})\left(\frac{h}{2}\sum_{j=i+2}^{N-1}(\overline{p}_i + D_i\overline{l}_i) + \frac{h}{2}\overline{p}_N + H_N\overline{l}_N\right), \quad i=\overline{2,N-3},$$

$$[B_3'q]^{*}\overline{q}^{i} = -\frac{h^2}{2}\sum_{j=1}^{2N-1} r_1^k \overline{r}_1^{k} - \frac{h^2}{2}\sum_{j=i+1}^{N}\sum_{k=j}^{2N-j} \overline{r}_i^{k}\left(r_1^{k+j-1} + r_1^{k-j+i}\right) - 2h\sum_{j=i+1}^{N-1}\overline{l}_j F_j r_j^{2j-i}$$

$$+\left(p_{i-1}+p_{i+1}\right)\left(\frac{h}{2}\sum_{j=i+1}^{N-1}\left(\overline{p}_j + D_j\overline{l}_j\right) + \frac{h}{2}\overline{p}_N + H_N\overline{l}_N\right) - \frac{h}{2}\overline{l}_i F_i\left(r_i^i + r_i^{i+2}\right) - 2h\overline{l}_N F_N r_i^{2N-i}, \quad i=\overline{2,N-2}.$$

$$[B_1'q]^{*}\overline{q}^{1,1} = -\frac{h}{2} l_1 \sum_{i=1}^{N} \overline{r}_i^{i} - \frac{1}{2} l_1(p_0\overline{l}_0 + p_1\overline{l}_1),$$

$$[B_1'q]^{*}\overline{q}^{1,3} = -\frac{h}{2} l_1 \overline{r}_2^{2} - \frac{h}{2} l_1 \sum_{i=1}^{N-1} \overline{r}_i^{i+2} - \frac{1}{2} l_1(p_0\overline{l}_0 + p_1\overline{l}_1) - 2 l_1\overline{l}_2\left(p_0 - \frac{h}{2} l_1(p_0+p_2)\right),$$

$$[B_1'q]^{*}\overline{q}^{1,k} = -\frac{h}{2} l_1 \sum_{i=1}^{\frac{k+1}{2}} \overline{r}_i^{k-i+1} - \frac{h}{2} l_1 \sum_{i=2}^{N-\frac{k-1}{2}} \overline{r}_i^{k+i-1} - 2 l_1 \overline{l}_{\frac{k+1}{2}} F_{\frac{k+1}{2}}, \quad k=\overline{5,2N-3},$$

$$[B_1'q]^{*}\overline{q}^{1,2N-1} = -\frac{h}{2} l_1 \sum_{i=1}^{N} \overline{r}_i^{2N-i} - 2 l_1 \overline{l}_N F_N, \qquad [B_1'q]^{*}\overline{q}^{i,i} = -\frac{h}{2} l_i \sum_{j=i}^{N} \overline{r}_j^{j} - \frac{1}{2} l_i \overline{l}_i F_i$$

$$[B_1'q]^{*} q^{-i,i+2} = -\frac{h}{2} l_1 \overline{r}_{i+1}^{i+1} - \frac{h}{2} l_i \sum_{j=i}^{N-1} \overline{r}_j^{j+2} - 2 l_i \overline{l}_{i+1} F_{i+1} - \frac{1}{2} l_1 \overline{l}_i F_i,$$

$$[B_1'q]^{*}\overline{q}^{i,2N-1} = -\frac{h}{2} l_i \sum_{j=i}^{N} \overline{r}_j^{2N-j} - 2 l_1 \overline{l}_N F_N, \quad [B_1'q]^{*}\overline{q}^{i,k} = -\frac{h}{2} l_i \sum_{j=i}^{\frac{k+1}{2}} \overline{r}_j^{k-j+i} - \frac{h}{2} l_i \sum_{j=i+1}^{N-\frac{k-1}{2}} \overline{r}_j^{k+j-i}$$

$$-l_1 \overline{l}_{\frac{k+1}{2}} F_{\frac{k+1}{2}}, \quad i=\overline{2,N-2}, \quad k=\overline{i+4,2N-i-2},$$

$$[B_1'q]^{*}\overline{q}^{N-1,k} = -\frac{h}{2} l_{N-1} \overline{r}_{N-1}^{k} - \frac{h}{2} l_{N-1} \overline{r}_N^{N} - \frac{1}{2} l_{N-1} \overline{l}_{N-1} F_{N-1} - \frac{3}{2} l_{N-1} \overline{l}_N F_N, \qquad k=[N-1,N+1],$$

$$[B_1^{'}q]^* q^{-N,N} = -\frac{h}{2} l_N \overline{r}_N^{-N}.$$

$$h\sum_{m=0}^{N-2}\delta p_m\left(\frac{h}{2}l_{m+1}\sum_{i=m+2}^{N}\overline{p}_i\right)+h\sum_{m=0}^{N}\delta p_m\left(\frac{h}{2}l_{m-1}\sum_{i=m}^{N}\overline{p}_i\right)+\frac{h^2}{2}\sum_{m=0}^{N-3}\delta p_m l_{m+1}\sum_{i=m+2}^{N-1}D_i\overline{l}_i+\frac{h^2}{2}\sum_{m=2}^{N-1}\delta p_m l_{m-1}\sum_{i=m}^{N-1}D_i\overline{l}_i$$

$$+h\sum_{m=0}^{N-2}\delta l_m l_{m+1}H_N\overline{l}_N+h\sum_{i=2}^{N}\delta p_m l_{m-1}H_N\overline{l}_N$$

$$\left[B_2'q\right]^*\overline{q}^i = -\frac{h}{2}(\overline{p}_i + D_i\overline{l}_i) + (l_{i+1}+l_{i-1})\left(\frac{h}{2}\sum_{j=i+2}^{N-1}(\overline{p}_j + D_j\overline{l}_j) + \frac{h}{2}\overline{p}_n + H_N\overline{l}_N\right), \quad i=\overline{2,N-3}.$$

$$h\delta l_i\left(\frac{h^2}{2}\sum_{k=1}^{2N-1}r_1^k\overline{r}_1^{-k} - \frac{h^2}{2}\sum_{i=2}^{N}\sum_{k=i}^{2N-i}\overline{r}_i^{-k}\left(r_1^{k+i-1}+r_1^{k-i+1}\right)+\frac{h}{2}(p_0+p_2)\sum_{i=2}^{N}\overline{p}_i\right)$$

$$-\frac{h}{2}\left(p_1\overline{l}_1+p_0\overline{l}_0\right)\left(r_1^1+r_1^3\right)+(p_0+p_2)\frac{h}{2}\sum_{i=2}^{N-1}D_i\overline{l}_i-\sum_{i=2}^{N-1}2hF_i\overline{l}_i r_1^{2i-1}-\overline{l}_N F_N 2hr_1^{2N-1}+(p_0+p_2)H_N\overline{l}_N$$

Conclusions

All calculations of receiving discrete analog of a gradient and its strictly corresponding differential interfaced problem was carried out at the discrete level in net space.

From this scheme of calculations of receiving approximation of the interfaced problem, there is no guarantee that $\widetilde{\Lambda}^*$ coincides with $\widetilde{\Lambda}^*$, in case of their discrepancy also the discrete analog of a gradient as a result will change, $B \neq A_h$.

From the point of view of the theory of differential schemes, using any choice of finite approximation [7] of the interfaced problems, it is possible to pick up exact approximation of the interfaced problem that gradients to them corresponding coincided.

References

1. Kabanikhin S.I., Bektemesov M.A., Nurseitov D.B. An initial and regional problem for the equation of elliptic type // Messenger TREASURY. – 2006. – Vol. 2. – P. 33-47.

2. Napke M., Neuhauer A., Scherzer O. A convergence analysis of Landweber iteration for nonlinear ill-posed problems // Numer. Math. – 1995. – Vol. 72.– P. 21-37.

3. Karchevsky A.L. The correct scheme of actions at the numerical solution of the return problem by an optimizing method //the Siberian electronic mathematical news. – 2008. – Vol. 5. – P. 609-619.

4. Kabanikhin S.I., Satybaev A.D., Shishlenin M.A. Direct methods of solving multidimensional inverse hyperbolic problems // The Netherlands: VSP. – 2004. – 179 p.

5. Samara A A. Introduction to the theory of differential schemes. – M.: Science, 1971. – 553 p.

6. Nurseitova A.T., Tyulepberdinova G.A. Skhodimost of a method of iterations of Landveber for the solution of a problem of determination of acoustic rigidity // The Messenger of KAZNPU of Abay. Almaty - 2008. – Vol. 21, No. 1. – P. 215-217. – Physical and Mathematical Sciences series.

7. Tyulepberdinova G.A., Adilzhanova S.A. Return problem of acoustics and its conditional correctness // International Journal of Mathematics and Physics, *Almaty,* Kazakhstan – 2014. – Vol.2, No 5. – P.7-13.